Beamforming in Multi-antenna Wireless Communication
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Lecture - 34
Problems on Grassed Waterways

Hello welcome to another module in this Massive Open Online Course. So, if so far we have been looking at the mathematical preliminaries or the concepts that form the foundation of the optimization framework all right. Namely convex sets we have looked at convex functions and now let us start looking at the practical applications of these concepts in the form of optimization problems that arise in practice and that can be solved using the framework of convex optimization.

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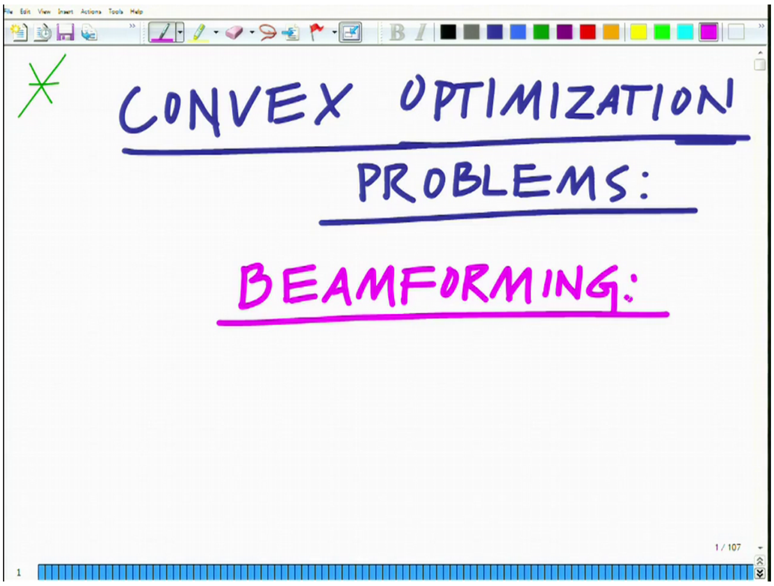

So, what we want to start looking at is convex optimization problems, various optimization problems that have practical application convex optimization. And now what happens in a convex optimization problem is well let us look at it through an example the first take the example that I want to look at through is an excellent and a very simple example. But yet very practically relevant and that is the example of Beamforming, which is one of the most important components or you can say one of the most important techniques in a modern day wireless communication system.

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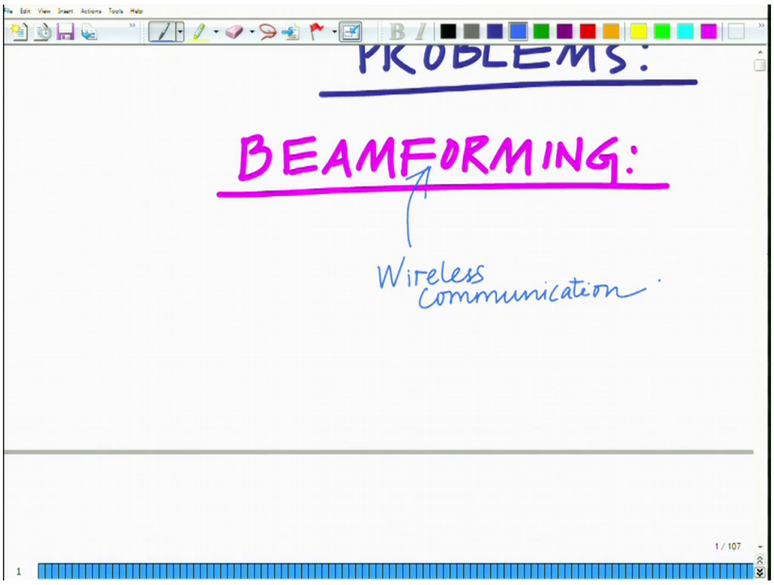

So, this has a lot of applications in the context of wireless communication, you can think of 3g, 4g wireless communication systems and so on wherever you have multiple antennas and the Beamforming problem is the following that is at a very high level if you have a multiple antenna system.

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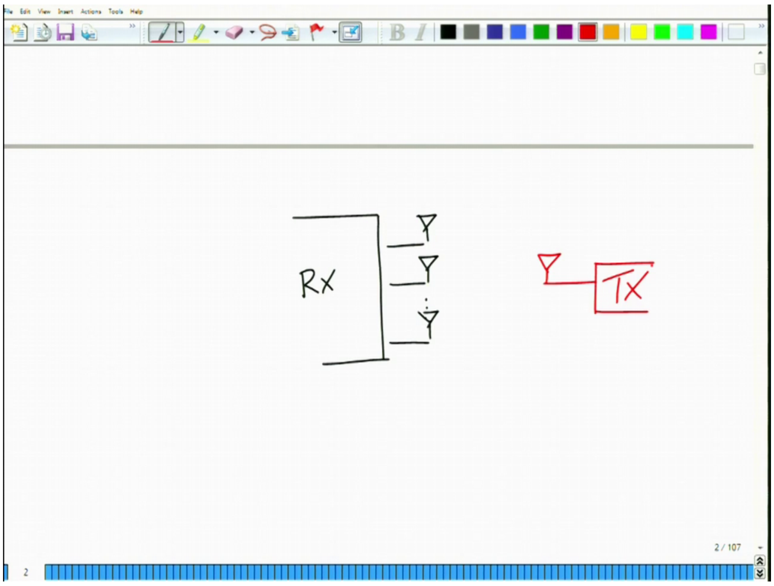

Let us say you have multiple antenna system each of these represents an antenna you have a transmitter or for that matter receiver let us say and beam forming can be done both at the transmitter and receiver and what do you want to do and what you want to do is, so this is let us say user 1 transmitter 1 and this is your transmitter 2.

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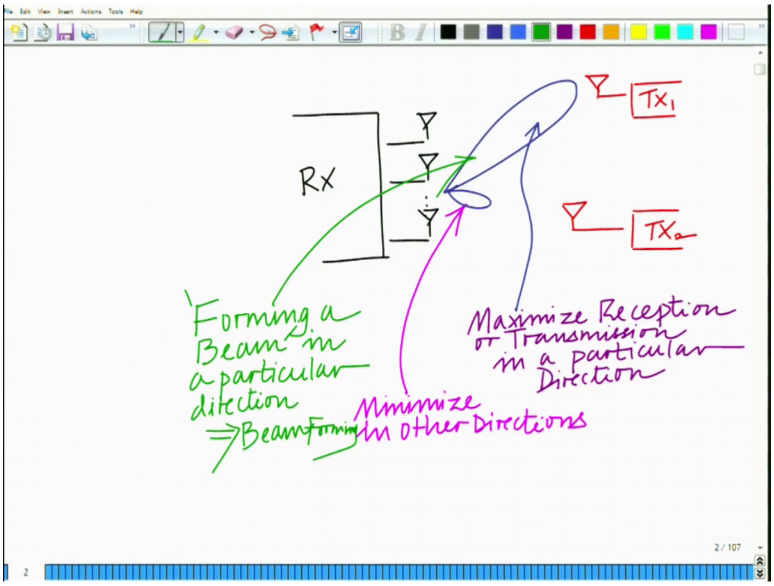

What you want to do in beam forming is essentially you want to form a beam or maximize the signal to noise power ratio in a particular direction that is let us say user one is the desired user. While minimizing the power so, you want to maximize the reception or transmission it is symmetric. Maximize reception or transmission; maximize the reception or transmission in a particular direction.

Maximize in a particular direction and minimize in the other direction the undesired is let us say user twos and minimize in other directions and therefore, you are forming a beam in a particular direction and this is what is termed as beam forming. So, what you are doing is you are forming a beam in a particular direction and this implies and this is what is termed that this forming this beam in this particular direction.

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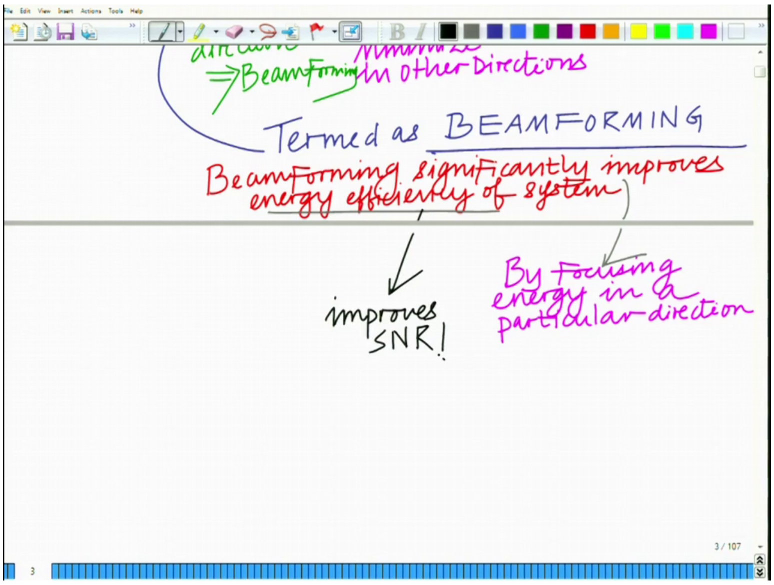

This process of formation of this beam this is termed as beam forming and as you can see this is unique to a setup with multiple antenna. So, you have multiple antennas what that helps you it helps you to form a signal beam in the direction of s of a desired user, while suppressing the signal in the direction of the interfering user or the other unintended users. And this significantly improves the energy efficiency why because, you are transmitting now energy only in a particular direction which was previously transmitted spread out in a diffused fashion over all directions.

So, by focusing the energy you are significantly improving the energy efficiency and you are significantly improving the SNR of the system. So, Beamforming the main idea behind Beamforming is that Beamforming significantly improves the energy efficiency of the system by focusing the energy in a particular direction and not transmitting in all directions by focusing energy in a by forming a beam. You are basically forming a beam by focusing the energy in a energy so it simply improves the SNR.

So, Beamforming improves SNR, ultimately Beamforming the main aim is it improves the SNR and therefore it is very important already there is a not as SNR improves the performance of communication efforts alright. So, Beamforming in that sense it is very important communication it is important even in radar right that is another interesting application where Beamforming is heavily used.

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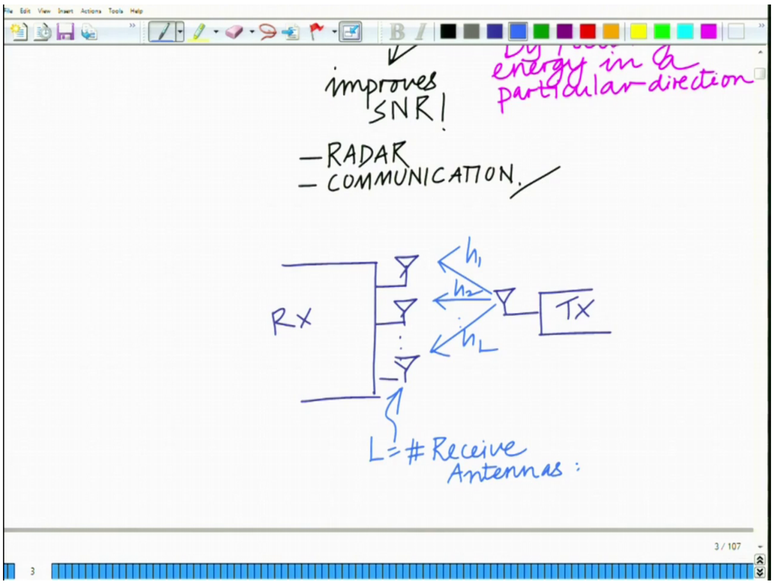

So, it is communication systems as well as radar. So, these are, wherever you have signal transmission wireless signal transmission and reception one can use beam form. So, how do we build a model for this Beamforming communication system that is what we want to develop and we have already seen this to some extent and the model is very simple, which we probably have seen in earlier modules also. That is you have the receiver let us say and remember this is for the receiver you can do something exactly identical for the transmitter.

So you have single transmit antenna multiple receive antennas this is also known as a single input, multiple output system or this is known as receive antenna diversity and so on. So, you have L receive antennas, L equals the number of receive antenna for simplicity number of transmit antennas equal to 1, otherwise you can generalize this.

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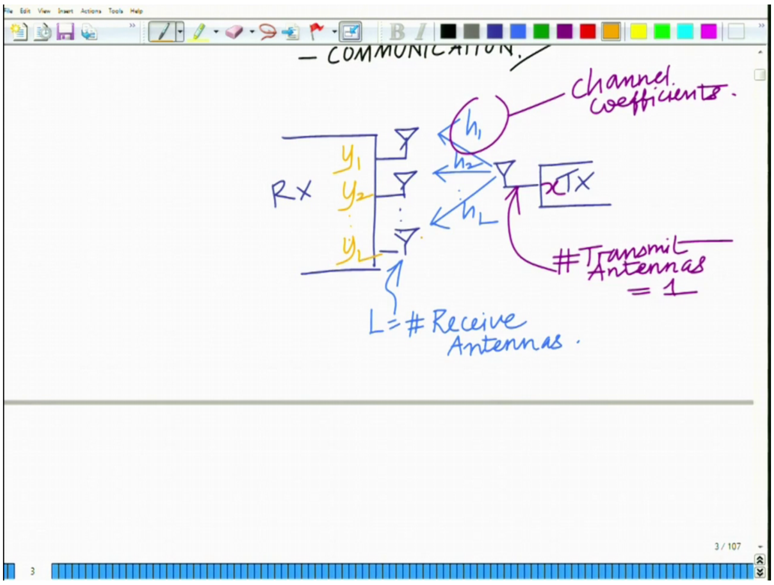

For simplicity h 1 h 2 h L these are the channel coefficients, fading channel coefficients, remember in a wireless communication system the channel is fading in nature because of the multipath scattering. Now, the transmitted symbol is x from the single antenna the received symbols at the different antennas are y 1 y 2 y L at the L antennas correct.

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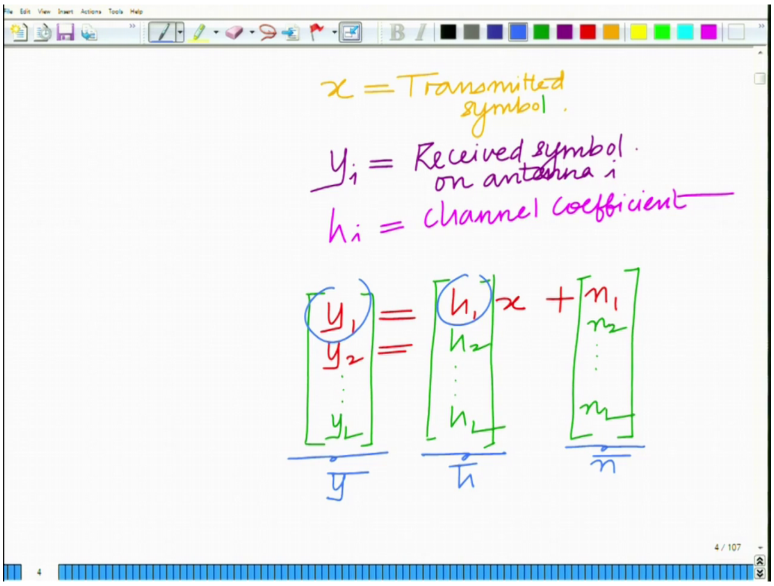

So, you have x equals transmitted symbol; x is the transmitted symbol; x is the transmitted symbol y i equals received symbol on antenna i and h i is the channel coefficient corresponding to antenna i. You can also say this is the; you can also say this is the fading channel coefficient and therefore I can express this system model as y 1 equals h 1 x transmitted fading channel coefficient times transmitted symbol x plus n 1 y 2 equals.

In fact, I can write this as a vector y 1 y 2 y L equals the vector h 1 h 2 h L plus n 1 n 2 up to n L, what this means is basically y 1 equals h 1 times x plus n 1 y 2 received similar and antenna 2 equals h 2 times x plus n 2 and so on. So, you have the l dimensional received vector y bar, which is equal to the transmit channel vector h bar times x the transmitted symbol plus n bar, these are L dimensional vectors y bar is the received vector.

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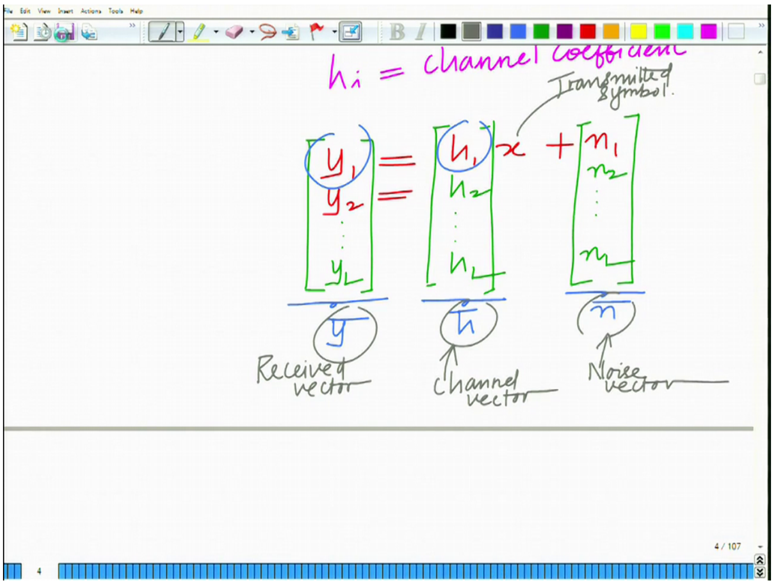

H bar is a channel vector n bar is the noise vector x is the transmitted symbol. Y bar is the received vector, h bar equals the channel vector, x bar is the transmitted symbol, n bar is the; n bar is the noise vector.

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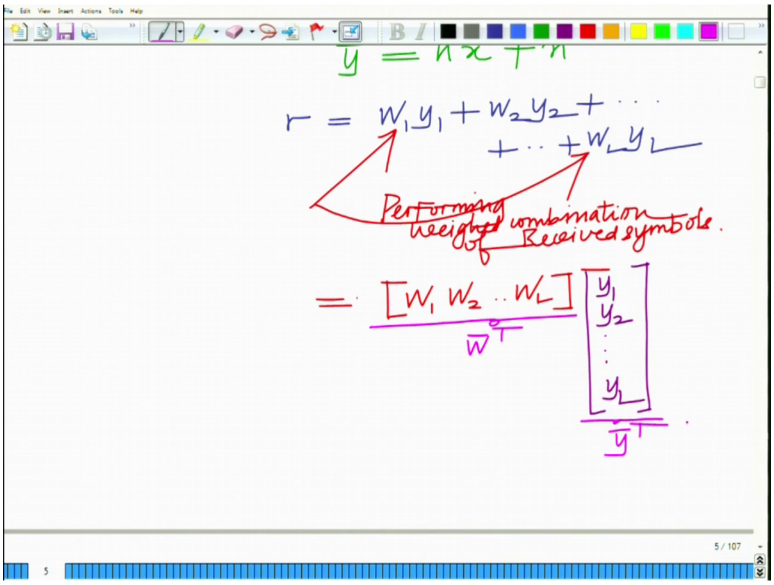

So, our system model is y bar equals h times x bar plus n bar. Now what do we want to do we want to, now remember we have the receiver this y 1 y 2 y L. What you want to do is we want to beam form or we want to combine these symbols and we want to perform a weighted combination. So, what we want to do is we want to combine these received symbol w 1 y 1 times w 2 y 2 plus so on. So, what are we doing we are performing a weighted combination we are performing a weighted combination of the received symbols. I can also write this as w 1 w 2 w 1 the row vector times y 1 y 2 y L the column vector. This is basically w bar transpose this is basically y bar the received vector the same thing that we have been seen before.

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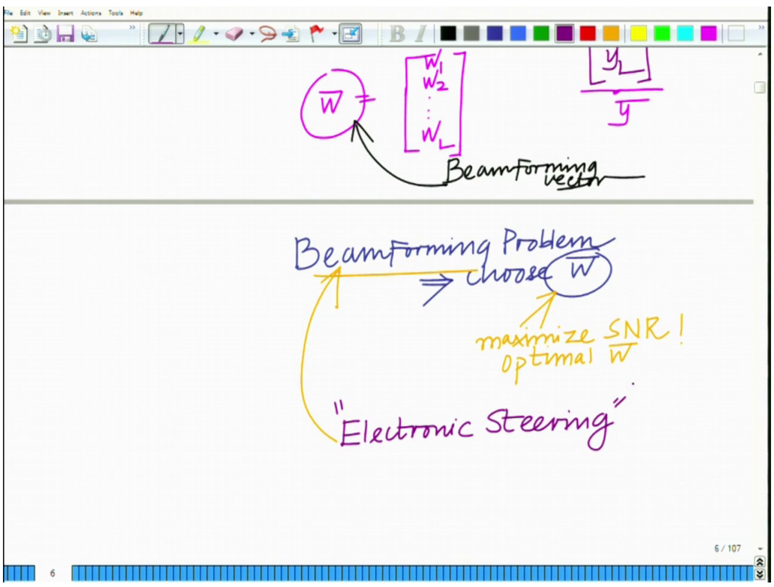

Now, what is this vector w bar? W bar is the vector of weights w 1 w 2 up to, you can think of this as a vector of weights this vector w bar is basically what is known as the beam forming vector. This process of multiplying the various samples receive samples y 1 y 2 y L with these weights and then combining this process is nothing but beam forming. What you are doing is by choosing the weights you are focusing the beam in a particular direction while suppressing the interference or not focusing the beam in other direction.

So, this vector w bar this is known as the; this is known as the beam forming vector and the beam forming problem is basically to choose this vector w bar. The beam forming problem is basically to choose; Beamforming problem is to choose this vector w bar. How do you choose the vector w bar that we have already said to maximize the SNR. The Beamforming problem is to choose find this vector, optimal vector and that is known as the optimum the optimization problem to find the optimal vector w bar.

That is the optimization problem to find the optimal vector w bar and this also what you are doing is basically by choosing these weights, by simply choosing this weights you are steering the beam in a particular direction, this is known as electronic steering. In literature this is known as electronic steering where you are simply electronically doing this, by choosing the this is in opposite or contrast or you can say this is not equal to mechanical or manual steering.

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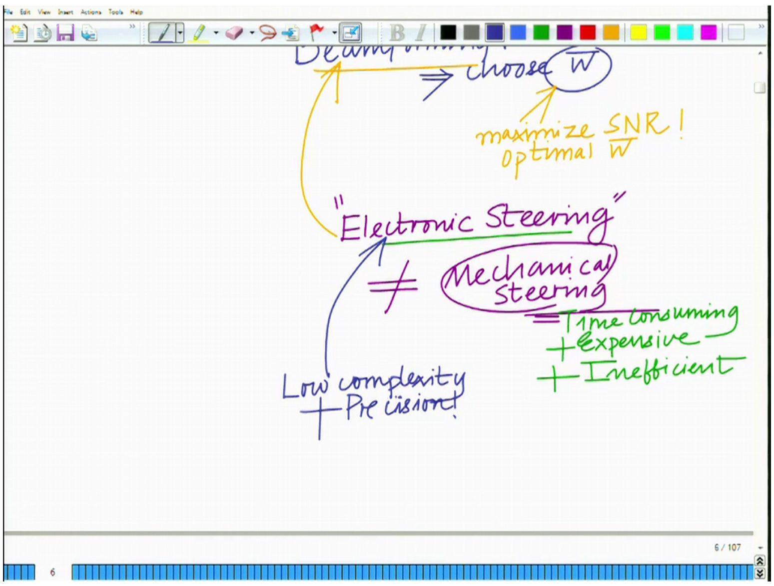

Now, what happens in mechanical steering? Mechanical steering is basically where you rotate the array or you tilt the array in a particular direction to change the direction of the beam. This was what was that conventionally and as you can see this mechanical steering is something that is extremely time consuming and also energy inefficient because it to tilt to the array and tilt it with precision is both something that is time consuming and it is also energy inefficient. So, mechanical steering this is basically time consuming plus expensive not to forget the cost plus inefficient, its energy it requires energy and of course it is time inefficient it is also energy inefficient. As against electronic steering the biggest advantage is it is easy is low complexity and precise because you do it in the digital domain plus precision.

So, not to forget that is the most important thing; for instance you want to tilt the array in direction 23.65 degrees mechanically that would require a lot of skill, but electronically that can be done with very good precision. So, that is the advantage of electronics steering and now how do we do that by simply adjusting the weights of the Beamformer by choosing all we are doing is choosing w bar that is what we are doing, by choosing w bar, by simply choosing the weights or simply adapting.

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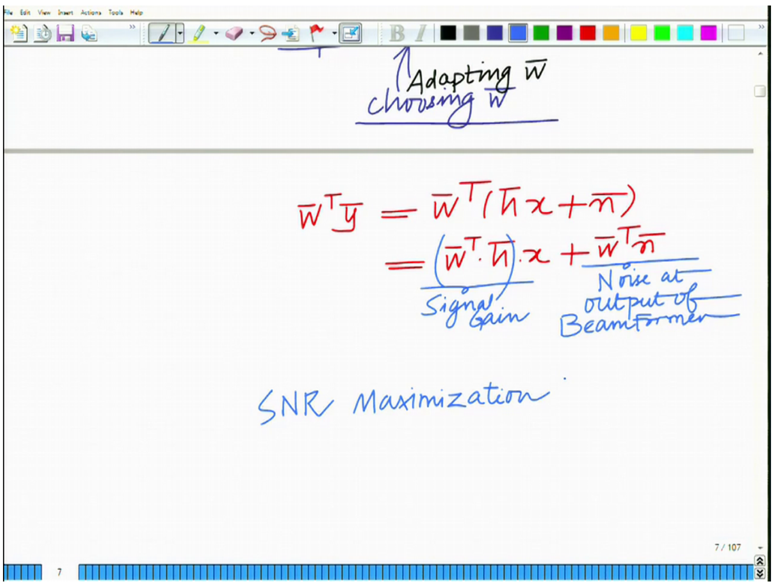

In fact you can also say this is adaptive, an important way in which this is done is by adaptive, adaptively changing w bar in a time varying scenario. So, this has a lot of applications including adaptive signal processing and the process is very simple. So, we perform y bar transpose w bar transpose y bar this is your beam former, substituting the expression for y bar, you have h bar times x plus n bar which is w bar times h into x bar plus w bar into n bar. Now if you can look at this component, now x is the transmitter symbol this component is basically the signal gain, w bar transpose h bar is the signal gain and w bar transpose n bar is the noise at the output of Beamforming, noise at output of; this is the noise at the output of the beam former. Now, what we want to do? We want to maximize the signal to noise power ratio.

Now, there are 2 ways of doing it one is either keep the signal gain constant, minimize noise power or keep the noise power constant maximize the signal here all right. We will choose the first approach that is keep the signal gain constant. Constant gain for the signal minimize the noise power that will result in maximizing signal to noise power ratio. So, SNR maximization can be achieved, that can be achieved as follows, implies what can we do? We can keep you can keep signal gain or signal power constant plus minimize the noise power.

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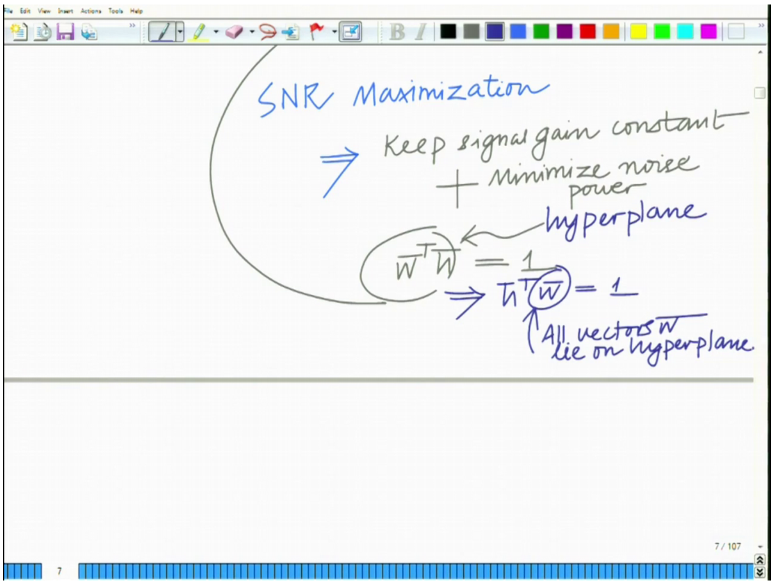

Now, what whatever signal gain is w bar transpose h bar you can see from here this is the signal gate. So, what signal gain constant means we can sit w bar transpose h bar equals 1 and this we have already seen this is your affine constraint or this is basically represents a hyper plane. So, there we have a convex function, so this represents hyperbola. So, all the vectors w bar, you know w bar transpose h bar equals 1 or h bar transpose w bar equals 1. So, you can also write this as h bar transpose w bar equals so on, all vectors w bar lie on the hyper plane.

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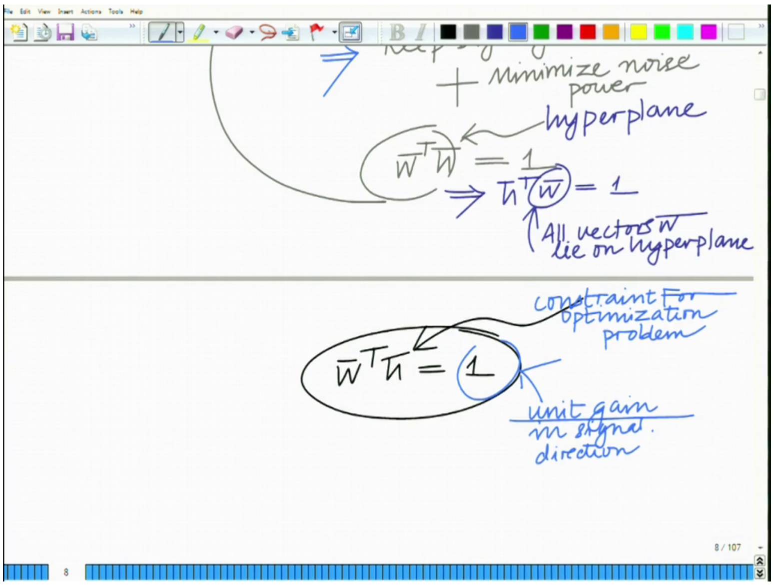

So, we can sit w bar transpose h bar equals 1 this is known as the constraint in the optimization problem, this is the constraint for the optimization problem. So, this is equal to 1 this implies unit gain in signal direction, this maintains unit gain in signal direction all right.

This is the constraint, this is the constraint for our optimization problem and now what we have to formulate is here to formulate the SNR maximization all right, which will form the objective function for our optimization. From then one can from the optimization problem and one can solve the optimized resulting optimization problem which we are going to do in the next module.

Thank you very much.