Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 48
Linear Program Practical Application: Base Station Co – operation

Hello, welcome to another module in this massive open online course. So, we are looking at convex optimization problems. We have looked at the canonical or the standard form of a convex optimization problem. In this module, let us look at an important subclass of convex optimization problems which is the, which basically linear or the class of linear programs, ok.

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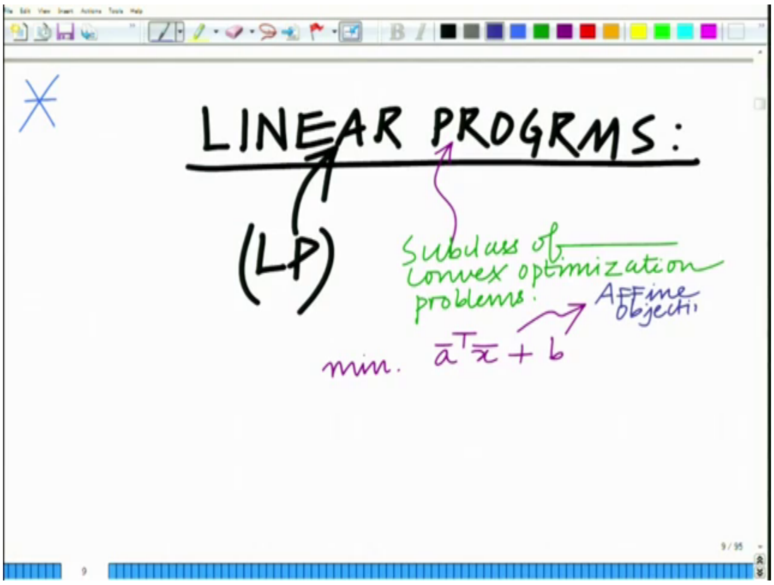

So, what you want to look at is basically linear programs, which is also referred to as LPs. And this forms it is a subclass as I already said, it is we can think of this as a special case or type of convex optimization problem or a subclass of or a subclass of convex optimization problems.

And now, a linear program, ok. And this is also referred to as then also referred to this as an LP as I already said, it is a linear program is also an LP and well, linear program can be expressed as follows: that is remember any convex optimization any optimization problem has an objective, let us say, as minimize a bar transpose x bar plus b, ok, which basically has a linear objective or an affine and affine objective function, subject to the constraints which are also affine.

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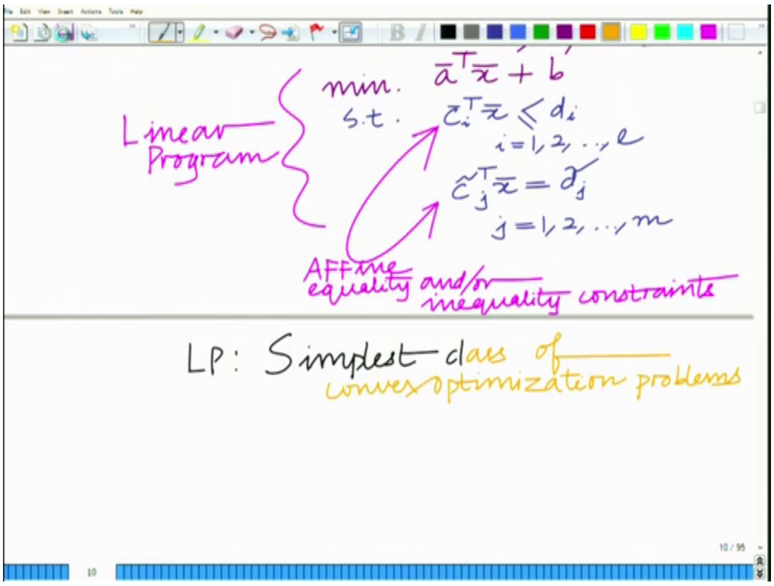

That is c bar transpose c i bar transpose x bar less than or equal to d I, for i equals to 1, 2 up to l and equality constraints which have to be affine in any case for a convex optimization problem as we have been seen in the previous module, that is c tilde j transpose x bar equals d tilde j, for j equals 1, 2 up to m alright

So, basically the constraints, the equality as well as the inequality constraints alright; so, all affine that is it has affine equality and slash or inequality affine constraints and slash or inequality constraints. So, basically linear objective, no, of course, an affine function is a convex functions. So, a convex function; so, objective function is convex alright. So, and an affine object or otherwise an affine constraint is also convex. So, therefore, this is a special class of convex optimization problem. So, linear program is a special is a subclass of convex optimization problems, in which the objective function as well as the constraints equality as well as inequality constraints are all affine in nature, alright.

And you can say, this is so, this is basically your linear program and well, you can say, this is a simplest because, everything is affine. This is the simplest class or category of convex optimization of convex optimization problems.

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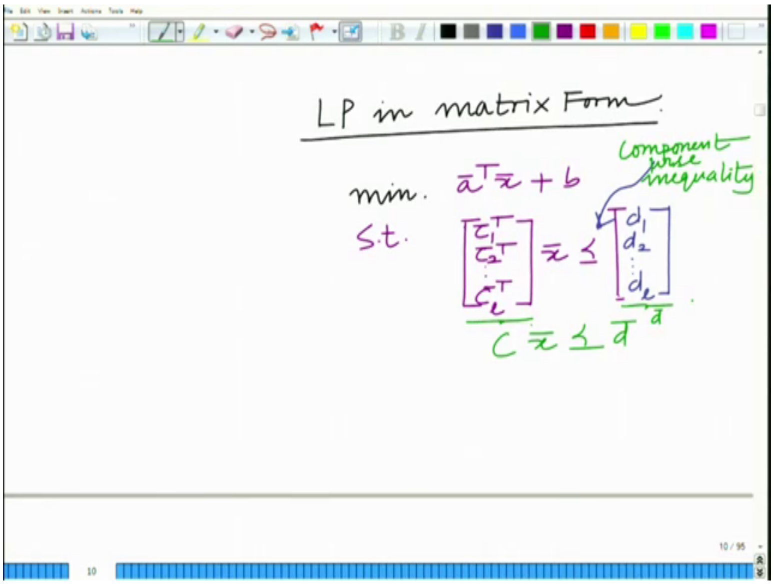

Now, just for convenience, I can write this in matrix form. So, I can represent the LP in matrix form as follows, is not very difficult. So, I have minimize the objective which is a bar transpose x bar plus b ah, subject to the constraints. Now, constraints I can represent them as a matrix. So, I can write this as c 1 bar transpose, c 2 bar transpose, c l bar transpose x bar. Now, this is component wise that is each component of the vector on the left has to be less than each component of the vector on the right. So, this is a component wise, is also remember is also termed as component wise this is the component wise inequality. So, I can write this now as your matrix c times x bar component wise less than, this is your vector d bar, alright.

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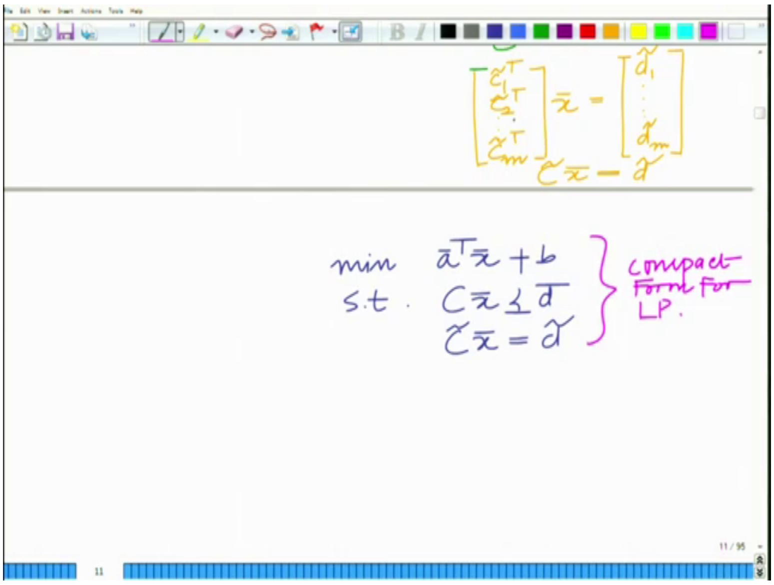

So, I can represent it in a compact form using matrices. Similarly, the equality constraints I can represent them as c 1 tilde transpose c 2 tilde transpose c m tilde transpose times x bar equals d 1 tilde up to d m tilde. So, this becomes your, c tilde x bar equals d tilde. And therefore, the compact this can be written in compact form as follows: minimum minimize a bar transpose x bar plus b subject to c x bar component wise less than d bar and c tilde x bar equals d tilde. So, this is a, you can say a compact form for a, this is the compact form for the linear program. So, this is basically expresses your linear program in a very compact form in a compact form using vectors and matrices, ok.

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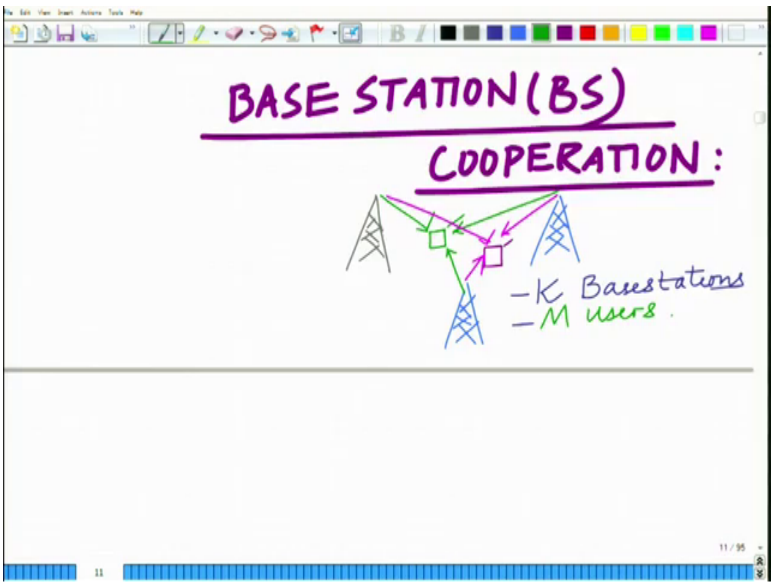

Let us look at an example for this linear program and we have already seen this before. That is, we have already introduce this before that is the example of base station cooperation where there are several base stations in a cellular communication scenario whereas, user at the edge of several cell or the intersection or the region that can be served by several cells alright or can be served by several base stations. So, the different base stations typically, when the user is at the edge of a cell or at the edge of cells the user can be served by several base stations belonging to the different cells which are overlapping at that particular point. And so, we consider a scenario in which this particular user is being served by several base station or not just one the single user. In fact, several users can be served when the by different base stations cooperating with each other, alright.

So, let us take a look at our example of base station cooperation or cooperative. And we have already seen what happens in base station corporation. In base station cooperation, we have a group of group of cells that are cooperating to transmit to one or many users. So, I have different users and the base stations are transmitting to the various users in a cooperative fashion. Normally, we have a single base station serving any particular user, but in this particular stay in, but this particular scenario base stations can cooperate to serve the various uses thereby enhance the SNR, enhance the reliability of communication in a wireless communicate scenario and wireless communication scenario, alright.

So, this is basically would be stations, where consider a scenario in which we have k. So, we have k base stations and m. So, k base stations are cooperating in this cooperative cellular scenario to serve basically m users, ok.

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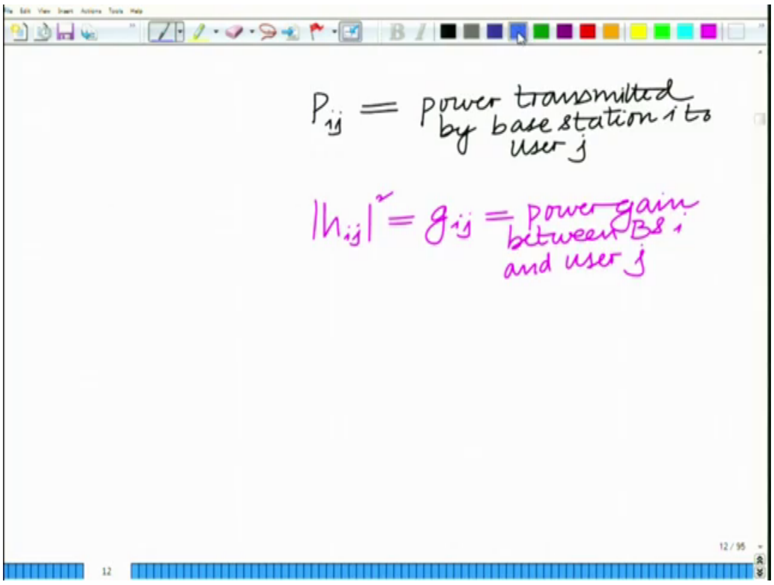

And in this scenario, let P ij, it could denote the power of power transmitted by base station i to user j, ok. And we will denote by h ij is the fading channel coefficient. So, magnitude h ij square equals g ij represents the power gain, the power gain between base station I base station I and user j. So, P ij denotes the power transmitted by base station i to user j, alright. And magnitude h ij ah, ah, h ij is the fading channel coefficient, magnitude h ij square which is g ij represents the power, the gain from base station i to user j, ok. And therefore, now if you look at the power that is received by each user i.

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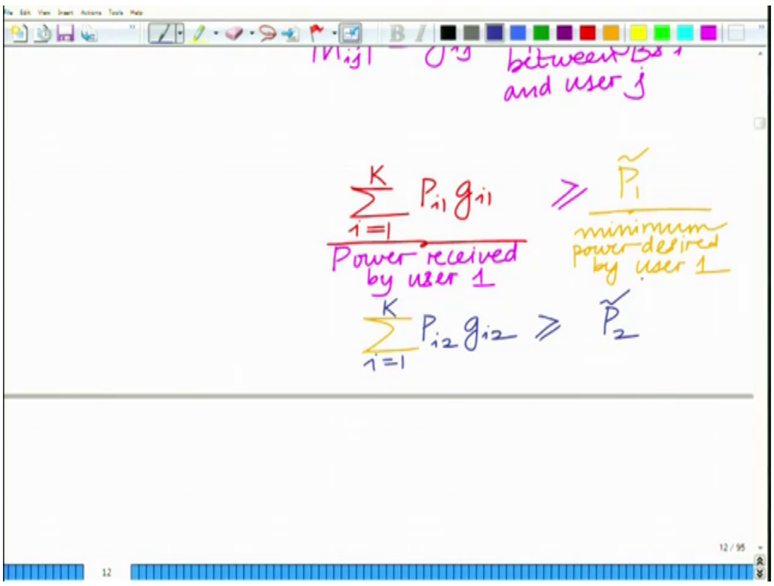

So, the power that is received by each user I, in this cooperative fashion is the power sum of the powers that is transmitted by all the base station. So, I have, i equals 1 to k, P i1 g i1. This basically is the power received at user 1. This is the power received at your, this is the power that is basically the sum of the powers, correct. That is the powers the sum of the powers received from all the base stations, alright. And remember we said this has to be greater than or equal to P 1 tilde, which is the minimum power desired by user 1 this is the minimum power desired by user 1.

So, the sum of the powers received from all base stations has to be greater equal to P 1 tilde. This has to also hold similarly for the other user. So, at user 2 we must have the sum of the powers received from all the stations, that is P i 2 g i 2 summation over i equals 1 to k over all base stations that has to be greater than or equal to P 2 tilde.

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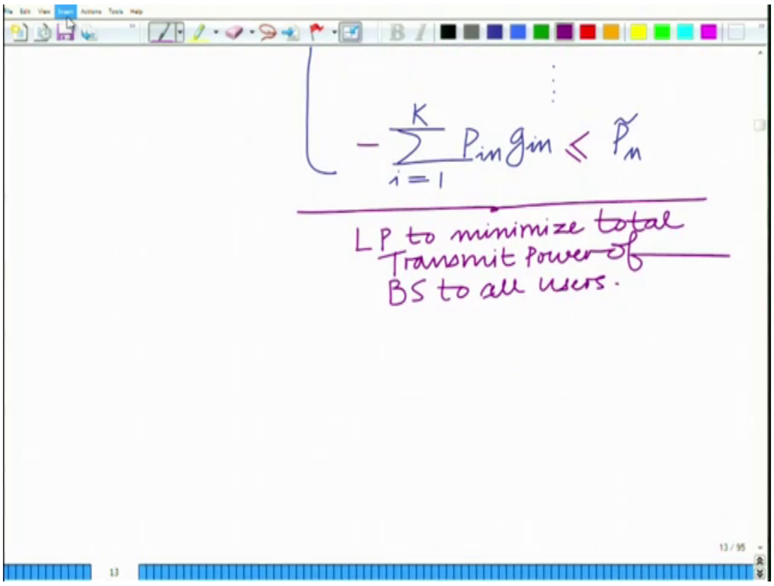

And so on and so forth, that is summation I equal to 1 to k, P i m at user m summation i equal to 1 to k P i m g i m greater than or equal to P m tilde, ok. So, these are your constraints ok. Ah, and you can see these are basically affine constraints.

And now, we need an objective. One of the objectives that one can consider is basically, we want to meet the desired power level at each user, but simultaneously we also want to transmit the minimum amount of power. So, what is the total, what is the minimum total power that can be transmitted by all base stations to all users to meet these desired criteria at the various users? So, the objective function can be to minimize the total power transmitted by all the base stations to all the users. So, now, that makes it a convex optimization problem. So, the objective function can be, minimize summation i equal to 1 to k summation j equal to 1 summation over all base stations summation over all users P i j, that is power transmitted by the base stations to all. So, this is your linear objective remember it is simply the sum. So, therefore, it is a linear objective linear objective and this represents remember this is the total power by all base stations to all users, alright. This is a total power by all base stations to all users.

So, what is our optimization problem? Our optimization problem is to minimize the total power transmit power of all the base stations to all the users subject to these constraints, that is the min that is the power received at each user is greater than a minimum threshold which is denoted by P j tilde, alright. At user 1, it is P 1 tilde, user 2 p 2 tilde. So, on at user m, it is P m tilde. And you can clearly see the objective is linear, constraints are linear; this is a linear program. So, this is so, cooperative base station transmission or base station cooperation this is basically this is a one can formulate of this linear program and you can see thus the variety or the interesting and very interesting applications of the simple yet very flexible and powerful optimization framework for that of a linear program. So, this can be obtained to minimize the total transmit power total power that has to be transmitted by all the base stations to all the users.

Now, also note that in the standard from convex optimization problem the inequalities are have to be less than or equal to. So, and one can readily convert it to the standard form by simply introducing a minus sign. So, I will have a minus in front of everything and the inequalities become less than or equal to. So, this becomes minus is less than or equal to and this is now your standard form linear program. So, this is your LP, to simply minimize total transmit power of base stations to all the users.

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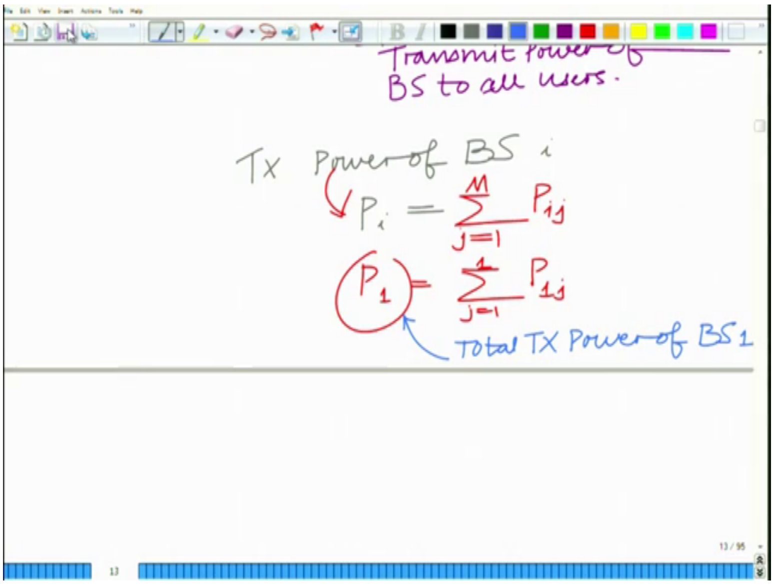

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Now, an interesting variation on this problem can be the following thing. Now, if you look at the transmit power of any single base station, now, let us look at the transmit power of base station I, that will be equal that can be represented as P of i.

So, the transmit power of base station i can be represented as P of P i equals summation that is the transmitted power transmitted to all the users summation over all users P i j, ok. For instance, P 1 power transmitted by base station 1 equals j equals 1 to M P i or P 1 j, that is the power transmitted by base station 1 to each user j summer over all the users, ok. So, this is power transmitted by total transmit power total transmit power of base station 1 in fact, that is what this is, ok.

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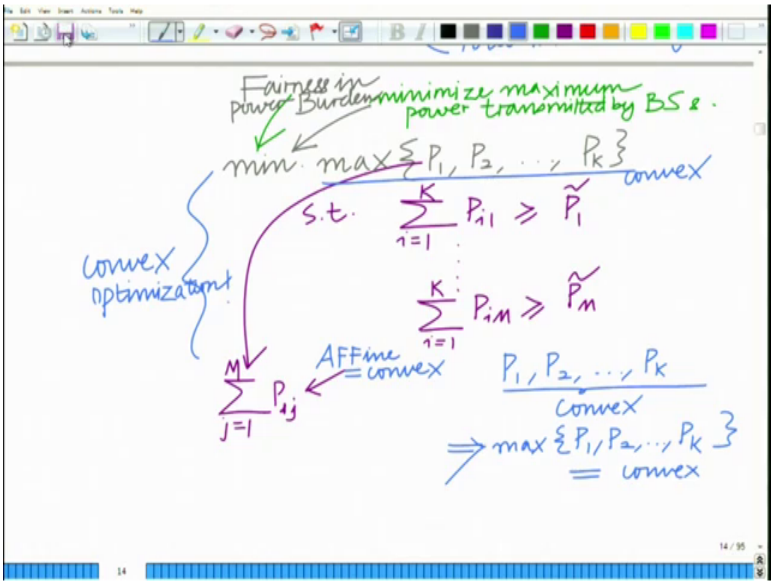

And now, what do we want to do is we want to consider an interesting optimization objective function, in which we want to minimize the maximum of the powers transmitted by the various base station. So, the total k base station, so, you have transmit powers P k. So, you want to minimize the maximum. So, what this is doing is minimizing the maximum power transmitted minimize the maximum power transmitted by the base station.

Now, what happens typically in this cooperative cellular scenario is that, ah, there are few base stations there are few there are several base stations and what when you minimize the total power transmitted by all the base stations together that might result in an undue burden on a single base station alright. So, one of the particular one particular base station which probably has good condition good channel conditions of with channels to the different users are good can be over burdened or over tax in comparison to. So, this does not ensure that the load is. So, the previous total power minimization does not ensure that the power is uniform that the power load that is the burden the transmitter power burden is not uniformly levied on all the base stations alright. There might be different base stations which are ah which are levied more or which have to transmit more power in comparison to others.

But, when your minimizing the maximum transmit power, what this does is that this ensures a sort of fairness in the power burden it sort of ensures that this power burden to the, for the different users is rather uniformly rather fairly distributed on all the base station. So, this is an important property of such problem. So, this ensures you can say min max the min max criterion basically ensures fairness in the power burden in the power burden or the power distribution minimizes the maximum power.

And subject to the constraints are all they are as usual. That is, if you look at the desired power at each user that has to be greater than the threshold. So, the constraints are there. So, this is, now, if you look at this min max, now, if you get now if you look at each P i and we have already seen this each P i is summation j equals 1 to m is summation j equals 1 to M P P 1 j and this you can see, this is affine or this is convex ok. So, each of these P 1, P 2, P k each of these powers, so, each of these is convex, alright. And therefore, we need take the maximum of this; remember the maximum of a set of finite or infinite convex function is convex, alright.

And therefore, the maximum of this set is convex. So, this implies the maximum of this is convex and therefore, this is basically convex and therefore, you are minimizing a convex optimization, convex objective function, constraints are as usual, I mean similar to the previous one they are convex. So, this is also a convex optimization problem, ok. And this is also therefore, convex optimization problem. Now, what is this relation to a linear program? Now, at present it seems unrelated to a linear program, but we are going to demonstrate that this in is in fact, can be written as an equivalent linear program and for that we will use the epigraph trick that we have seen in the previous module.

So, how can this be written as a linear program.

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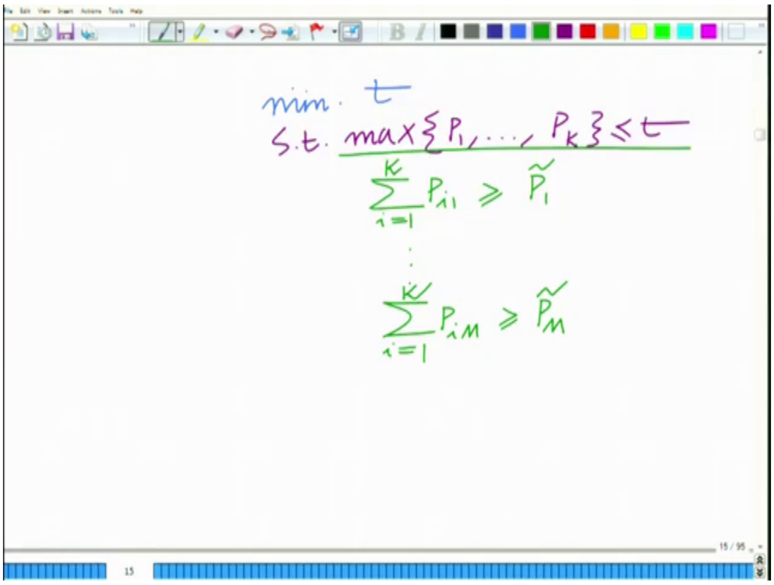

So, we have minimize now we will use the epigraph form. So, I can write this as minimize t subject to the object to maximum of P 1, P 2, P k this is less than equal to t and the rest of the constraints are as usual there. That is summation i equal to 1 to k P i 1 greater than or equal to P 1 tilde and so on and so forth, summation i equal to 1 to k P i M greater than or equal to P M tilde. Now, the maximum of P 1 P 2 up to P k less than or equal to t, the maximum of a set of quantity is less than equal to t if and only if each of these is less than or equal to t.

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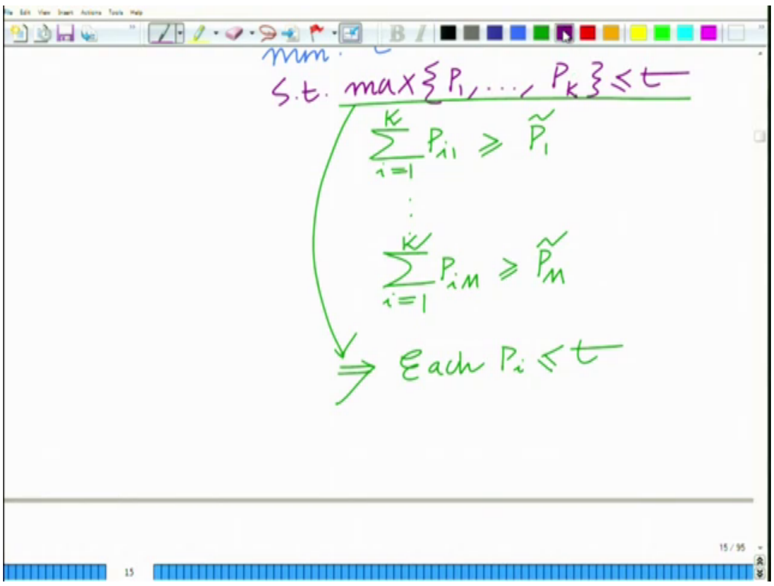

So, this implies each is has to be each P. This implies each P i has to be less than or equal to t.

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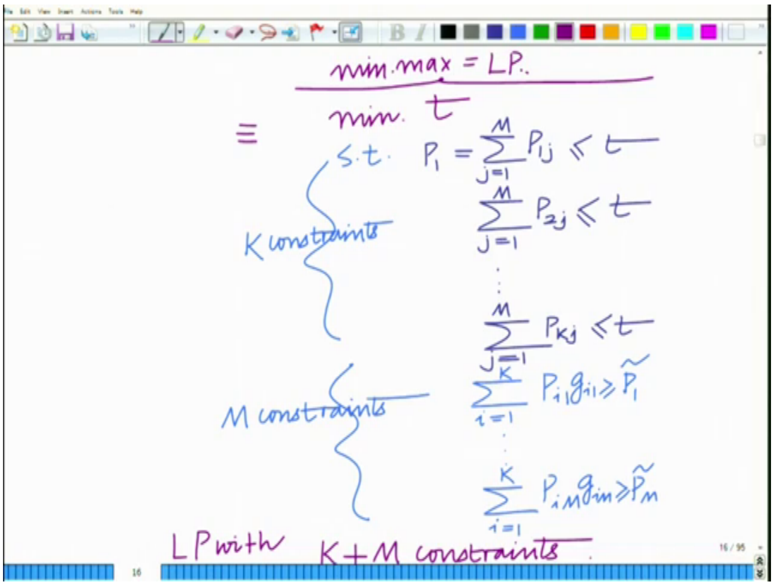

And now, I can write the equivalent optimization problem. Therefore, this is optimization problem is equivalent to minimize t subject to, now, we need that each P i is less than or equal t. So, this means P 1 equals summation over j equals 1 to M P 1 j less than or equal to t j equals to 1 to M P 2 j summation less than or equal to t, this is the power transmitted by base station 2 to all users and finally, summation j equals 1 to M P k j less than or equal to t. These are the base station power constraints. And you have the user constraints as well similar to previous. So, summation i equal to 1 to k P i 1 greater than or equal to P 1 tilde summation i equal to 1 to k P i M greater than or equal to P M tilde. I am sorry this has to be ah, I am missing I am missing here ah, missing the channel gains the channel gains are very much there. So, this has to be g i 1 g i M greater than equal to P 1 tilde P M tilde.

And similarly this has to be P i 1 g i 1 so on forth, summation P i M g i M greater than equal to P M tilde. Ah, similarly here also, and summation i equal to 1 to k, P i M g i M greater than equal to P M tilde. Just, let me make sure that we are not missing this at any point yes. In fact, I think that so that ah, so, want to minimize the maximum correct using the epigraph form I can write this as minimize t such that the maximum is less than or equal to t, the desired power at each user has to be greater than or equal to P j tilde that is there as usual and if the maximum is less than t then we have each P i is less than t, which implies that is why I can write this as.

So, now we have this is basically the k constraints and you can see each is an affine constraint ok. And these are your previous M constraints. So, the mini max optimization problem has k. This has a total of K plus M constraints alright. So, this has a total of so, this has a so, you can write the min max. So, this is also an LP. So, you can see each of these all these constraints are affine. So, therefore, the mini max problem is also a linear program and it is not obvious the first instance, but using a clever trick alright or by manipulating this you can write the min max problem as an equivalent linear programming.

And therefore, the linear program has can be written in various forms and has very interesting application. So, not only can be it be used to minimize the total transmit power, but you can also be used to as I have said, it also it can also be used to minimize the maximum transmit power maximum power transmitted by any base station any of these base stations ah, thereby ensuring that this power burden or the transmitted power burden is fairly or it is sort of evenly distributed of all the cooperating base stations, alright. So, that basically introduces the linear program and basically demonstrates it is demonstrates it is application in a practical scenario in a practical wireless scenario for base station corporation, alright. We will stop here and continue in the subsequent modules.

Thank you very much.