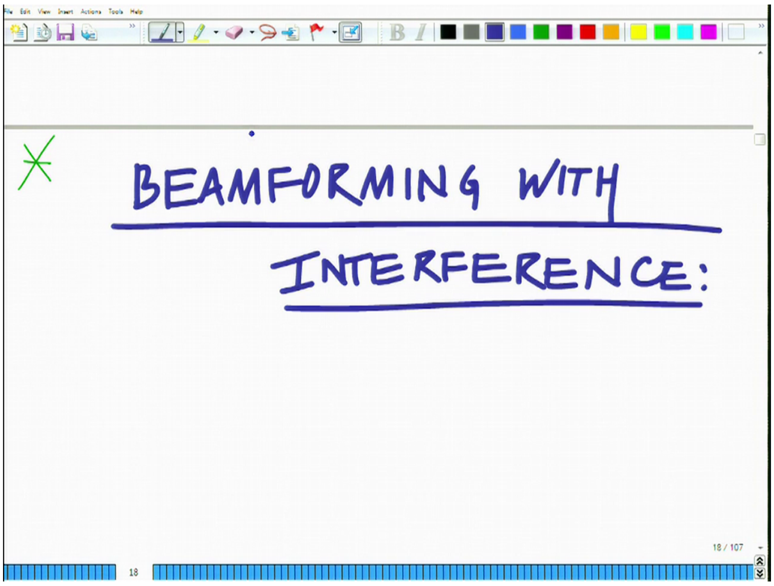
Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 36
Practical Application: Multi-antenna Beamforming with Interfering User

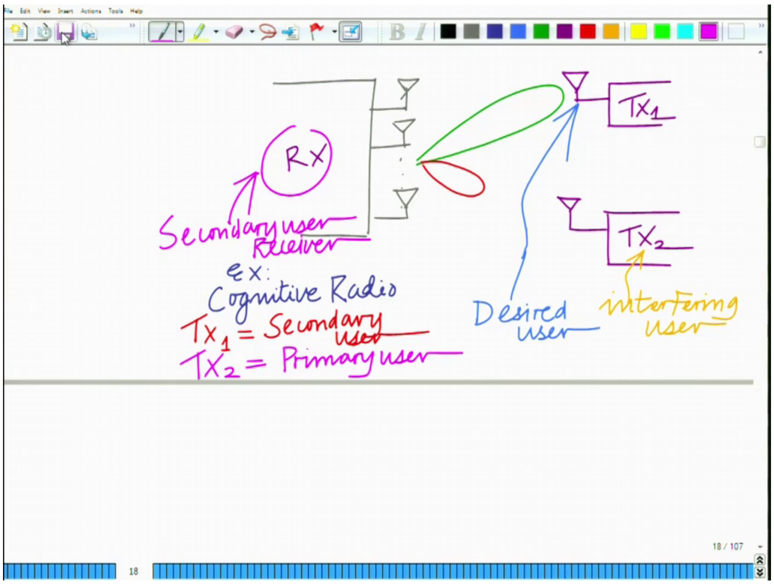
Hello, welcome to another module in this massive open online course. So, we are looking at practical applications of optimization in particular we have looked at beamforming that is how to focus the beamforming, the beam in the direction of a particular user correct a system that is beamforming. Let us now extend this paradigm to include to look at beamforming with interference that is what happens when you have a desired user and also an interfering user.

(Refer Slide Time: 00:40)



So, we want to look at beamforming with interference, something that you not looked at so far we have simply considered a single user interference. So, what happens when you have a desired user as well as an interfering user.

(Refer Slide Time: 01:12)



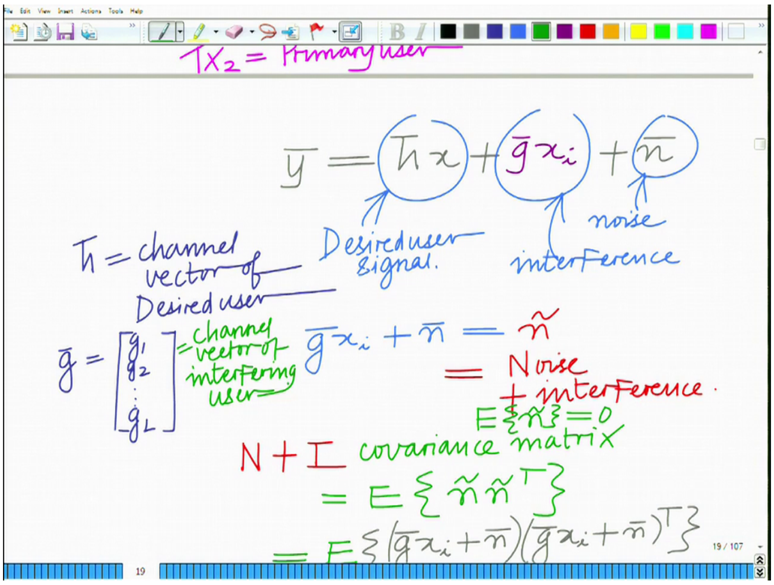
So, you have again as usual you have this multiple antenna array. And now, you have a user to which you want to transmit let us call this, let us or user who is the desired user whose signal you want to receive and you have another user who is an interfering user whose signal you want to reject. So, this is the receiver you want to form a beam in the direction of the desired user; at the same time you want to reject correct, you want to reject the signal from an undesired user. So, this is let us say your interfering user. This is your interfering or this is your desired user actually. So, this is your desired user, and this is your undesired or you could call this as your interfering user.

So, you want to receive the signal or focus your energy to in the direction of the desired user while the same time rejecting correct, rejecting the signal from the undesired or the interfering user. And this can occur in several scenarios. For instance, you have several users, you want to focus on one particular user; or you have for instance of cognitive radio scenario in which case you have the secondary user and then you have the interference from the primary user all right. So, you want to reject the interference from the primary user while at the same time focusing your energy while transmission or receiving the signal from your secondary user.

So, for instance, you can think of this as an interesting application in the evolving paradigm of cognitive radio in which you have T X 1 equals your secondary user. So, you want to receive the signal from the secondary user, but there might be an ongoing transmission of the primary user T X 2 that is undesired user that is at the secondary receiver is the primary user. So, you want to reject at the secondary. So, this R X can be the secondary receiver. Secondary user the receiver or you can also think of this as a secondary base station.

At the secondary base station you want to get the signal correct, you have to beamform such that you receive the signal from the secondary user while rejecting the interference from the primary user, because the primary user is the licensed user hand the primary user has priority for the transmission. So, when there is an ongoing primary transmission, how do you make sure that this does not impact the signal reception the signal quality at the secondary user. So, this has many applications, there are several applications in fact beamforming in the presence of interfering users.

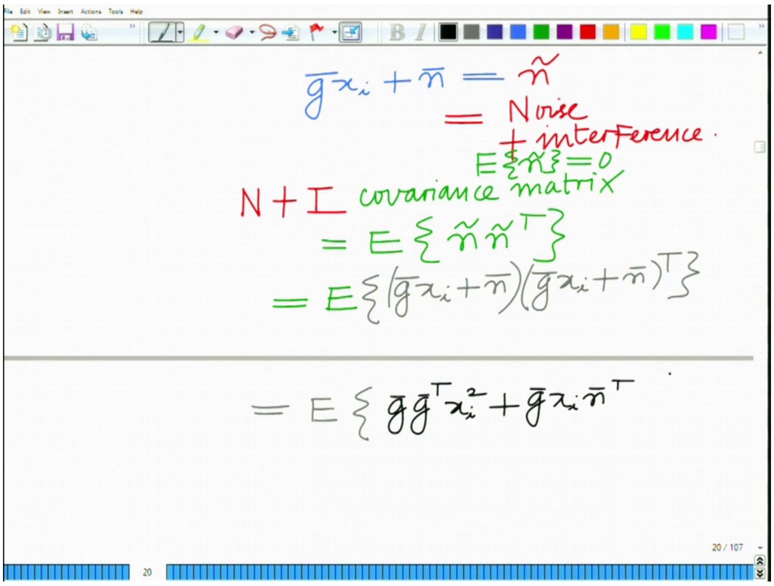
(Refer Slide Time: 04:54)



The received signal or remember the signal model is as follows previously, we had without interference we had y bar equals h bar x plus you can think of this as n bar. So, desired signal plus noise, now you will also have the interference. So, previously we simply had h bar x plus n bar, now you have this additional component which is basically your interference from the interfering user. So, this is as usual this is your noise all right, additive white Gaussian noise; this is your desired signal.

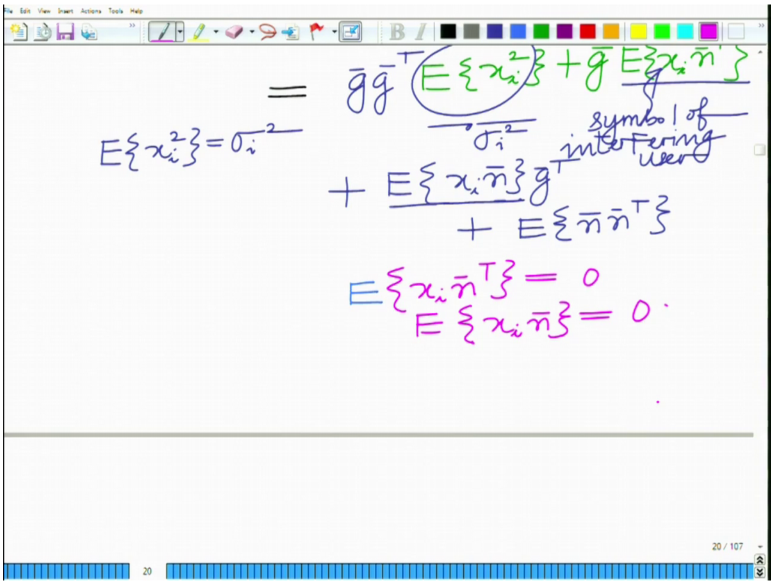
And so therefore, g bar x i plus n is the noise plus interference. You can also call this as n tilde, this is equal to you can call this as n tilde; this is your noise plus you can also think of this as multiuser interference because this interference is arising because of other users. So, this is noise unlike for instance, it is a inter symbol interference, this is your multiuser interference, interference arising from the other users.

(Refer Slide Time: 06:37)



So, you have noise plus interference, and now we want to compute the noise plus interference covariance, the covariance of the noise plus interference. Now, this is simply your expected value of n tilde n tilde transpose, remember this is the definition of the covariance matrix. Assuming that expected and tilde equals 0, this is the typical assumption that is the noise mean we already seen that the noise is zero mean also assume that the interference is zero means or the noise and plus interference will have zero mean. And then the covariance matrix will be expected value of n tilde plus n tilde transpose which is expected value of well that will be g bar x i plus n tilde or g bar x i plus n bar times g bar x i plus n bar transpose which is the expected value of, value you have g bar g bar transpose x i square plus let me write this as follows g bar x i n bar transpose plus x i into n bar into g bar transpose plus n bar n bar transpose.

(Refer Slide Time: 08:00)



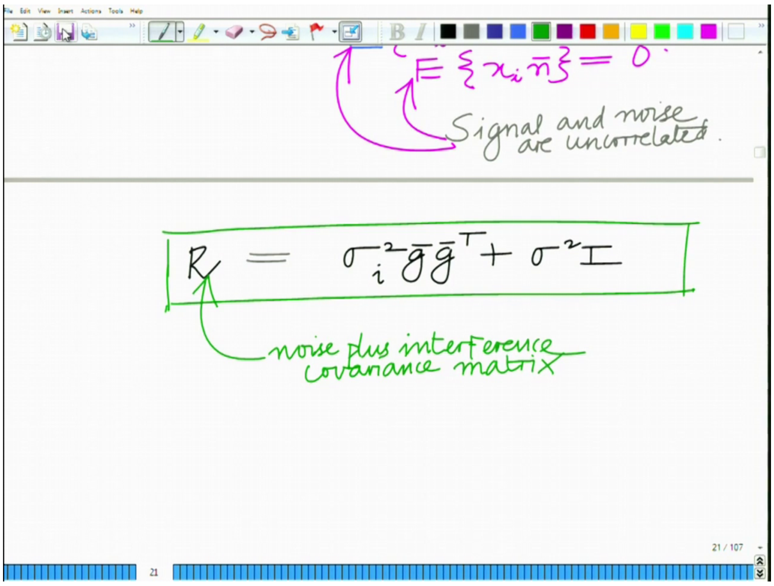
Now, this can be simplified as follows taking the expected value inside this is g bar g bar by the way g bar now remember h bar is the channel vector of the desired user, as per our this is all according to previous notation channel vector of desired user. And similarly g bar which is the L dimensional vector corresponding to L antennas this is the channel vector of the interfering user. This is the channel vector of the interfering user, so that is your g bar.

So, you have g bar g bar transpose times expected value of x i square plus g bar expected value of the symbol of the interfering user that is x i times n bar transpose. Remember x i this is basically your signal or you can think of this as symbol of the interfering symbol of interfering user plus you have expected value of x i into n bar into g bar transpose plus expected value of this is the noise covariance that is what we have previously seen n bar n bar transpose.

Now, what we will do is we will assume the symbol power to be p which means expected value of x i square our p i or sigma i square let me just write this as sigma i square. So, we are assuming that the signal power or the interference power is sigma i square, expect value expected value of x i square is sigma i square. Now, we have this quantity which is interesting, the cross correlation this is expected value of x i n bar transpose expected value of x i into n bar that is it looks at the correlation between the signal and the noise.

Typically the signal and the noise are uncorrelated because the noise arises from the system and the signal from the information that intense is the information that corresponds to the particular either desired user or interfering so user. So, the signal and noise are typically uncorrelated, in fact, they are independent. So, if they are both zero mean then expected value of x i into n bar expected value of x i into n bar transpose both are 0. So, this is the other assumption which is of course, very intuitive, but needless to say, but nevertheless it can be better I think these are worth clarifying this that is expected value of x i n bar equals or these are not equal. In fact, this is a one is a row vector the other is a column vector. So, expected value of x i n bar transpose is 0; expected value of x i n bar equals 0. So, both these quantities are 0.

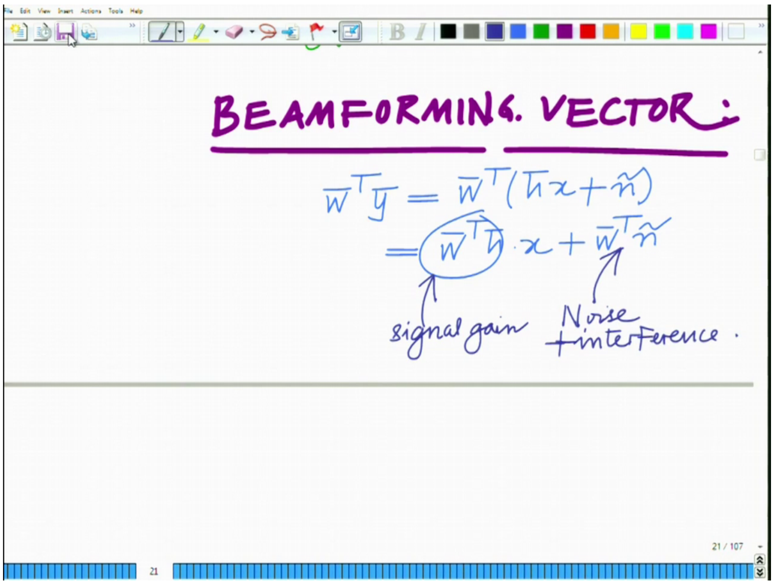
(Refer Slide Time: 11:45)



And the reason is the same because signal and noise are independent. In fact, what we need is simply uncorrelated. And that signal and noise are uncorrelated typically that is what you have. And therefore, what you have is you will have sigma i square. And this we know noise covariance this is simply sigma square times identity matrix, because the noise samples are we have assumed the noise samples are the different antennas to be IID - Independent Identically Distributed.

So, if you look at the covariance that is simply sigma square times identity. So, finally, what you will get is if you call this noise plus interference covariance matrix as R, what you will get as sigma i square times g bar g bar transpose plus sigma square times identity. So, this is your noise plus interference covariance matrix. This is your noise plus interference covariance matrix corresponding to this scenario of multiuser beamforming with interference. Now, again we want to find the beamforming vector W bar. How are we going to find the beam forming vector W bar?

(Refer Slide Time: 13:37)



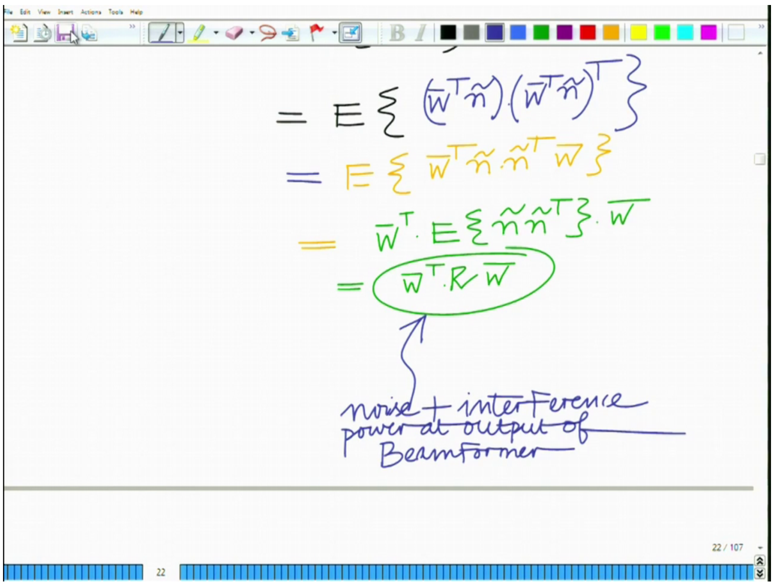
So, our intention is to now perform beam forming and for that we need to have the beam forming vector. How are we going to find the beam forming vector well let us say W bar is the beam forming vector, W bar transpose y bar you are doing beam forming remember it is also known as electronic steering that is you are simply linearly combining the samples of the received signal it is a weighted combination a weighted linear combination of the samples of the received signal that is W bar transpose y bar. W bar is the beam forming vector, W bar transpose substituting y bar this is y bar times h x plus.

Now, n tilde with n tilde is the noise plus interference noise plus interference, this is well again this is W bar transpose h into x plus W bar transpose n tilde. This is the signal, this is the signal gain in fact W bar transpose h bar. And this is the noise plus interference now not simply the noise. So, this is the noise plus interference. Now, what we want to do is we want to minimize the effect of this noise plus interference at the output. So, we have to calculate its power.

So, while setting the gain in the direction of the signal, now this is something that you have to pay attention to the reason being the follows. If you minimize, simply minimize the noise plus interference without consider this optimization problem, it is simply minimize the noise plus interference or the noise without paying attention to the signal all right. So, all you want to do is minimize W bar transpose n tilde, then the optimal value of W tilde or W bar beam forming vector will simply be 0, because if you use beam forming vector to be set it to be 0, then the output noise is 0. But the problem with that is the output signal is also 0 all right, and therefore, and you cannot have that all right.

So, therefore, you have to minimize the noise or noise plus interference while restricting it constraining it in such a way that the signal is not affected, that the signal gain is still unity all right, so that is the important aspect to pay attention here.

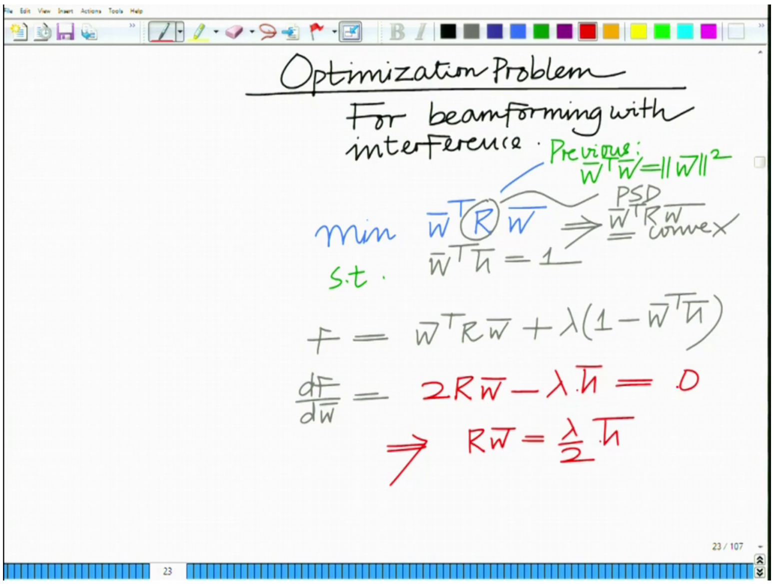
(Refer Slide Time: 16:22)



So, want to minimize the noise plus interference what is the noise plus interference power that is W bar transpose n tilde will come calculate this in a compact fashion expected W. Now, this is a scalar quantity. So, I can do all kinds of manipulations that is W bar transpose n tilde times itself scalar quantity is scalar quantity transpose. So, I am going to simply write it as W bar transpose n tilde transpose which is nothing but W bar transpose n tilde square. Now, this is expected value of W bar transpose n tilde n tilde transpose into W bar.

And now you have something interesting if you take the expected value inside you have W bar transpose expected value of n tilde n tilde transpose W bar which is W bar transpose expected value of n tilde n tilde, this is nothing but the noise plus interference covariance. So, this is W bar transpose R expected value of n tilde n tilde transpose this is the noise plus interference. So, this is the net noise plus interference power at the output of the beam former. So, this is your noise plus interference power at the output, noise plus interference power at the output of the beam former. And now what is our optimization, now again we want to have remember signal gain W bar transpose h bar this has to be 1. So, signal gain has to be unity.

(Refer Slide Time: 18:22)

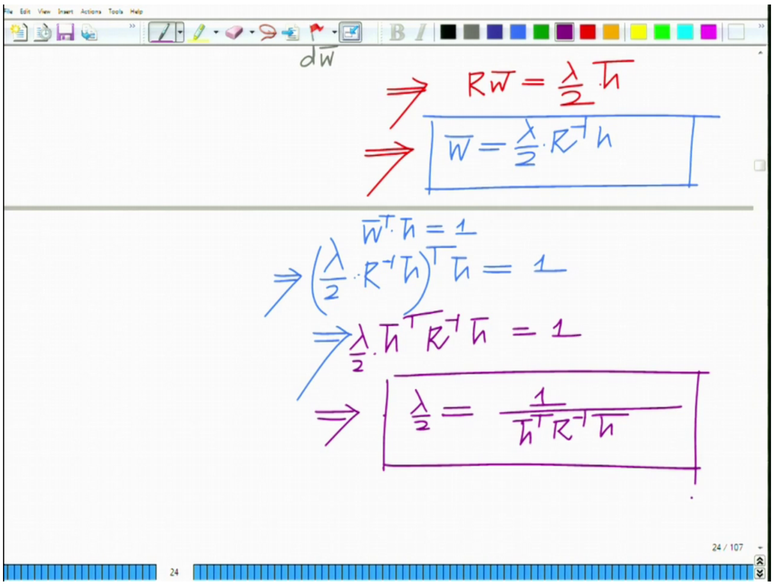


So, what is our modified optimization problem for this beam forming with interference. So, the optimization problem for beam forming with interference, so the optimization problem for beamforming with interference is minimize that is what we have seen minimize the noise plus interference power which is what we have calculated W bar transpose R W bar. Previously this was simply W bar transpose W bar if you remember. Previously this was simply W bar transpose W bar equals norm W bar square with the noise when you simply add noise.

Now, you have the noise plus interference therefore, is W bar transpose R W bar where R is the noise plus interference covariance that is the only difference. Subject to the constraint, what is your constraint? Constraint is well W bar transpose h bar equals 1 gain in the direction of the signal equals unity. All you have to do is now form the Lagrangian F which is W bar transpose R W bar. Now, again remember the covariance matrix is positive semi definite this is a positive semi definite matrix implies.

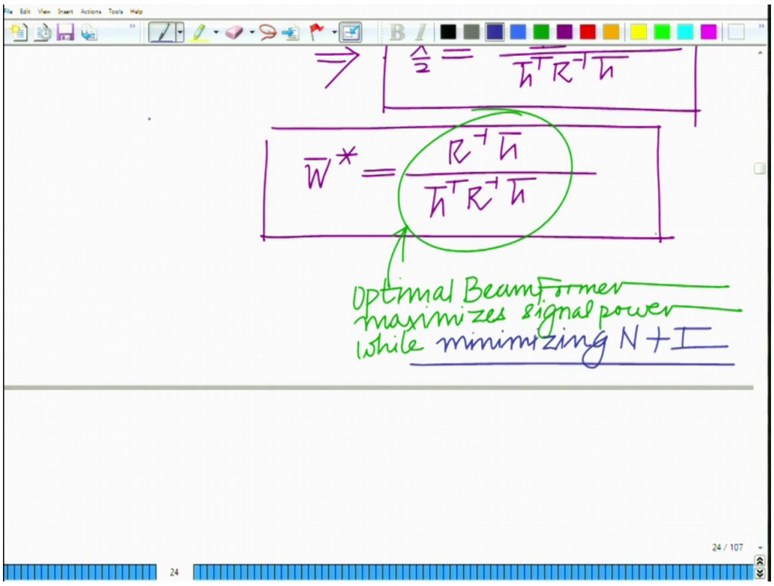
Now, that is always important to check implies W bar transpose W bar is convex, so that is an important thing. So, this is a convex optimization problem because the objective function is convex that is something that you have to verify at each and every stage that you are solving indeed solving the right kind of problem. So, this is W bar transpose R W bar and then usually have you; as usual you have the Lagrange multiplier 1 minus W bar transpose h bar 1 minus W bar transpose h bar. And now differentiating this W bar transpose R W bar because R is symmetric matrix this is simply twice R W bar minus lambda derivative one is 0, W bar transpose h bar derivative is h bar set it equal to 0 which implies the optimal beam former is lambda by 2.

(Refer Slide Time: 21:20)



So, what you have is R W bar equals lambda by 2 h bar which implies that W bar equals lambda by 2 R inverse lambda by 2 R inverse h bar. So, this is lambda by 2 R inverse h bar. And therefore, now, we have to find the Lagrange multiplier lambda, and how do we find that we simply use the constraint. So, lambda by 2, so we have W bar transpose h bar equals 1, this implies lambda by 2 R inverse h bar transpose times h bar equals 1, this implies that h bar transpose lambda by 2 h bar transpose R inverse into h bar equals 1, this implies lambda by 2 equals 1 over h bar transpose R inverse h. So, we have found out the value of lambda by 2, lambda by 2 is 1 over h bar transpose R inverse h substitute this value of lambda by 2 above.

(Refer Slide Time: 22:44)



And therefore, your optimal beam former you can call that as W star equals lambda by 2 1 over h bar transpose R inverse h bar times R inverse h bar. So, this is your optimal beam former with interference. Now, you have also incorporated. So, this is the optimal beam former which maximizes signal in the desired direction or you can say this maximizes the signal power while minimizing noise plus interference and that is the important aspect; while minimizing noise plus interference that is important aspect.

And therefore, you can see with the slight twist slightly modifying the objective function how we can change how you can make it even more comprehensive alright to tackle the tackle a much more general problem. Previously we only had the noise power, now you had a noise, now you have the noise plus interference where the interference can arise for a variety of reasons. This can be different users in a cellular scenario. The interference can also be used be due to a harmful user or and a or a malicious user rules trying to interfere with the base station or it can be in a cognitive radius scenario where there is an ongoing interfere; ongoing transmission of the primary user, and therefore, this causes interference at the secondary user interference. So, this has several practical applications in that sense all right. So, we will stop here, and continue in the subsequent modules.

Thank you very much.