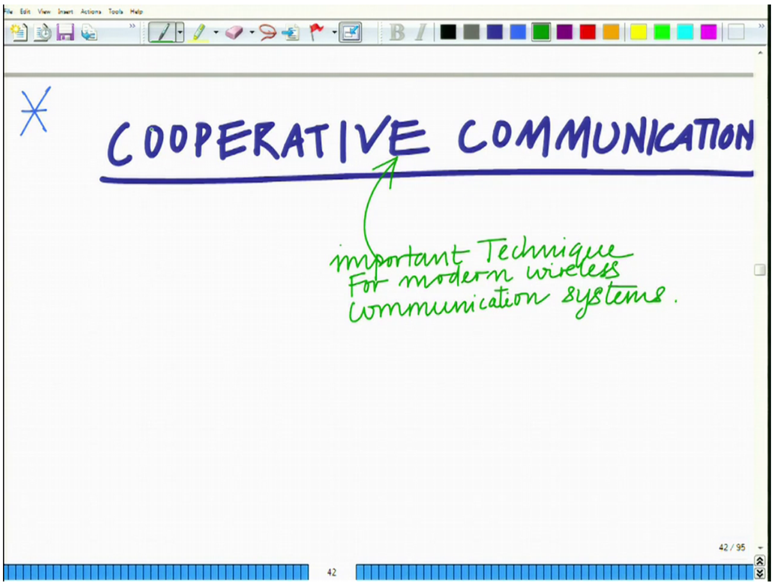
Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 52
Practical Application: Co-operative Communication, Overview and Various Protocols Used

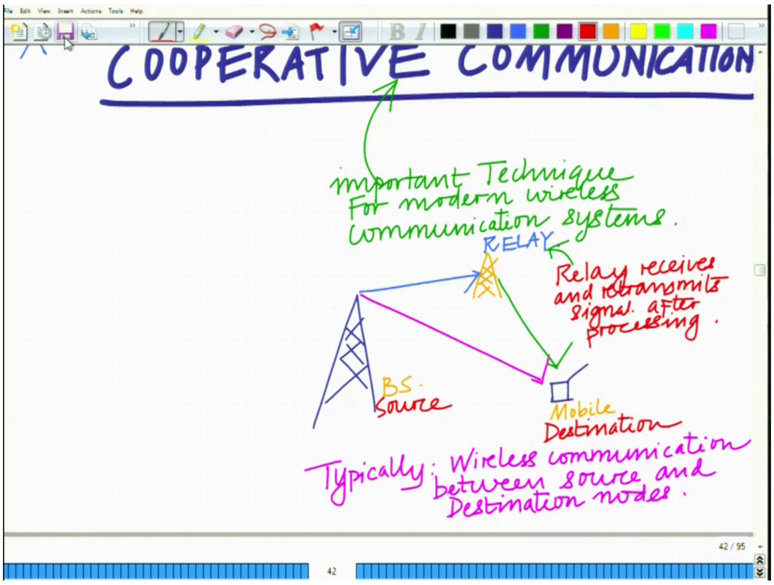
Hello, welcome to another module in this Massive Open Online Course. We are looking at several optimal optimization paradigms, in fact several practical applications of the optimization theory that we have seen so far. Let us we look at yet another interesting and novel application of the optimization framework and that is with respect to cooperative communication.

(Refer Slide Time: 00:36)



So, starting this module in this and probably the next couple of modules, we want to look at another very novel communication paradigm or communication framework and that is of cooperative communication or corporation in wireless communication. This is in fact a very recent idea and is emerging to be very popular modern wireless communication systems especially for high data rate and you can say next generation wireless communication. So, very important you can say a mode very important technique. And this is also something that is latest it is probably not more than about 5 to 10 years also important technique for modern wireless for modern wireless communications systems.

(Refer Slide Time: 01:59)



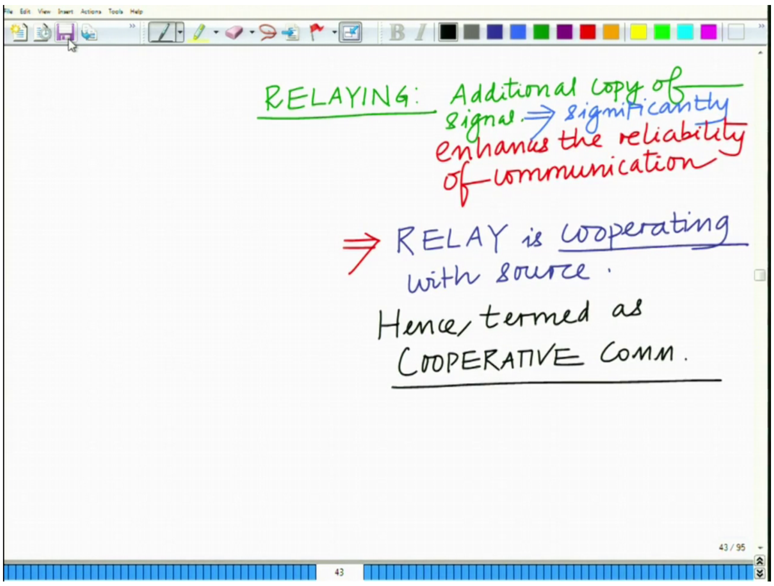
Now, what happens in cooperative communication system, now typically if most of you already know, what typical wireless communication system at a very high level, what happens is you have a base station correct. And we must have seen this in many many times before and even in this course that is you have a base station. And then you have a mobile which is also termed as the user equipment and so on. These can basically be termed as the source of the communication signal that is S. And this is the destination and typically what we have is we have communication from the source to destination.

So, in typically we have or frequently in a wireless communication system, we have there is wireless communication between the source and the; the source and the destination node. Now, what happens is in addition what you can have to enhance the quality of communication is to have what is known as a relay, remember what a relay does or the meaning of this, what relay is basically something that takes something and relays it relays in relay in the sense that it hands it over all right that is the principle of relay. And it is it is not with respect communication, but that is a general definition of a relay. For instance, you have a relay race format in which one of the runners takes the baton carries it forward and then hands it over to the next runner and so on that is the concept of relay.

Specifically, in the context of wireless communication what a relay does is it receives the signal from the source; can perform, can manipulate it can perform some operations on it some basic signal processing operations or complex and advanced signal processing operations that depends on the nature of the relay and then it relays the signal forward it forwards this signal to the destination. So, the relay, so what happens is the source relay, so this is your relay.

And again there can be a single or multiple relay there can be a single relay or more relay. So, this relay receives the signal it makes some signal processing operations on it and forwards it. So, relay receives and retransmits the signal and this is the important operation; relay receive and of course, relay receives and retransmits after suitable operation after processing some processing some after some processing. Now, so relay retransmits the signal basically that is the point. Now, this can significantly enhance.

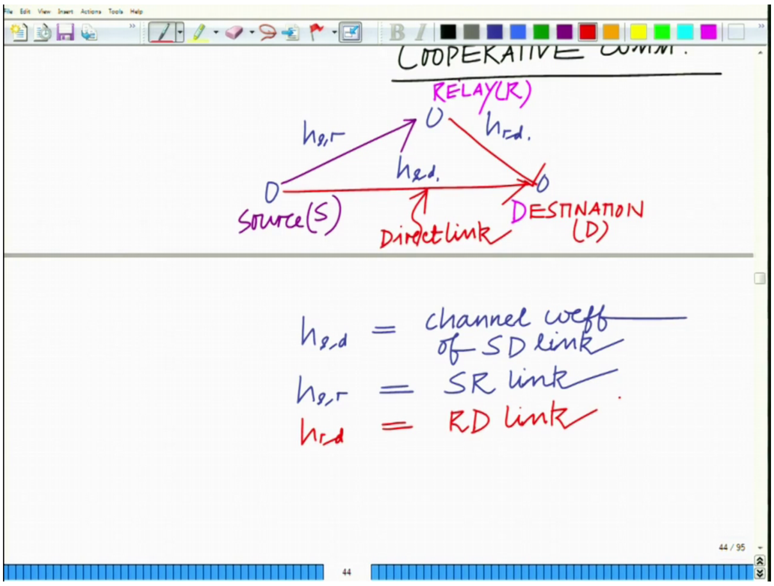
(Refer Slide Time: 05:28)



So, the relaying this process, because now you have an additional copy of the signal you have you have an additional copy of the transmit signal, which results in significantly; this significantly enhances the quality of communication or we can say enhances the reliability. We are going to define that in a little bit this significantly enhances the reliability of communication in the wireless communication system. What do you mean by reliability, we are going to define that what is the metric for reliability, but you can see that you have an additional copy. And this process what the relay is doing this is termed as cooperation; relay is cooperating, what is the relay doing, the relay is cooperative communication cooperating.

Hence, this is termed as this paradigm is hence typically termed as hence termed as cooperative that is the title of this module. Hence, this is frequently termed as this paradigm, where we have a relay takes gets the signal from the source. Manipulates it performs some operations on it and transmits to the destination there by aiding in this process or cooperating with the source in this process of communication wireless communication is termed as cooperative communication. Now, what is the relation of cooperative communication to optimization we are going to see that shortly all right, but first we have to set up this framework for cooperative communication.

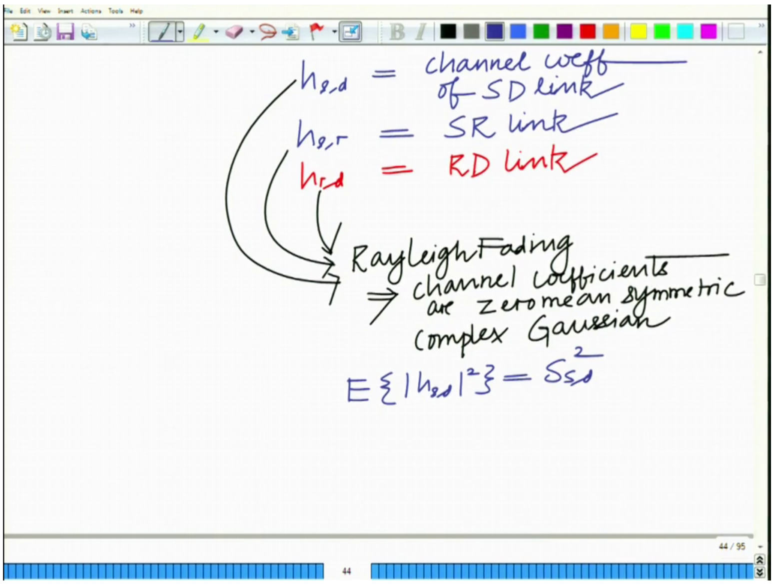
(Refer Slide Time: 07:47)



Now, so schematically, let me just describe my cooperative communication system as the combination of these three nodes. So, I have the source which is S, destination source sorry this is your relay and this is the destination, which is D. So, source to destination this is also termed as the direct link, this is termed as the direct SD link it can be present, sometimes it can be absent, if it is for the shadowed and so on, then you have a link where that is source to relay and then the relay to destination.

And let us denote the channel coefficients, remember each of this is a wireless link therefore each of these will be characterized by a fading wireless channel coefficient corresponding to each link source destination source relay and relay destination. And therefore, each of these will have a channel coefficient all right. Let us, denote the channel coefficients by the terms or by the symbols h. And these are self explanatory h s d is source destination, h s r is source relay, h r d is relay destination. So, h s d equals the channel coefficient of the direct or S D link h s r is for the SR link the channel coefficient. And h r d is for the RD link it is self explanatory all right. Just writing it out explicitly, but then the notation is self explanatory.

(Refer Slide Time: 10:10)

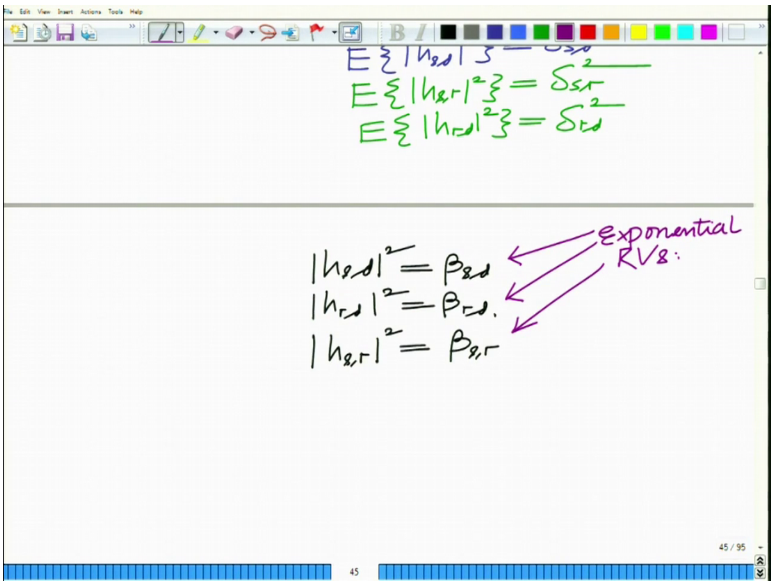


And let us say these are relay fading and let us make a simplest let us make simplistic assumption I mean these can be a follow any distribution when depending on the various kind of distribution you will have different results, but let us make assume that these are Rayleigh fading in nature, which implies basically that these coefficients are symmetric complex Gaussian implies that the channel coefficients are 0 mean symmetric complex Gaussian in nature. These are symmetric 0 mean complex Gaussian.

And let us say, these average powers expected magnitude h s d square equals again these are self explanatory that is the source destination links has average power delta s d square, source relay link has the average power delta s r square, and relay destination link has average power r d square. And these powers can vary depending on the distance between source destination distance.

For instance, if the source relay link is has a very the source and relay are very far or the source and destination are very far, let us say then the source destination average gain will be much smaller compared to for, let us say source relay and relay destination all right. So, there can be several scenarios depending on where the relay is located with respect to source, and where the destination is located with respect to the source, and where the destination is located with respect the relay.

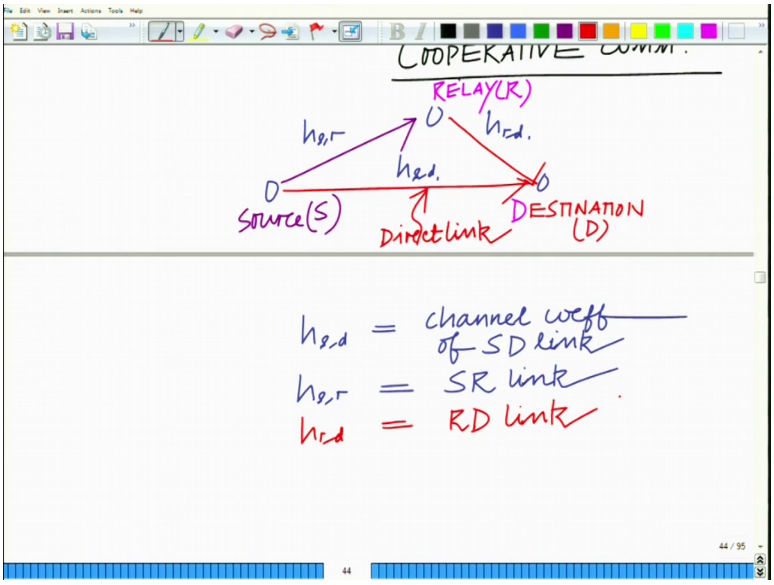
(Refer Slide Time: 12:00)



Similarly, you have expected magnitude h s r square equals delta s r square, and expected magnitude h r d square equals delta r d square. And we said these are, now if you denote these quantities by now these are of course, random in nature, because the fading channel coefficient is random. So, if you denote the gains of this fading channels by betas that is h s d, h r d square equals beta r d gain of the relay destination link, and gain of the source relay link by beta s r.

Now, these will have an exponential distribution, because these are Rayleigh fading, which means the amplitude is Rayleigh fading, the gain that is magnitude, so the magnitude is Rayleigh fading the gain that is magnitude square is exponentially fading random variable. These are exponentially fading I am going to give the exponential distribution, these are exponential random variables.

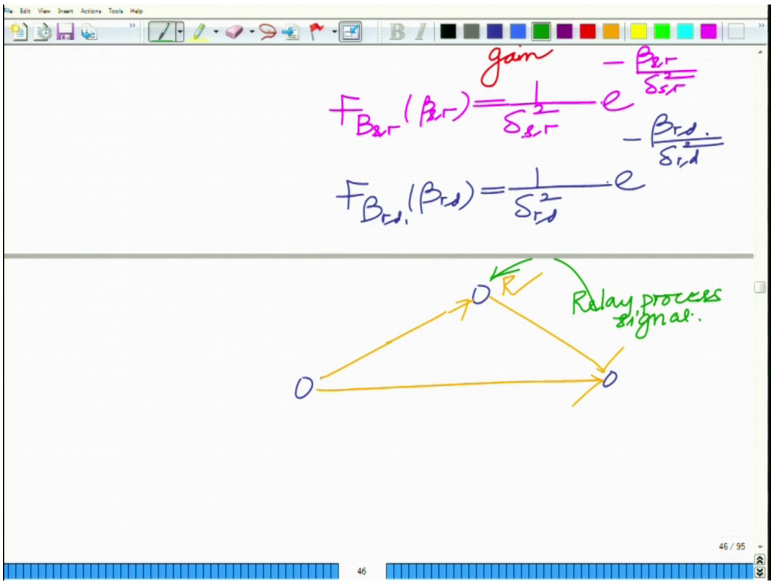
(Refer Slide Time: 13:30)



For instance, the distribution of this will be given as if you look at source destination equals one over the average gain delta s d square w power minus beta s d divided by delta s d square. So, this is what we mean by exponential distribution of the source definition. This is the exponential distribution for the source destination gain. So, if you denote that random variable magnitude s d square by beta s d, we are denoting that by beta s d and beta s d is exponentially distributed.

Similarly, if you denote the magnitude s r square by beta s r, beta s r is also exponentially distributed, magnitude of h r d square by beta r d that is also exponentially distributed and these can be given similarly. So, what we have is and these will come in handy later. So, I am just setting up its slightly as you can see elaborate, and as a result and that is, because most of these things are these are the latest, and these are fairly sophisticated ideas. So, these require some amount of analysis.

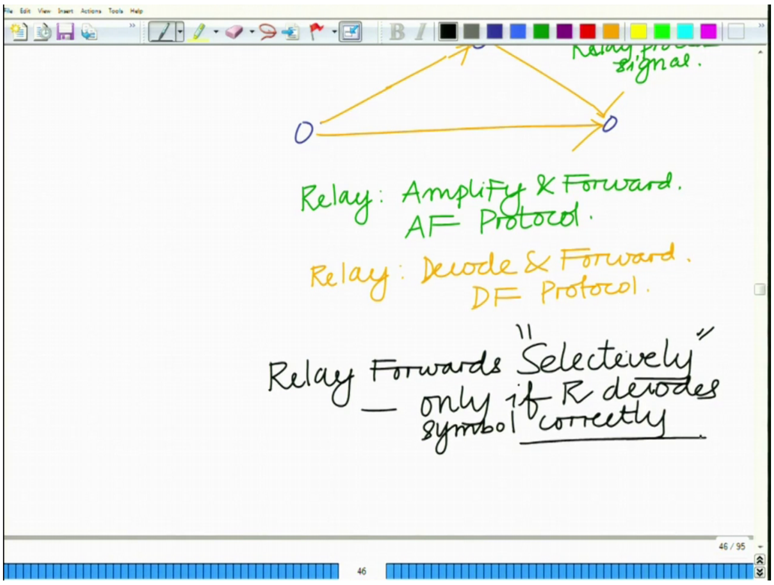
(Refer Slide Time: 15:25)



So, minus beta s r so this is the source relay link, which is average, which is exponential with average power delta s r square. And similarly beta r d 1 by delta r d square this is exponential with average gain delta r d square. So these are the relevant exponential distribution. So, exponential distribution for source destination s d, exponential distribution for source relay, exponential distribution for relay destination.

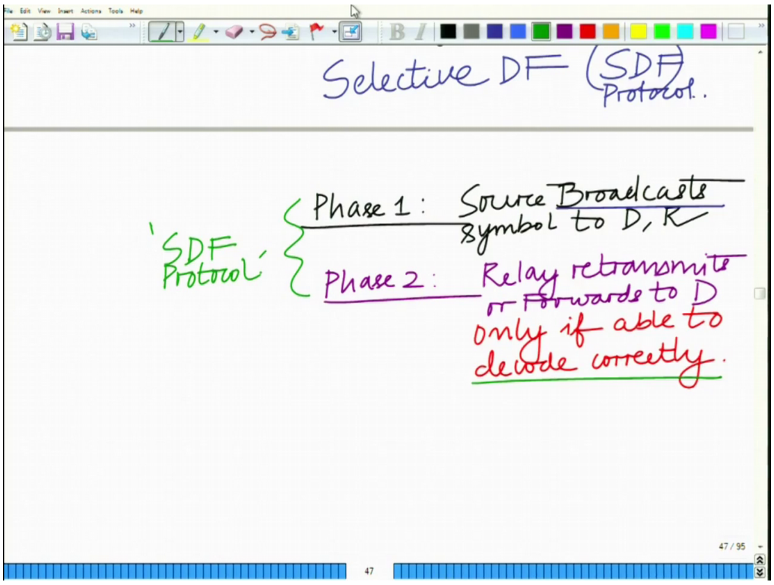
Now, there are several protocols remember we said that the relay does some processing. So, when you source destination and relay, so the relay processes the signal perform some signal processing operation on the (Refer Time: 16:34) perform some operations on the signal. So, it performs some it performs some signal processing. Now, depending on the nature of the signal processing you have a different protocol. For instance, relay can simply amplify the signal and forward it that is termed as amplify and forward; the relay can simply decode the signal and forward that is termed as decode and forward.

(Refer Slide Time: 16:58)



So, relay performs can perform amplify and forward this is known as the AF amplify and forward protocol or the relay can perform decode and forward the system as the decode and forward protocol. Now, what we are going to do, so there can be many many protocols, and the analysis for each protocol is going to be different. We are going to look at one simple protocol which is termed as the selective decode and forward protocol. What happens in the selective decode and forward protocol, it is a version or an extension of the decode and forward in which a relay receives the signal, it decodes the signal it forwards it selectively only if it is able to decode the signal or the symbol correctly, that is why this is known as selective decode and forward.

(Refer Slide Time: 18:47)



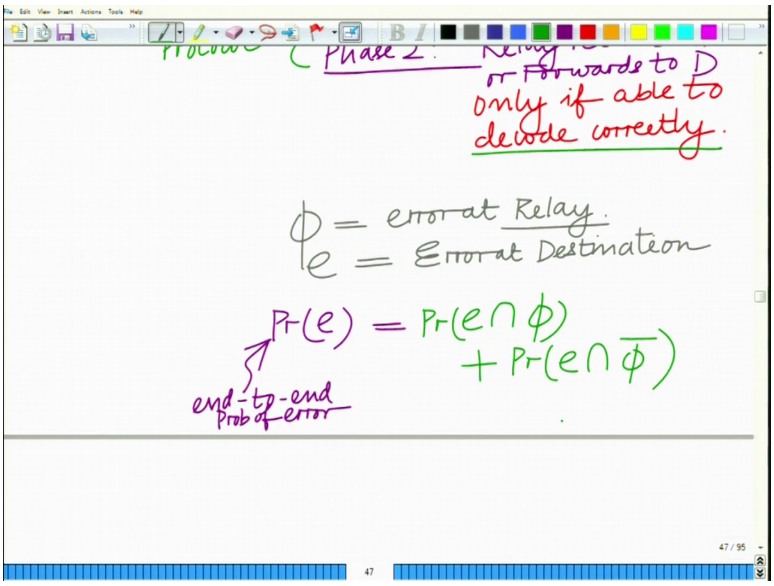
So, relay forwards selectively what do you mean by selectively, only if r relay decodes only if relay decodes the symbol correctly, this is known as selective DF or a selective decode and forward this is known as the SDF protocol. So, this is a very simple protocol which is of particular interest was this is known as the SDF or the selective. The selective it is also one of the popular protocol. There are several protocols and one can analyse several protocols one can consider and construct or develop optimization problems for several protocols. In particular, we are interested in this selective decode and forward protocol.

What and we have said as I have already said in selective decode, and protocol selective decode and forward protocol, the relay forwards the symbol to the destination, only if the relay is able to decode the transmitted signal by the source accurately. So, this happens in two phases so the SDF so any communication in general happens over multiple phases. So, here this SDF phase 1 source transmits or rather source broadcast symbol, which is received by destination and relay. So, the source in the first phase broadcasts the symbol to the destination and the relay.

In the second phase phase-2, thus relay retransmits or forwards relay retransmits or forwards to destination, only if it is able to decode correctly, only if the decoding at the really success only if it is able to decode correctly all right. So, this is the 2 phases of the SDF, so these are the two phases of the SDF protocol. Phase 1, and phase 2, phase 1 source transmits to destination and relay. So, destination is receiving it we are assuming there is a direct source destination this relay link.

In phase 2 relay retransmits, if it able to decode correctly. So, if relay transmits in phase 2, then destination is copies from both source and relay. Otherwise it simply as the copy from the source and that is similar to what you have in conventional wireless communication or the classical form of wireless communication non cooperative. And therefore, now you can see that this performance get typically critically depends on the decoding at the relay.

(Refer Slide Time: 22:02)

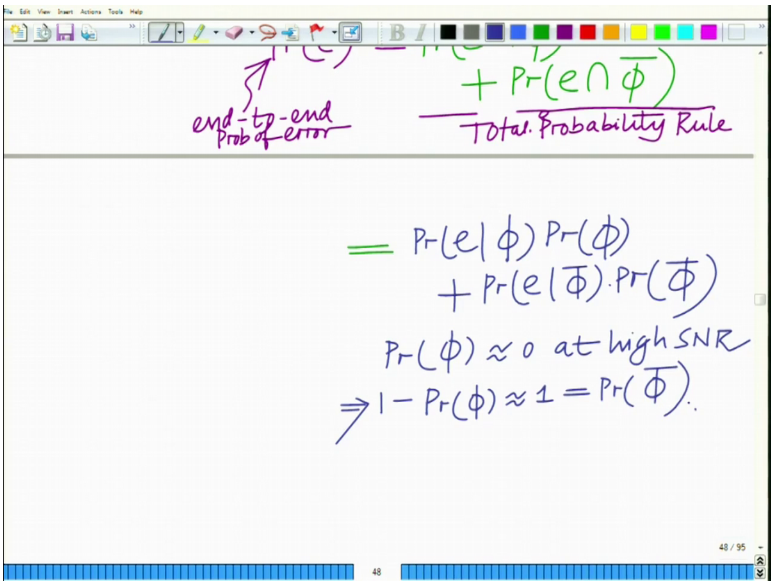


So, if phi denotes the error phi denotes that the event error event at relay, e denotes the error at destination, now node that probability of e that is the this is known as the probability of error at the destination or this is also known as the end to end probability of error. This is equal to probability of e, now we will use the total probability rule here we have to know little bit about probability. So, this is probability of intersection phi plus probability of intersection phi bar, because phi bar union phi union phi bar is the total set total event space S.

So, I can write it. So, we have a partition mutually exclusive and exhaustive all right. So, phi and phi bar are mutually exclusive and is an exhaustive partition. So, I can write using the total probability, I can write this as probability of the error at destination is probability of error intersection phi, and probability of error intersection phi bar right.

And it is also intuitive this is a probability of error with probability of error at destination and error at relay or probability of error or plus probability of error at destination and no error at relay all right that is basically the total probability rule.

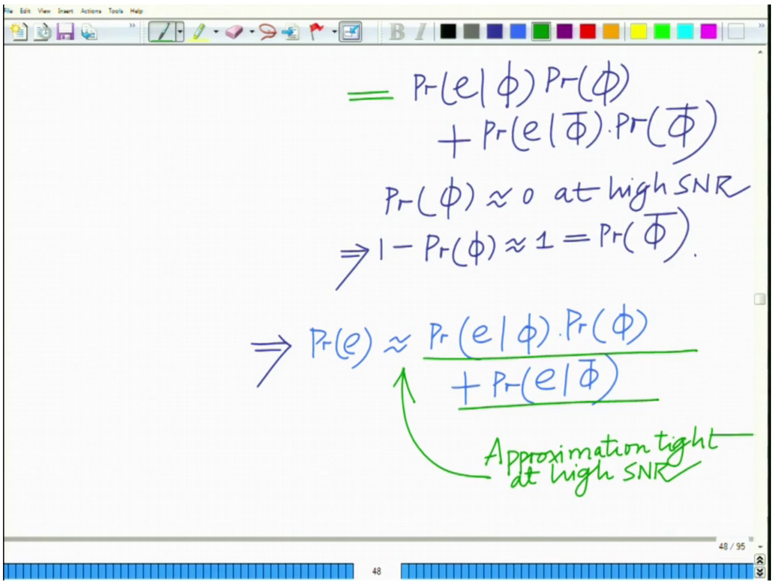
(Refer Slide Time: 23:55)



And now using the conditional probability I can write this probability of e intersection phi is probability of e given phi into probability of phi. This is the definition of conditional probability plus probability e given phi bar into probability of phi bar. So, this follows, the first one follows basically from so total probability rule. So, this is probability rule.

Now, probability of phi remember is the probability of error at destination. So, probability of phi is close to 0 probability of error at relay, so this is close to 0 at high SNR. When the signal to noise power ratio is high, probability of phi that is a probability of error at the relay is close to 0, which means 1 minus probability that is probability of phi bar which is 1 minus probability of phi that is approximately equal to 1. So, I am going to make that approximation.

(Refer Slide Time: 25:18)



So, this implies 1 minus probability of phi approximately equal to 1, this is probability of phi bar. So, what we have is probability of phi bar is approximately equal to 1, which means using the above approximation you can approximate the probability of error approximately as probability of e given phi into probability of phi plus probability of e given phi bar. What is probability of e given phi, probability of e that is error at destination given that there is also error remember phi denotes the error even at relay. So, e given phi means error at destination given error at relay.

Times probability of phi, what is the probability that error occurs at relay plus probability of error given phi bar that is probability of error at destination given that there is no error at the relay in which case the relay retransmits. And this is an approximation, but this is a good approximation which is very tight at high SNR, remember this is known as a high SNR approximation all right.

So, this is tight or is going to be very close to the actual P r. So, this approximation is tight the approximation is tight at high SNR all right. So, now we have to calculate these various quantities all right. So, now what we have to do is we have to calculate these various quantities to basically derive the expression for the overall probability of error. And this is something that we are going to look at in the subsequent modules.

Thank you very much.