Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 54
Practical Application: Optimal power allocation factor determination for Co-operative Communication

Hello, welcome to another module in this massive open online course. So, we are looking at optimization for co-operative communication all right. And in this module, we are going to get to the optimization problem all right.

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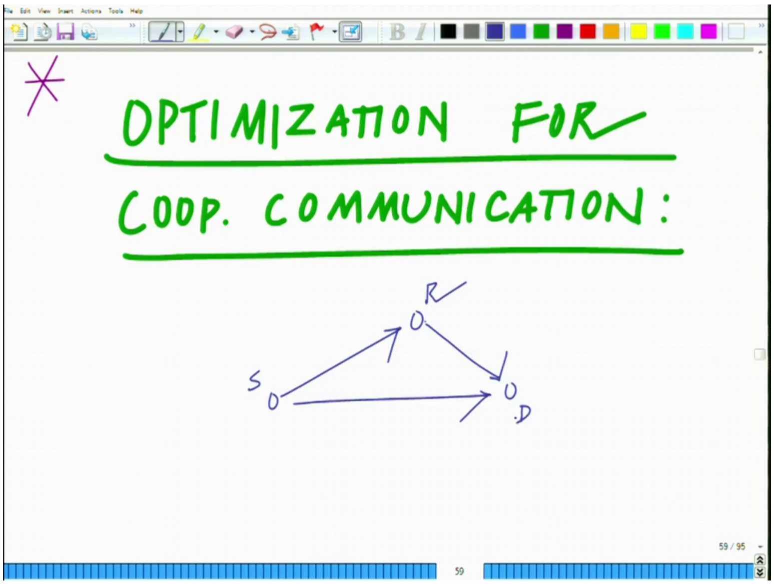

So, we are looking at optimization for your co-operative communication system. And well what we have is, we have a source, we have the relay, and we have a destination. So, this is my source, my relay and destination.

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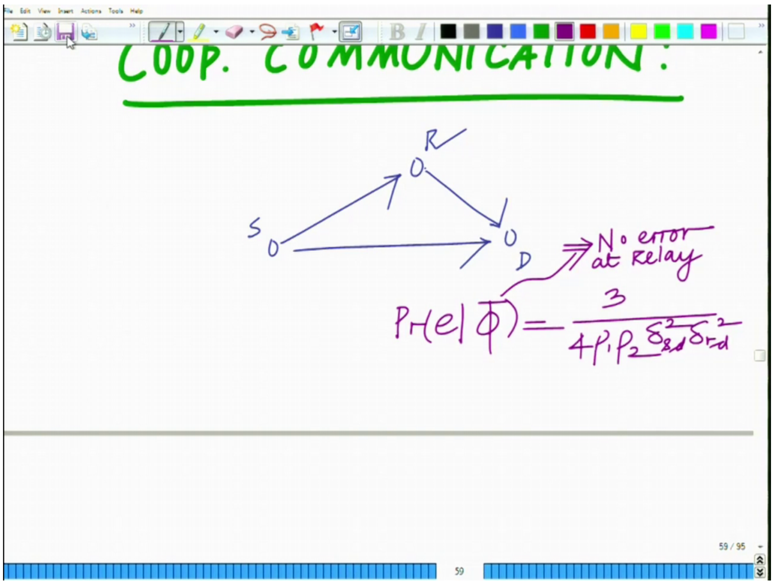

We also derive what is the expression for e given phi bar that is probability of error at relay error at destination given, given no error at the relay.

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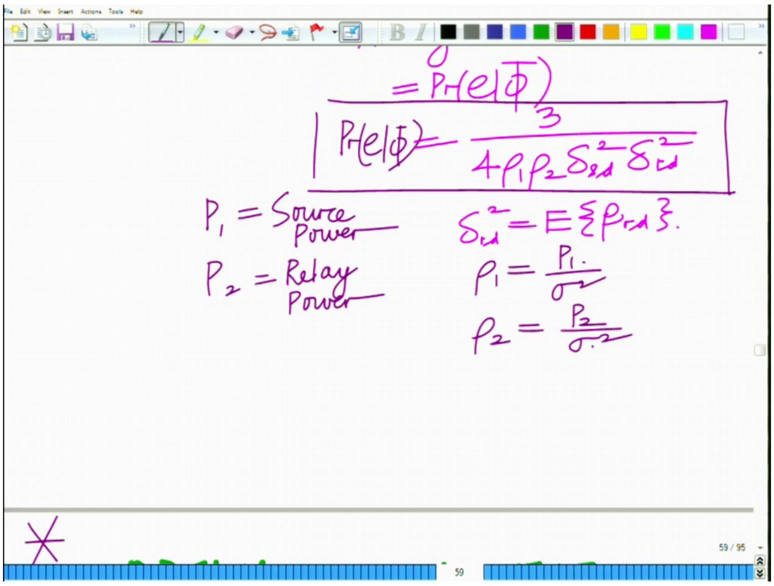

That is phi is the event of error at relay, phi bar s the event of no error at relay. And this we can see is we have seen is 3 by 4 rho 1 rho 2 delta s d square delta r d square. We are not explicitly derived this or you said that this can be shown ok, so phi bar implies no error at relay ok.

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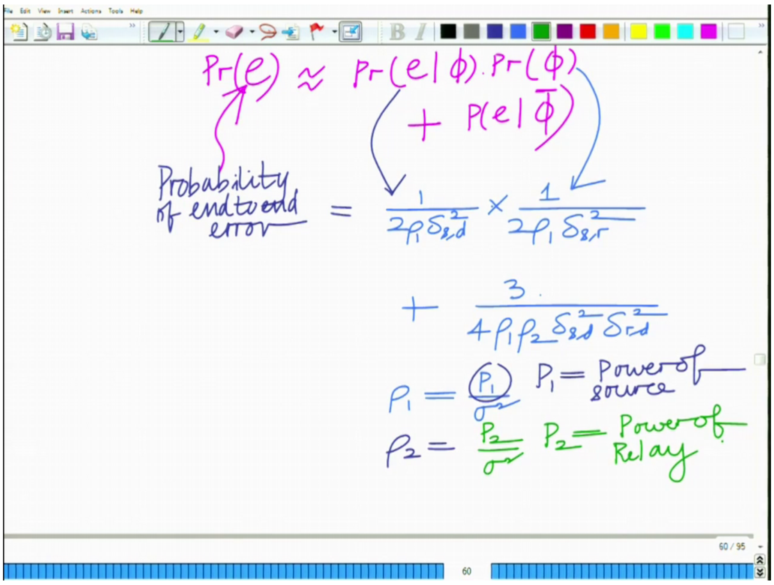

And now probability of error, so this implies and now we already know the expression for the error at the destination. So, probability of error at the destination, we have seen this is probability of e given phi times probability of phi is a proxy and plus probability of e given phi bar. We said this is an approximations, but this is a good approximation we just tied at highest you know.

Now, what we are going to do is, we are going to substitute the expressions, we have derived for each of this quantities and derived the probability of error at the destination. This is also known as the end to end error that is the end is one of the end of ends is the source, the other end is the relay, another end is the destination. So, this is the probability of the error at the destination that is for the end to end communication ok, because the communication happens in two phases, one is the source to destination, then source to relay, relay to destination ok. So, this is the probability of end to end error, this is the probability of end to end error.

And now substitute each of these quantities, so this is equal to, so first I am going to substitute probability of e given phi that is the error at destination given error at relay. So, this is 1 over 2 rho delta s d square times probability of error at relay, this is 1 over 2 rho 1 delta s r square plus probability of error given phi bar, this is 3 over 4 rho 1 rho 2 delta s d square delta r d square.

Now, what is rho 1? Remember rho 1 equals P 1 by sigma square. And what is P 1? P 1 equals power of source; rho 2 equals P 2 y sigma square; P 2 equals power of relay all right this is the transmit power of relay. And now what we want to do is now we want to make an optimization problem, where we want to minimize the bit error rate the end to end rate of course, we cannot just simply minimize. And because minimizing the error rate simply means increasing the power infinitely; and of course, when the power becomes infinite the bit error rate becomes 0. So, naturally that is not what we want we want to minimize this subject to constraint, because the transmit power cannot increase infinitely. So, what will impose is will impose up power budget on this corporative communication system.

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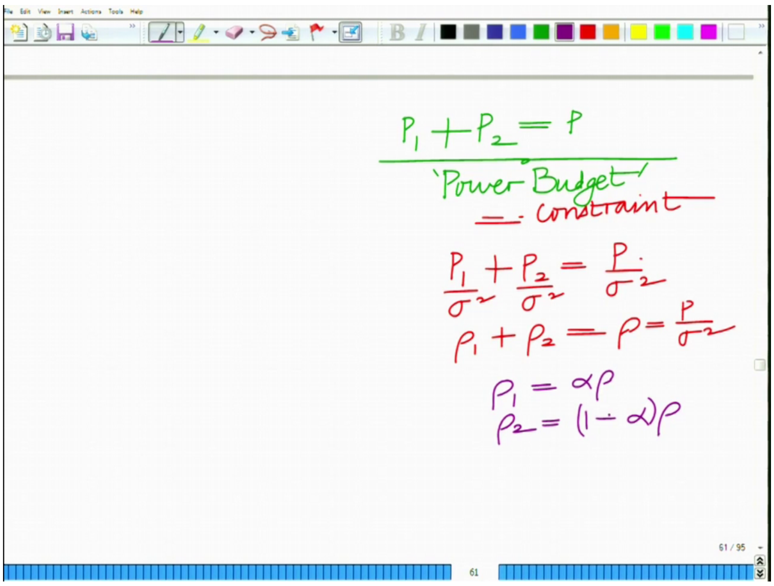

What is the power budget? The power budget is that the power of the source plus the power of that relay, this is the constraint. So, this is the constraint which is the power budget for the system. So, this is the power budget which is the constraint for this co-operative wireless communication system. Now, P 1 plus P 2 equals P. Now if you divide this by all sides by sigma square, we can get rho 1 plus rho 2 equals well P over sigma square equals rho equals P over sigma square. Further to simplify this, because there only two parameters, I can set rho 1 equals alpha times rho, rho 2 equals then becomes 1 minus alpha times rho because rho 1 plus rho 2 is equal rho.

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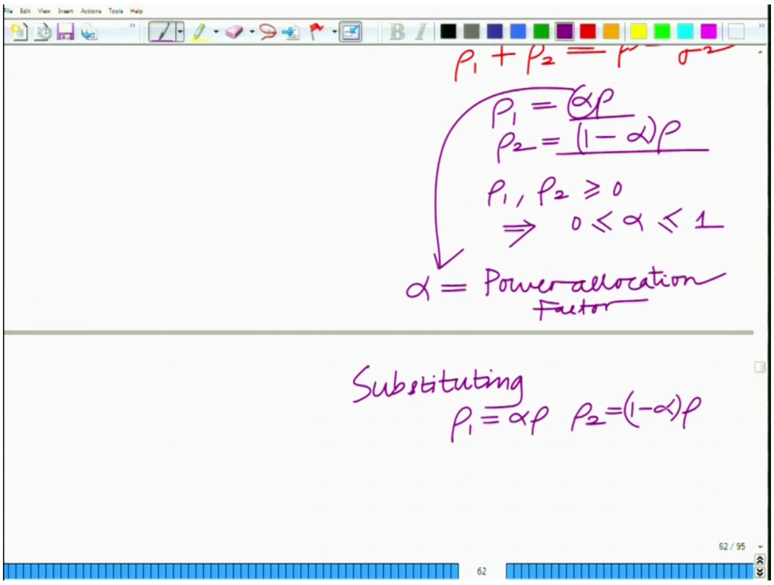

Where of course, because rho 1 comma rho 2 are greater than or equal to 0, this implies 0 less than equal to alpha must be less than or equal to 1 this. Now, is what is the what is this alpha you can think of this alpha as the power allocation factor, this is the power allocation factor which lies between 0 and 1; and rho 1 equals alpha times rho, rho 2 equals 1 minus alpha times rho. Now, substituting this values ok, substituting what are we substituting rho 1 equals alpha times rho, rho 2 equals 1 minus alpha times rho. And, if we call this expression, if we call this as star that is the probability of end to end error.

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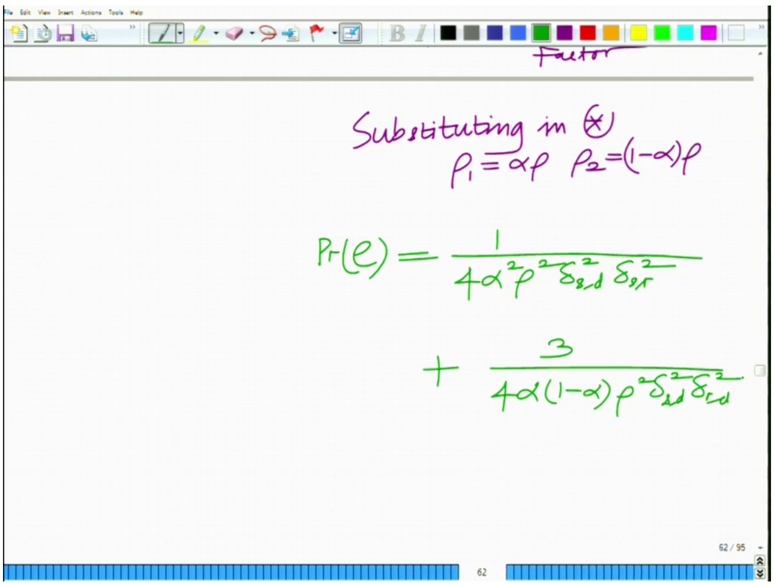

So, substituting in star what we get is the probability of error probability of end to end error, this equals 1 over 4 alpha square rho square delta s d square delta s r square plus 1 over or not 1 over in fact, this is 3 over 4 alpha into 1 minus alpha into rho square delta s d square delta r d square.

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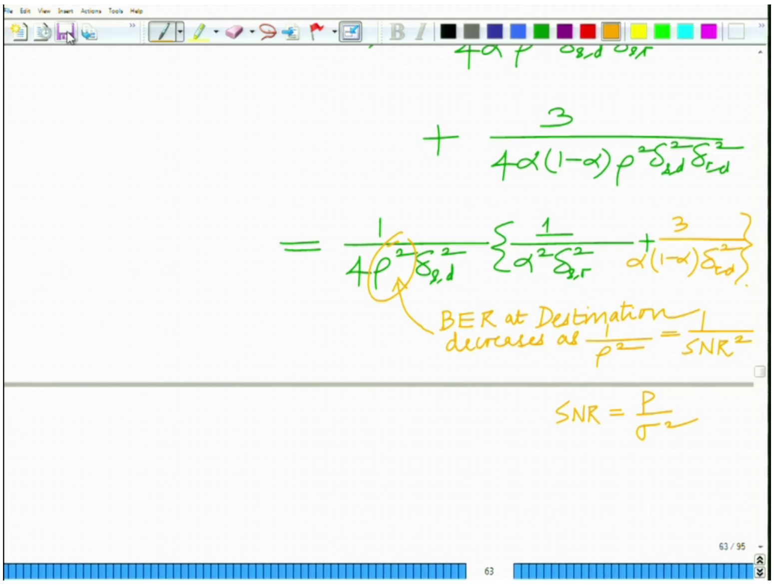

And now if you take this 1 over rho square common or in fact I can take 1 over 4 rho square delta or 4 square delta s d square I can take this common into 1 over alpha square s r delta square plus 3 over alpha into 1 minus alpha into delta r d square. Now, if you look at this bit error rate expression, now if we attention to this bit error rate expression, you will notice something interesting you will notice that the effective end to end bit error rate decreases as 1 over rho square. So, so or the bit error rate at destination equal to 1 over SNR square, what is SNR, SNR equals P over sigma square.

So, what you are observed is normally in a wireless communication system, we have seen in the absence. Now, if you go back ok, and if you go back and simply look at the source destination link I urge you to look at the source destination link. If you look at simply the source destination link, the probability of error decreases 1 over rho 1, it only decreases as 1 over SNR all right. Therefore, this is known as diversity order 1, which is the exponent of the SNR, is simply 1 in the bit error rate expression.

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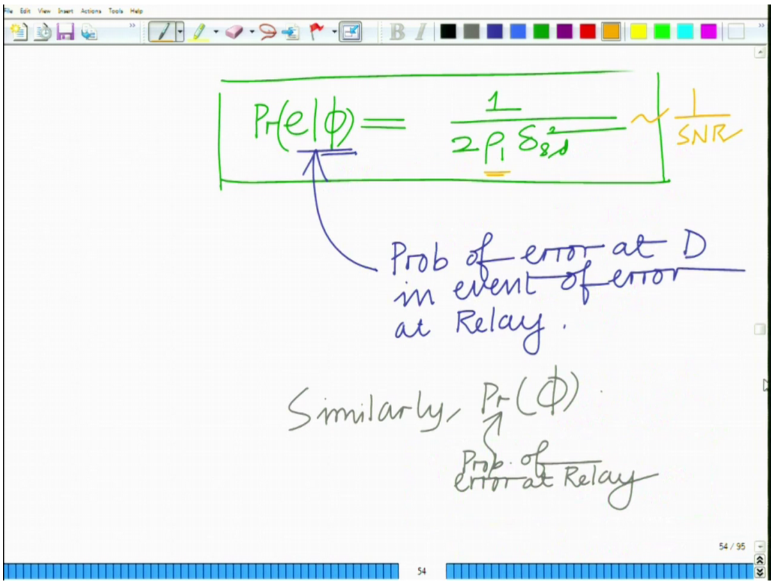

However, so this is basically you can say this is the 1 over SNR decrease, this decreases as 1 over SNR.

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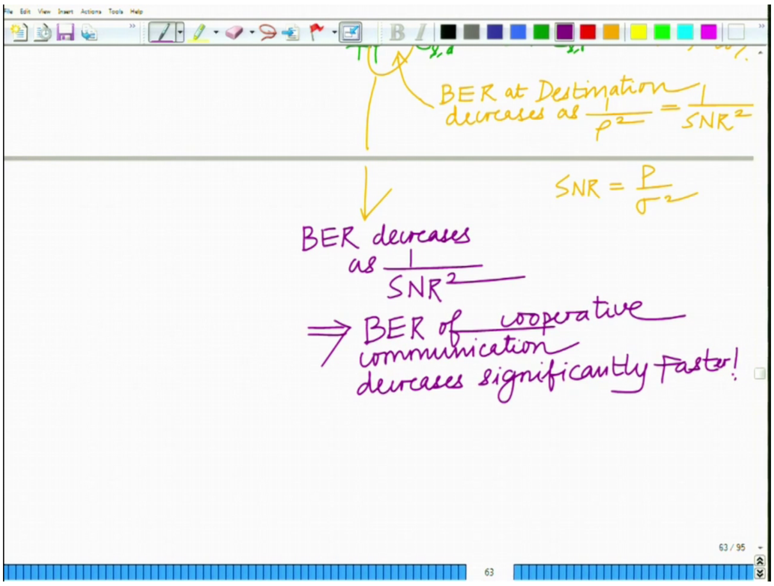

However, now once you are adding a relay in this co-operative communication system, the bit error rate in co-operative communication system, BER decreases as 1 over SNR square. And this is very important, because the bit error rate is decreasing as 1 over square of SNR. So, the BER bit rate decreases much faster, this is the impact of corporative communication. Thus corporative communication leads to a significant decrease in the bit error rate of a wireless communication system thereby improving the reliability ok. So, this implies BER of co-operative decreases, this is implies bit error rate to co-operative communication decreases significantly faster.

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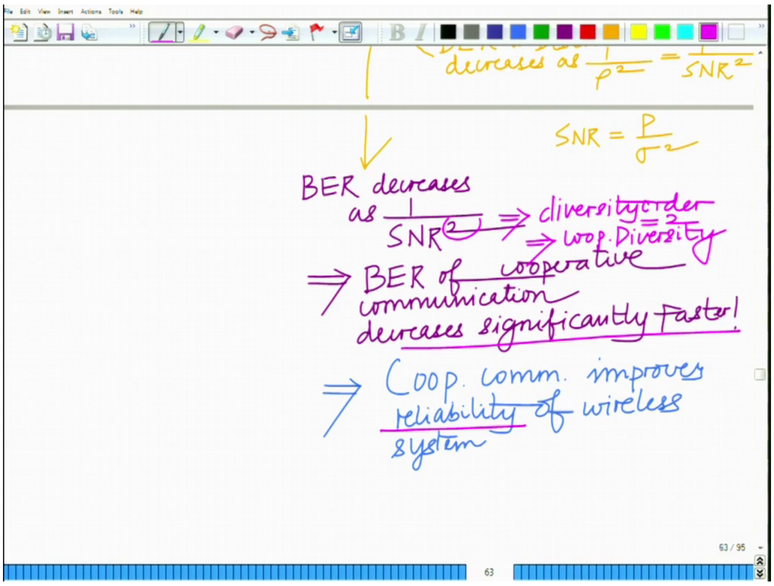

This implies co-operative communication significantly co-operative communication improves reliability, improves the reliability of a wireless system. So, this significantly improves the reliability of the wireless system. So, BER decreases significantly faster, because remember 1 over SNR versus 1 over SNR square for a co-operative communication system. So, these 2 implies diversity order equals 2; and this is also termed as co-operative diversity. So, co-operative diversity helps to improve the reliability for wireless communication system by making the bit error rate at the destination decrease significantly faster. Then it would have happened in the presence of only a source destination link that is when the relay is absent so that is the important point to realise here ok.

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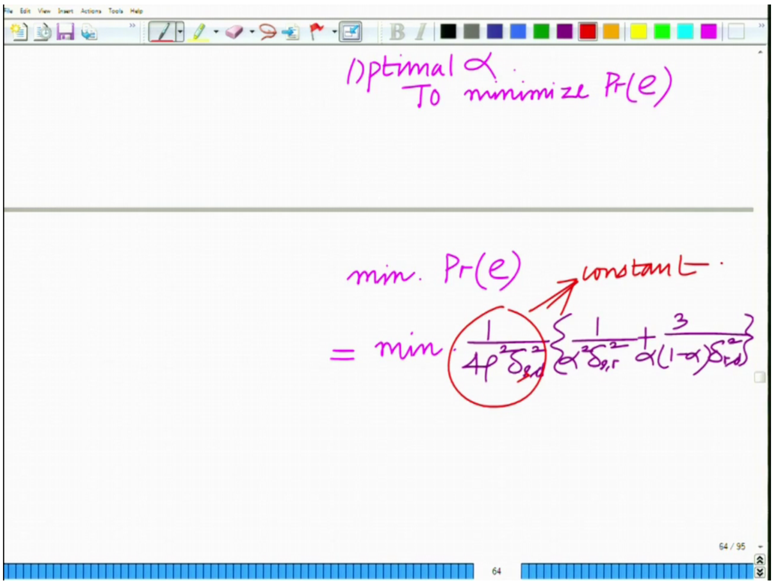

And now therefore, what you want to do, we want to find the optimal power factor alpha to minimize naturally, what you want to minimize is we want to minimize the bit error rate at the definition or probability of error. So, my optimization problem is, now minimize probability of error remember the constraint is now incorporated in alpha, because constraint was P 1 plus P 2 equal to p, which have written in terms of alpha rho and 1 minus alpha rho by writing P 1 and P 2 in terms of alpha and alpha rho and 1 minus alpha rho or P rho 1 and rho 2 in terms of alpha rho and 1 minus alpha rho ok.

And, so this is minimize probability of error, there has to be a dot minimize, which means minimize your earlier relay expression that we derived 1 over 4 rho square delta s d square times 1 over alpha square delta s r square plus 3 over alpha into 1 minus alpha delta d square. Now, observe that this is a constraint rho is fixed, P is fixed, rho is fixed, delta d square delta s d square is fixed. So, this implies this is a constant.

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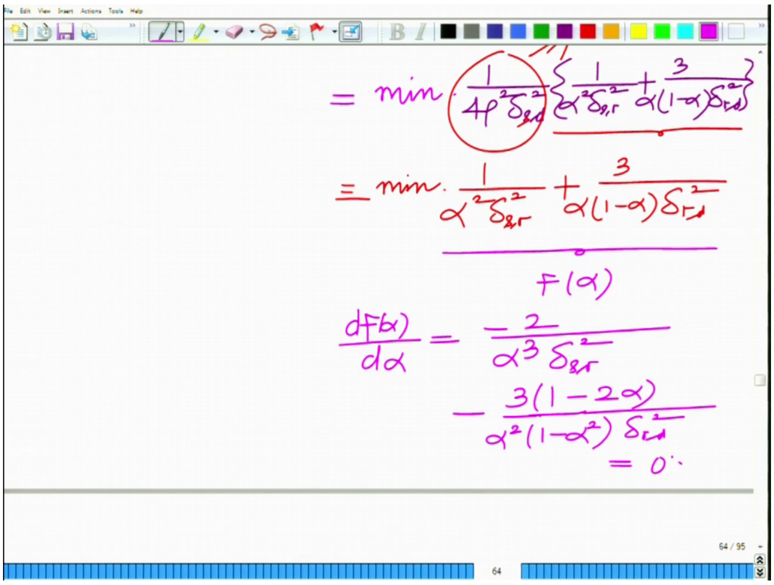

So, we need to only minimize this part which is therefore, equivalent to minimization of 1 over alpha square delta s r square plus 3 over alpha into 1 minus alpha delta r d square. Let us call this as a F of alpha d F of alpha over d alpha this is equal to minus 2 over alpha cube delta s r square plus or minus rather minus 3 1 minus 2 alpha by alpha square 1 minus alpha square delta r d square. Now, equate to 0 to find the optimal value.

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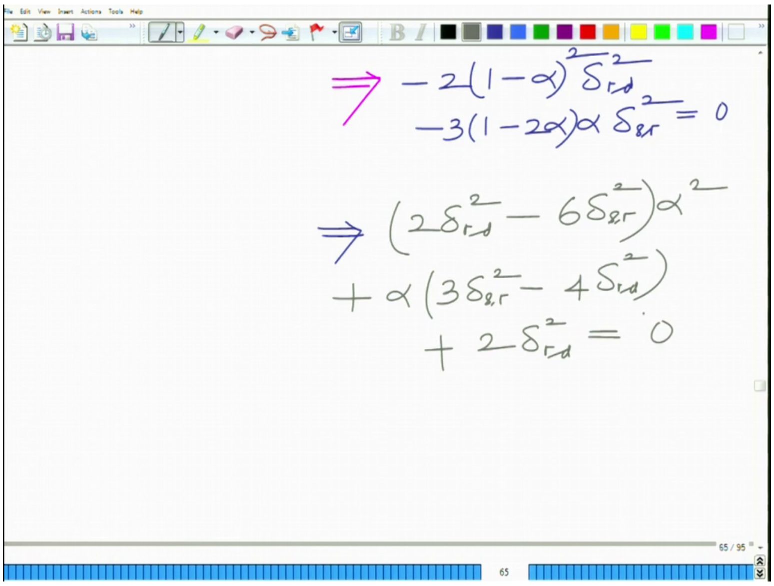

This implies minus 2 1 minus alpha square delta r d square minus 3 into 1 minus 2 alpha alpha delta s r square equal to 0; this implies the 2 delta d square minus 6 delta s r square into alpha square plus alpha 3 delta s r square minus 4 delta r d square plus 2 delta r d square equal to 0.

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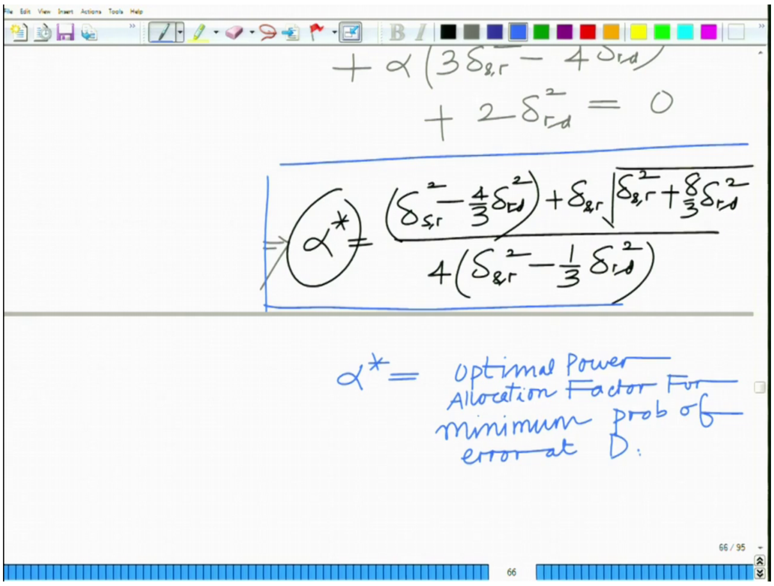

And you can solve this; it is a simple quadratic equation. And what you will get is alpha star is equal to delta s r square minus 4 by 3 delta r d square plus delta s r into square root of delta s r square plus 8 by 3 delta r d square 4 delta s r square minus 1 by 3 delta r d square. And this is basically your optimal you can say this is alpha star equals optimal power allocation factor for minimum, minimum probability of error at destination.

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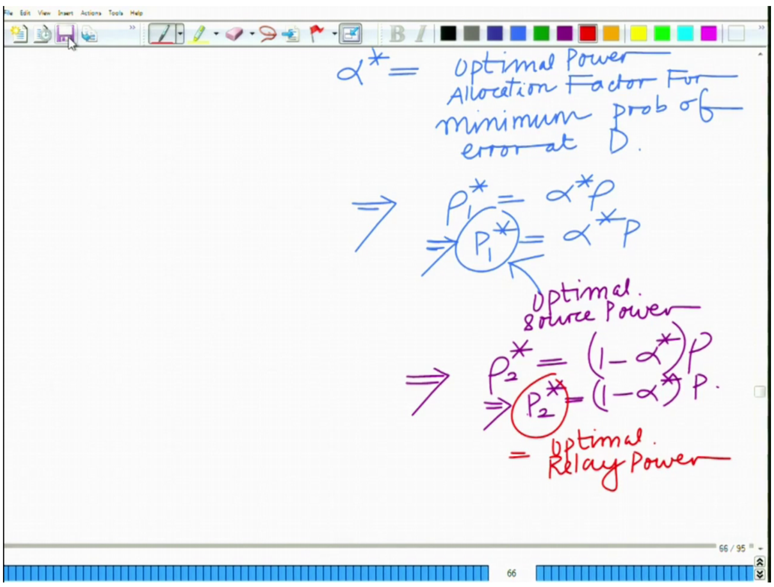

This implies that your rho star P 1 star or your rho 1 star equals alpha star rho optimal power optimal SNR. So, this implies also that P 1 star equals alpha star P. What is P 1 star optimal source power, and this also implies rho 2 star equals 1 minus alpha star into rho implies P 2 star equals 1 minus alpha star into P, and what is P 2 star this is equal to optimal relay power, so that gives the optimal source relay power allocation ok. So, what we have obtained is optimal source relay power allocation.

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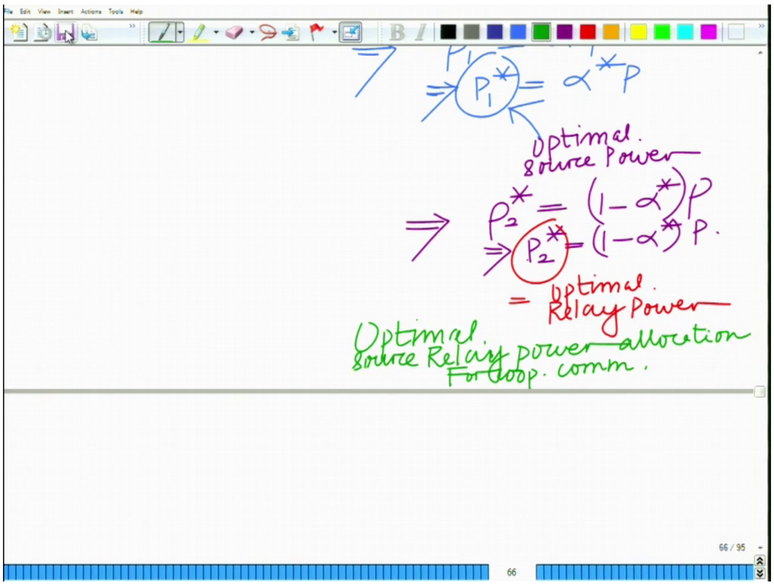

So, this is the optimization connect, so optimization arises everywhere in signal processing and communication. What we are saying is only a few salient and most relevant modern exams optimal source relay power allocation for co-operative communication, which minimum co-optimal in the sense that it minimizes the end-to-end bit error rate all right. So, the basically that completes our discussion on this co-operative communication system source, relay and destination nodes all right. In the relay implies that that is like decode and forward there is a protocol very transmits only if it is able to decode that respected simple correctly all right.

And we have shown that because of corporative diversity the bit error rate decreases as 1 over SNR square, which implies that the bit error rate of co-operative communication is significantly lower and therefore, the reliability significantly higher in comparison to that of having only a source destination link. This is co-operative diversity. And we have also derived the optimal source relay power allocation or power distribution.

Thank you very much.