Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 50
Practical Application: Multiple Input Multiple Output (MIMO) Beamforming

Hello, welcome to another module in this massive open online course. So, we are looking at various convex optimization problems, and especially focusing on their practical applications all right. We are looking at this from an applied perspective all right. In this module, let us look at another interesting problem with a very important, and I would like to say novel practical application that is of beamforming in MIMO systems.

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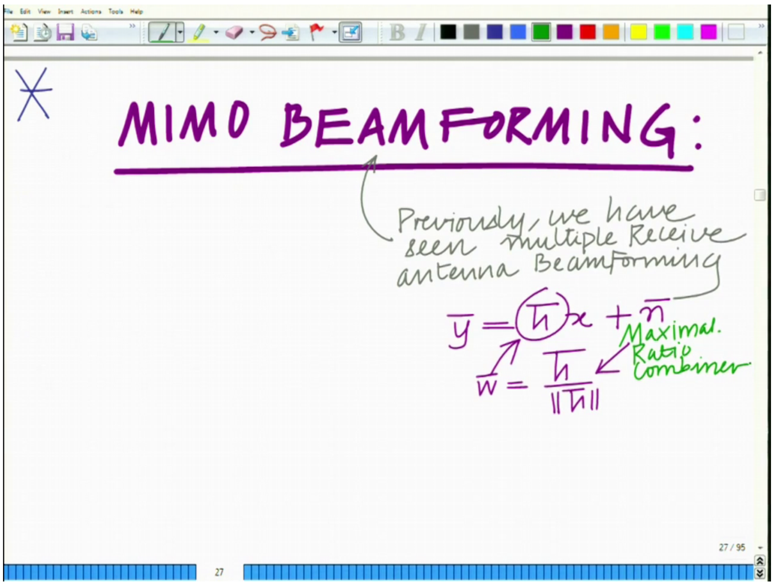

So, what you want to look at in this module is the problem of MIMO. Now, what happens in a MIMO, now what we have looked at previously, we have seen only multiple antenna previously we have seen when you have multiple beamforming, when you have multiple receive antennas. You might recall that we consider a system, where which is modelled as y bar equals h bar times x plus n bar. And then we derived the beamformer for this system, the beamformer for this system is given by W bar equals h bar divided by norm of h bar; the system the MRC or the maximal ratio of beamformer ok, the maximal ratio combiner. We have seen this before.

Now, what we want to do is we want to extend this to a MIMO system. Remember, you might recall MIMO system is a system, which has not just multiple receiver antennas, but also multiple transmitter antennas. So, your multiple antennas is the transmitter, as well as the receiver that is why it is known as the MIMO system or the multiple input multiple output system.

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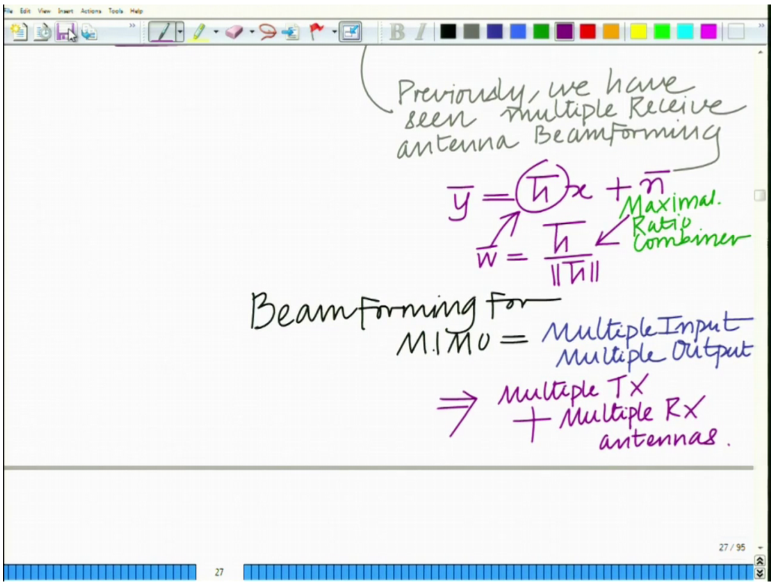

So, what we want to consider is this interesting problem of beamforming. And this is not so trivious straight forward beamforming for MIMO ok, which basically simply stands for Multiple Input and Multiple Output multiple input, which basically implies that you have multiple transmit and multiple receive antennas multiple T X plus you have plus you have multiple R X antennas multiple receiver, yeah multiple transmit and multiple receive antennas.

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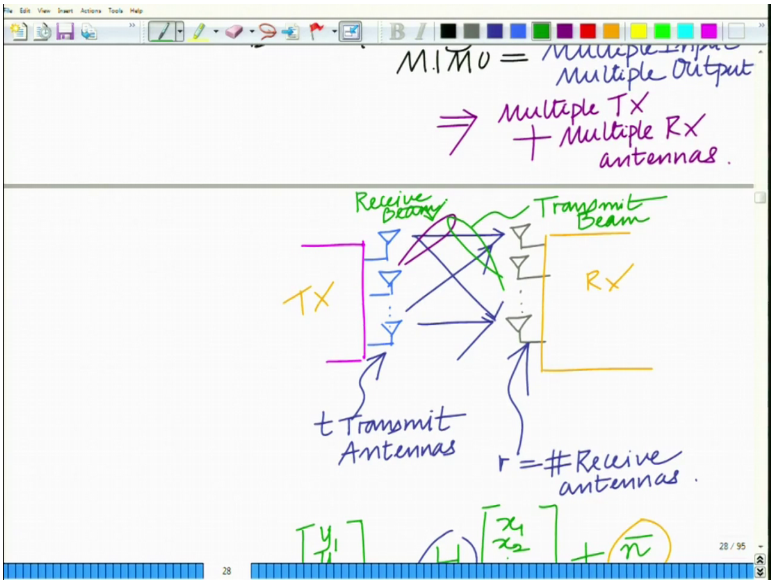

And one can represent this is from let us just, so you have let us say this is your transmitter, you have multiple transmit antennas. And the same time, you have multiple receive antennas. Now, these need not be equal ok. And this is your transmitter, this is your receiver, and you have the different possible paths, you can say or links between the transfer. Yeah, let us say t antennas at the transmitter, these are your t transmit antennas, and this are your r equals to the number of receive antennas. And this is your transmitter, this is your receiver.

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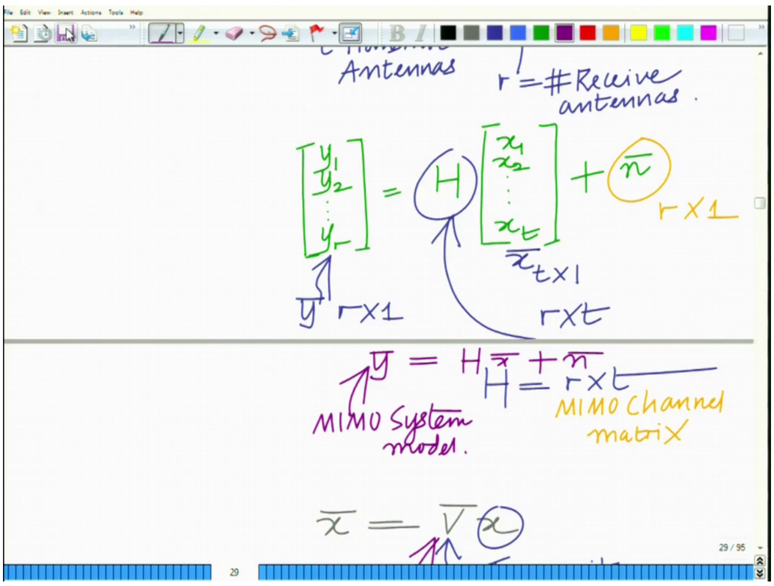

And the system model can be represented as follows. You are received vector of symbols y 1, y 2 up to y r, since there are r receive antennas. You are going to have r receive symbols y 1, y 2, y r equals H, this is your channel matrix times x 1, x 2 up to x t, because your t have t transmit antennas, therefore you can transmit t symbols plus you have your noise vector.

Now, as you can see this is y bar, which has r symbols ok, which is basically r cross 1 vector. This is x bar, which is t cross 1 that is it has t symbols, because you have t transmit antennas. And now naturally, you have r dimensional output, t dimensional input, so the channel matrix that transforms the t dimensional input, r dimensional output, this is r cross t. This is known as the H equals the r cross t that is r rows and t columns in the MIMO channel matrix. So, this is your r cross t MIMO channel matrix. And n bar that is the noise vector, this is an r cross 1, this is an r cross 1; this is an r cross 1 noise vector.

And what we want to do is we want to explore, how can we use beamforming? Remember, beamforming means transmitting in a particular direction that is focusing the transmit power in the particular direction, and receive beamforming means focusing that is looking in a particular direction to receive the maximum amount of energy that is what we mean by beamforming all right.

So, beamforming in a multiple antenna system remember that is basically you are looking in a particular direction, and at the transmitter also you have transmitting in a particular direction. So, this is your transmit beam so as to speak, and this is your received beam. Now, there can be multiple beams, but when people talk about beamforming that typically talk about a single beam.

So, you have a transmit beam that is you are focusing the energy in the particular direction, and you are have a received beam that is you are looking for the signal that is you are receiving the signal in a particular direction. By doing that, you are achieving several things as we have seen in multiple antenna beam forming that is one is you are maximizing the signal to noise power ratio. Two by using only a particular direction, you can be you can avoid the interference that is the interference, which is caused by the interfering uses. By avoiding those other directions, you can basically suppress the interference at the receiver. So, beamforming has many uses and very useful practically. Now, how do we do beamforming in a MIMO system that is what we want to explore.

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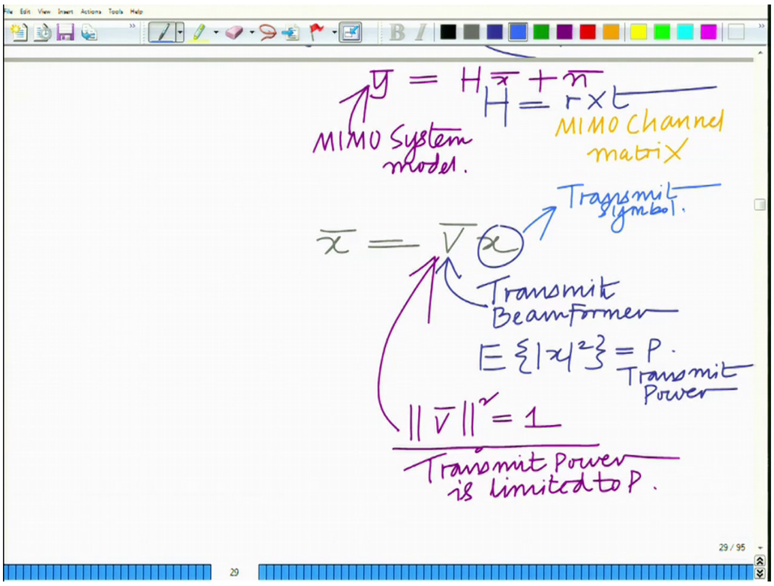

So, let us say we want to transmit the vector x bar, we want to use a beamforming. Remember, beamforming vector is central to the idea of beamforming. So, this v bar this is the vector of so this is your beamforming vector. And this is more specifically since you are doing this at the transmitter, this is also known as the, this can also be known as the transmit the transmit beamformer or the transmit.

So, what you are doing is you are using v bar at the transmitter all right. To focus the energy or transmit the energy or transmit the signal in a particular direction, this is known as transmit beamforming. And this v bar gains the transmit beamformer ok. And x is the transmitted symbol, which is transmitted with the aid of this transmit beamform. Remember, this is the concept of electronic steering, where you are adjusting the weights, such that you focus the signal in a particular direction. So, v bar x; this is the transmit.

Now, what we now we can have the transmit power equals can be P that is this is the transmit power. Now, the beamformer itself is simply focuses the signal, therefore it should not amplify it or attenuate the signal. So, what we will do is we will fix the power of the beamformer that is norm beam w square (Refer Time: 09:52) so the transmit power is fixed. So, the energy of the beamformer there is norm (Refer Time: 09:55) bar square is fixed, so that basically this imposes a transmit power constraint. So, transmit power is limited to P. So, this limits the power that has to be transmit.

So, this is also known as the transmit power constraint. So, norm v bar square equal to 1 ok. So, this is to basically this constraint is to limit, the transmit power we want to limit it to a, we want to make limit it to a unit norm beamforming. So, now remember, we have this system model here, which is basically your y bar equals H times x bar plus n bar, this is your MIMO, this is your MIMO system model.

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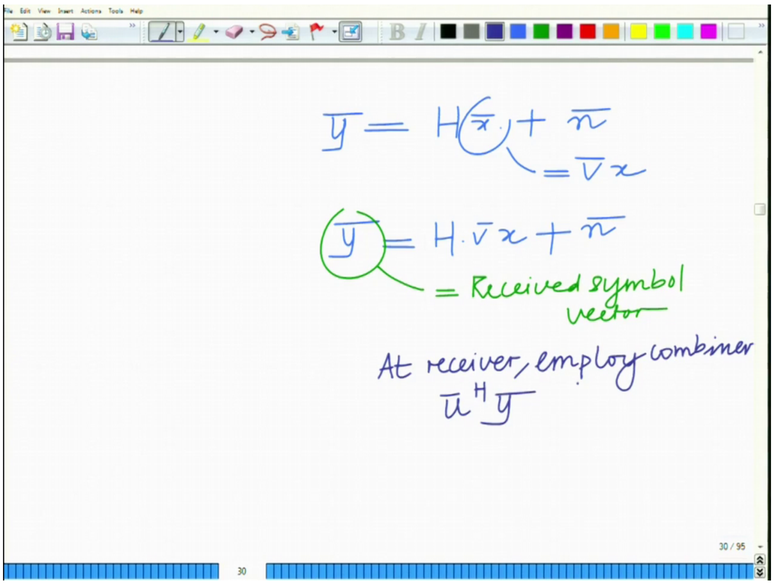

Now, what we want to do is we have y bar equals H x bar plus n bar. Now, here we want to substitute x bar equals v bar times x that is what we said, v bar is a transmit beamformer, x is the transmitted symbol, v bar is a transmit beamformer, x is the transmit symbol. And therefore, now this is going to be H times v bar x plus n bar, all I am doing is of so instead of x bar, I am substituting x bar equals v bar times x. Now this is y bar.

And now, what we are going to do at the receiver is at the receiver, we are going to employ a combiner. So, this is your received symbol vector. At receiver at the receiver, we employ a combiner. Now, at the receiver, one can employ a combiner, which is of the form that is you perform u bar Hermitian y bar. So, this is basically your receive beamformer.

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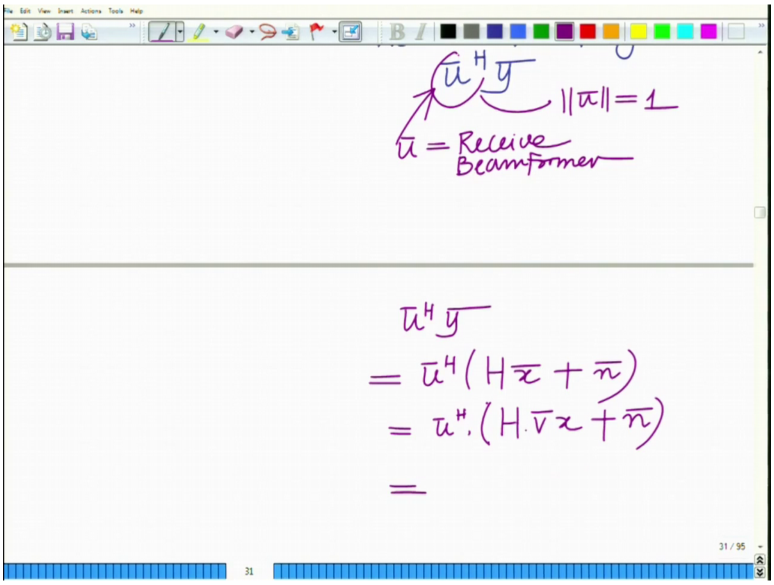

Now, this becomes your u bar equals to your. Well, you can think of this as the receive filter, you are combining with u bar, so you are performing u bar Hermitian y bar. And also, we can restrict without loss of generality, we can restrict norm u bar to 1 that is the norm of the received beamform is also being restricted or limited to 1.

And therefore, now if I substitute this model, I have u bar Hermitian y bar. Now, I am going to substitute expression for y bar equals u bar Hermitian H x bar plus n bar equals u bar Hermitian. Now H x bar of course, x bar we replaced by H times v bar into x plus n bar, which is now if we simplify this.

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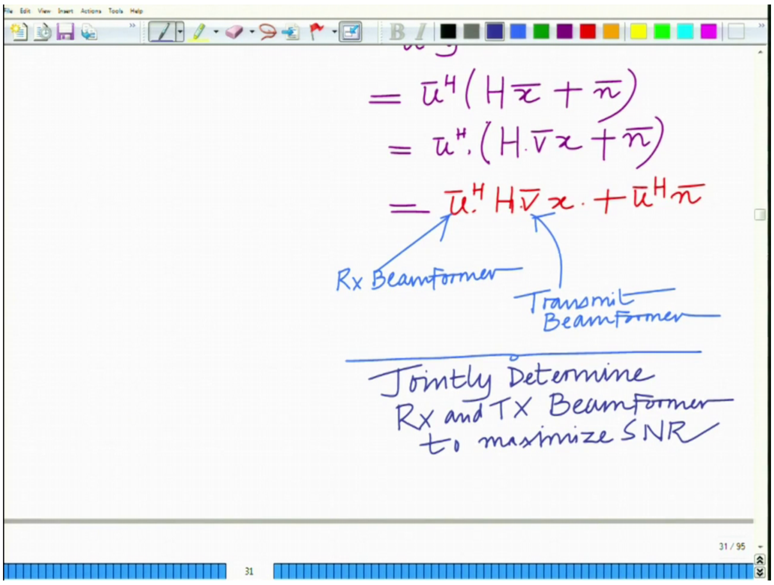

This is u bar Hermitian H v bar into x plus u bar Hermitian. And now what we have to do is we have to determine both u bar and v bar that max. So, this is my receive beamformer, and this problem is challenging, because you can see you have to determine not one, but two beamformer. So, this is your beam for receive beamformer, this is your transmit beamformer. And so, now we have to jointly, so now it what is the problem in MIMO beamforming, we have to jointly determine both R X and T X beamformer that is u bar and v bar v bar to maximize the SNR all right.

So, what we want to do is basically we have to determine both these beam formers that is both your u bar and also v bar. That is remember normally in a multiple antenna beamforming, when you have multiple antennas only at the receiver, you are simply determining the single beamformer, there is beamformer at the receiver that is what we have done so far.

But, here because you have multiple antennas both at the transmitter as well as the receiver, one has to determine both the optimal beamformer that is v bar that is the optimal beamformer of the transmitter, and u bar that is optimal beamformer at the receiv[er]. And these problems are interlinked, because depending on v bar, one has to choose u bar; and depending on u bar, one has to choose v bar. So, what is the what are the optimal beamformers u bar and v bar that maximize the SNR at the receiver that is the challenging problem that one has to address, which we will solve in the subsequent module.

Thank you very much.