Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 08
Example problems in Signals and Systems – Energy, Properties of Impulse, RL Circuit

Hello, welcome to another module in this massive open online course. So, let us continue doing examples to illustrate the various principles of signals and systems and particularly with respect to the properties and classification of signals and systems.

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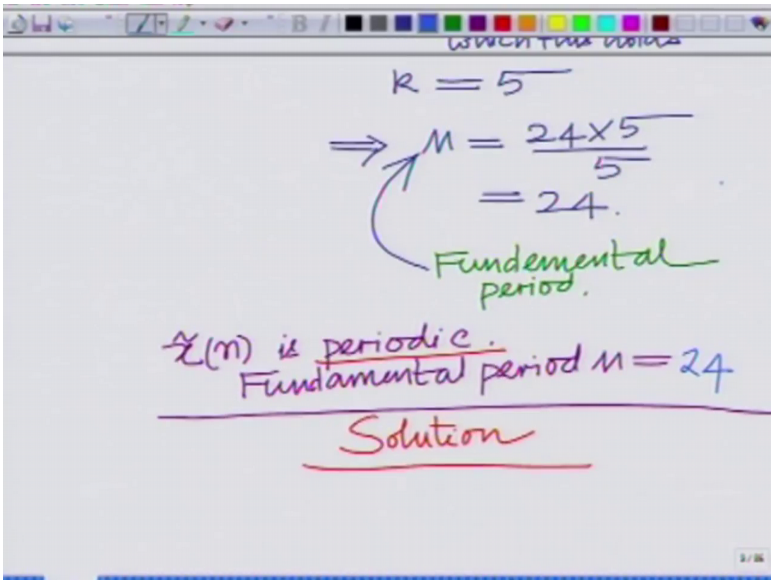

Let me just correct a minor typographical error that is M should be equal to 24; I think as we heard. So, this is periodic and the fundamental period is M equals 24 that was for the previous problem.

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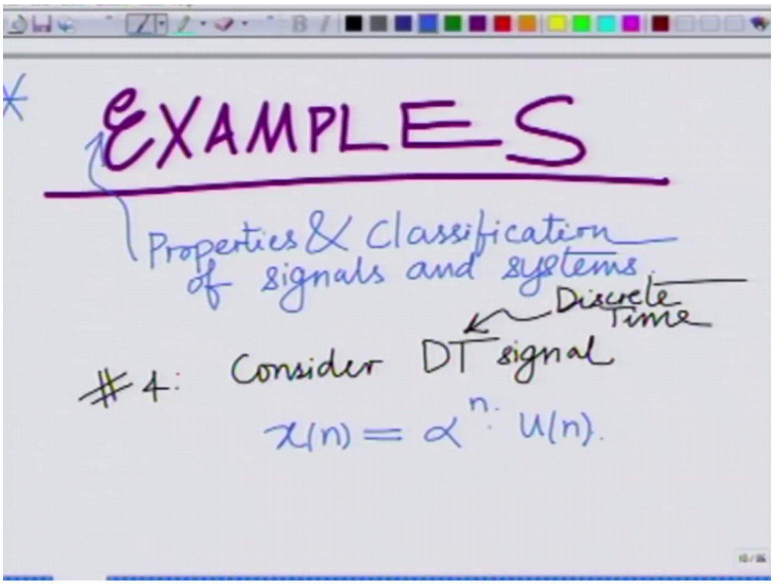

And now today let us continue our discussion of various other examples for signals and systems based on the concepts that we have covered so far; I think you well remember that is with respect to; I think the basic properties and classification; various properties and classification of signals and systems.

So, let us now consider the concept of an energy signal. So, let us consider a discrete time signal write DT for discrete time signal that is convenient. So, this is a discrete time signal x n equals alpha n u n. Now, the question is; is this an energy signal? If so what is this energy?

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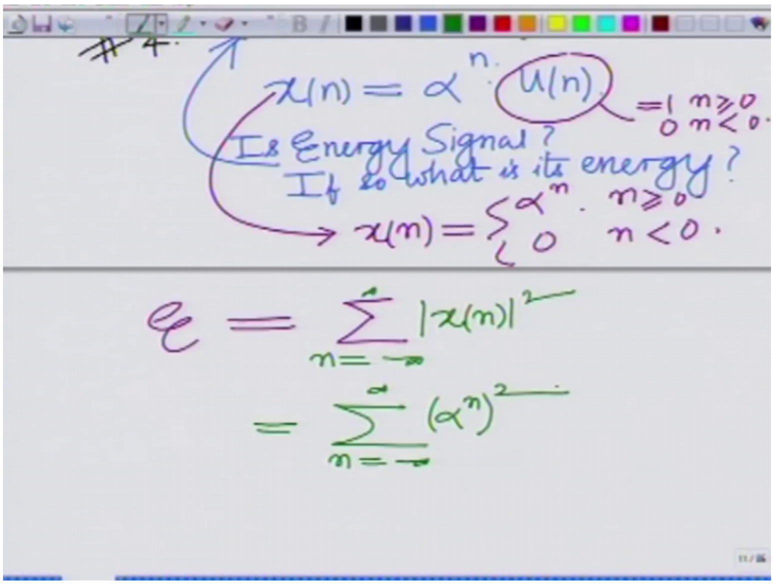

Now, we have seen that the energy of a discrete time signal; so the question is alpha n into u n. Now mind you u n is the unit step function; which is equal to 1 if n is greater than or equal to 0 and equal to 0 otherwise. So, this is basically equal to 1 for n greater than or equal to 0; 0 or n less than 0, implies basically x n equals alpha to the power of n for n greater than equal to 0; 0 for n less than 0.

The energy of a discrete time signal is summation n equal to minus infinity to infinity; magnitude x n square which is summation n equal to minus infinity to infinity alpha to the power of n whole square.

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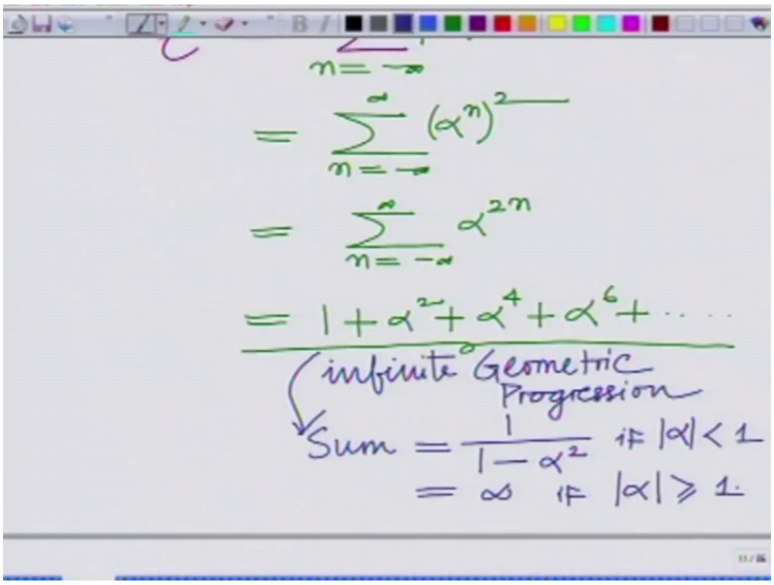

Which is summation n equals minus infinity to infinity alpha to the power of 2 n, which is basically 1 plus alpha square plus alpha to the power of 4 plus alpha power 6 plus so on. And recall that this is basically an infinite geometric progression with the ratio or the constant power with the incremental factor of the geometric progression given as alpha square.

So, this is an infinite geometric progression correct and therefore, the sum equals well sum equals 1 over 1 minus alpha square, if magnitude of alpha is less than 1 that is important. And this is equal to infinity or the diverges; if magnitude of alpha greater than or equal to 1.

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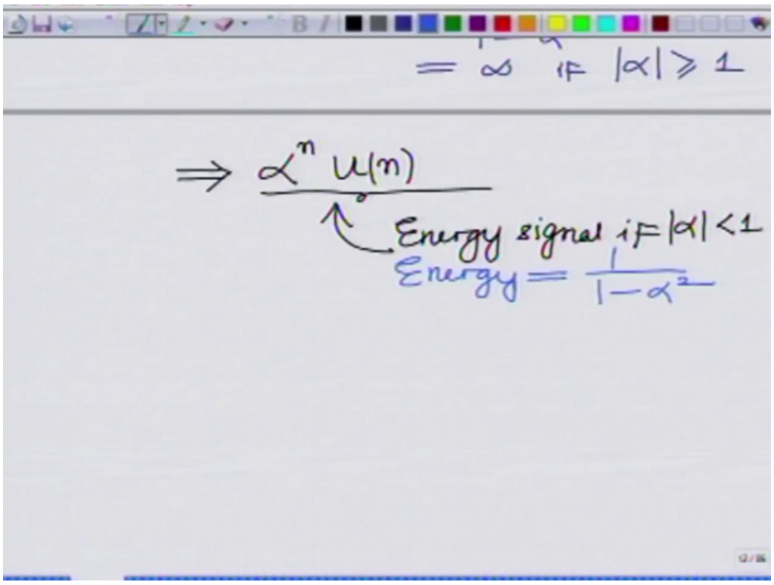

So, implies this is an energy signal equals n or alpha to the power of and u n remember that is equally important. This is an energy signal; if n if magnitude alpha is less than 1 and in that case the energy is equal to 1 over 1 minus alpha. So, this is an energy signal only if magnitude alpha is strictly less than 1 and in that case its energy is 1 over 1 minus alpha squared; otherwise it is not an energy signal, it is energy diverges; it means it does not have a finite energy; energy is infinite.

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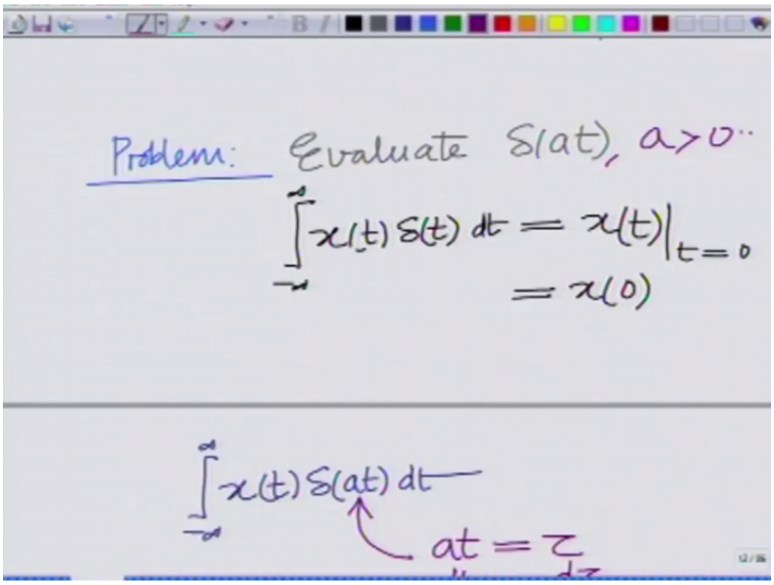

Let us now come to another problem and the problem is to evaluate the function delta a t; you know what is delta t; remember delta t is defined such that x of t, delta t for any function is equal to x of t evaluated at t equal to 0, this is equal to x of 0.

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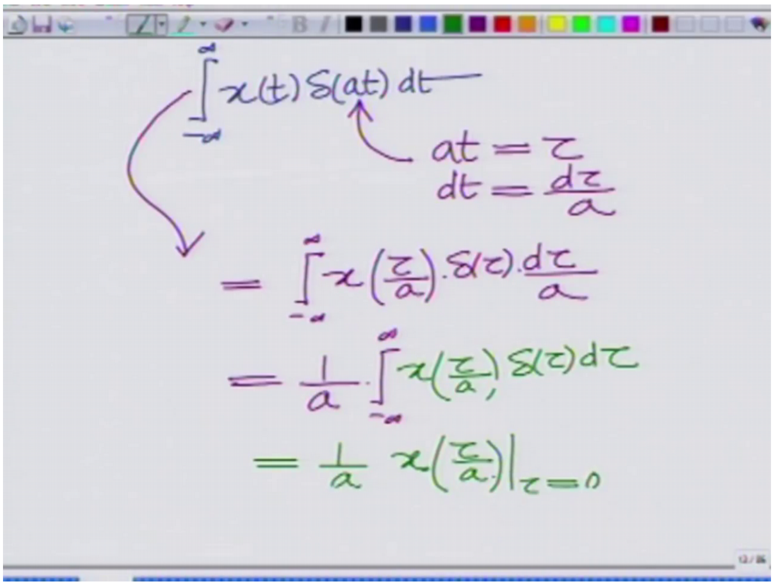

Now, similarly let us look at integral minus infinity to infinity; for any function x of t times delta a t times dt. Now; obviously, if you look at this I can substitute at equals tau which means dt equals d tau by a. For simplicity let us also consider a greater than 0, you can also clearly evaluate what happens when a is less than 0.

For a is greater than 0; we are considering a strictly positive a; this integral therefore, becomes integral minus infinity to infinity, t is equal to tau divided by a; delta at is delta tau, d tau divided by a the limit still remain minus infinity to infinity because dividing infinity by a positive constant a still gives infinity same with minus infinity.

So, limits become minus infinity by a; to because tau is a times t. So, limits become minus infinity times a to infinity times a, which are again basically minus infinity to infinity. So, limits remain unchanged integral becomes minus infinity to infinity x of tau divided by a; tau over a delta tau; d tau over a. Which is now very simple, if you look at this; this is 1 over a, x of tau over a; delta tau d tau, which is 1 over a; x of tau over a.

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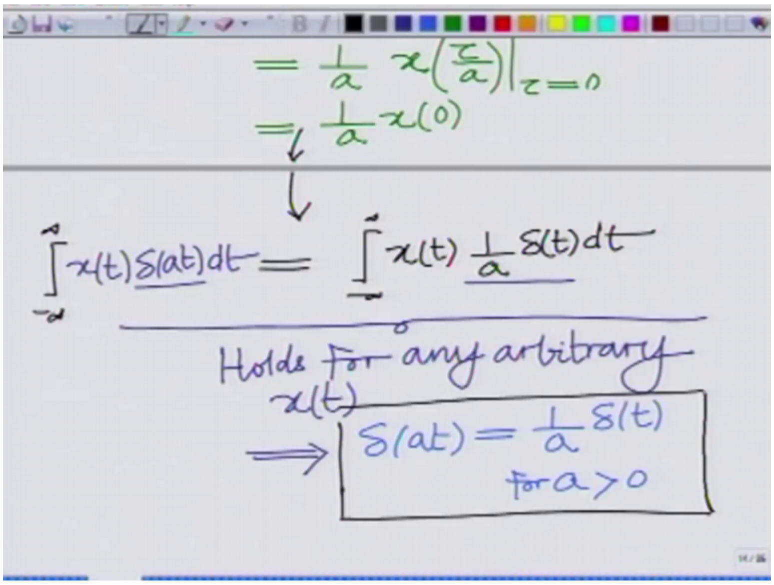

Now, evaluated at tau equal to 0; which is equal to 1 over a, x of 0, but recall that 1 over a; x of 0 this is also equal to simply integral minus infinity to infinity; well x of t, delta t over a or 1 over a times; delta t times dt. This is basically equal to your minus; minus infinity to infinity well x of t; d of t or delta of at holds for all x of t, this equality holds for imagine this is for any arbitrary x of t.

This only implies that this quantity must be equal to this quantity; this implies delta a t equals 1 over equals; 1 over delta tau for any a greater than 0, so that is basically your solution. So, delta a t equals 1 over a; delta tau for any a greater than 0.

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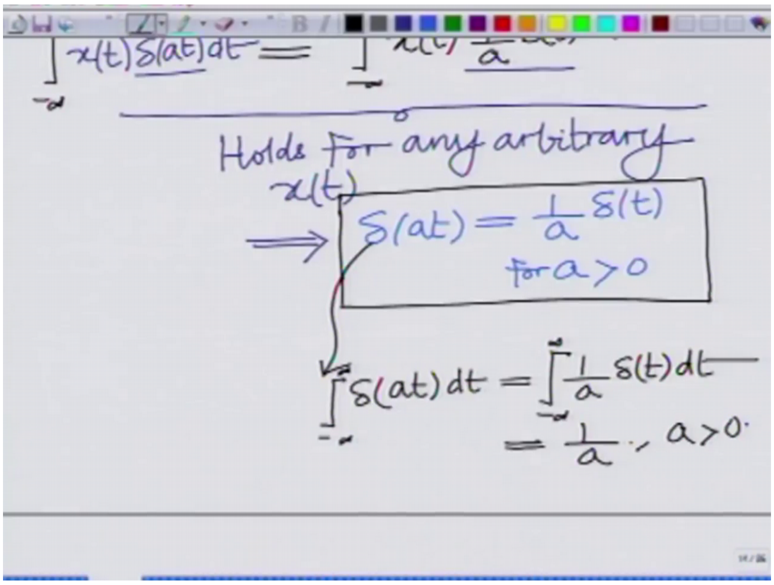

So, this holds for any a greater than 0 and therefore, naturally this implies that infinity minus infinity to infinity delta at; at, dt is equal to minus 1 over infinity 1 over a delta t; dt equals basically 1 over a; for a greater than 0, so that can be immediately shown. So, delta at; if a is greater than 0 is simply 1 over a delta of t, that is the impulse function.

Let us proceed on to the next example which is even more interesting.

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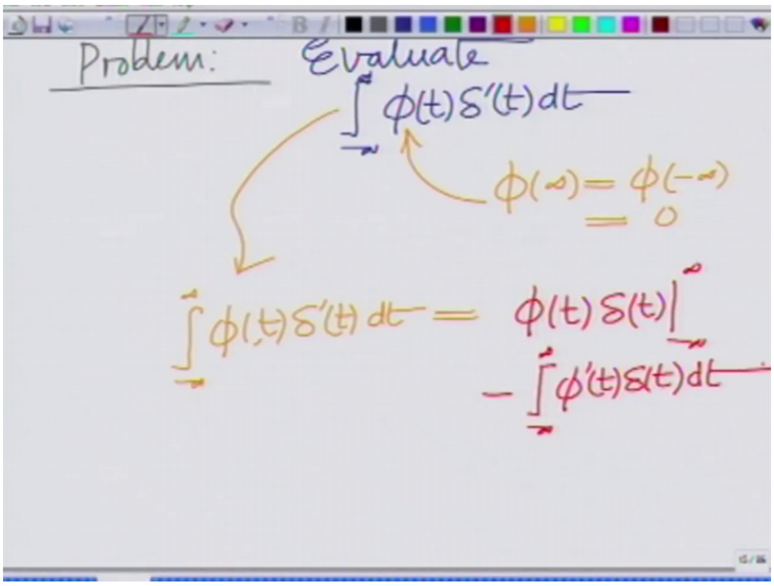

Let us look at just writing this as problems because I am not numbering; them I am just going to write this as problem to indicate that this is a fresh problem. So, we want to evaluate integral by t delta prime dt; where phi t is any function such that, let us consider phi t is any function such that phi of infinity or basically limit t tends to infinity phi of t equals phi of minus infinity equals 0.

Or in general this can be any constant that is phi of t does not blow up to infinity or does not increase to either infinity or minus infinity as t tends to either infinity or minus infinity. Now, if you look at this; what I can do here is I can evaluate this by parts. So, what I will have; integral phi of t delta prime t dt, this is equal to well integrating by parts this is equal to well integrate delta t for delta prime t.

So, that gives me a phi of t delta of t between the limits minus infinity to infinity; minus integral minus infinity to infinity differentiate phi prime of t dt phi prime of t that is derivative of phi of t into delta t dt. Now this is equal to well delta t at t equal to infinity is 0; we have assumed phi of infinity is 0. So, this is phi of t delta t evaluated at infinity is 0.

Similarly, phi of t delta t evaluated minus infinity equal to 0. So, this quantity is basically equal to 0 minus of course, minus infinity to infinity phi prime of t delta t d t.

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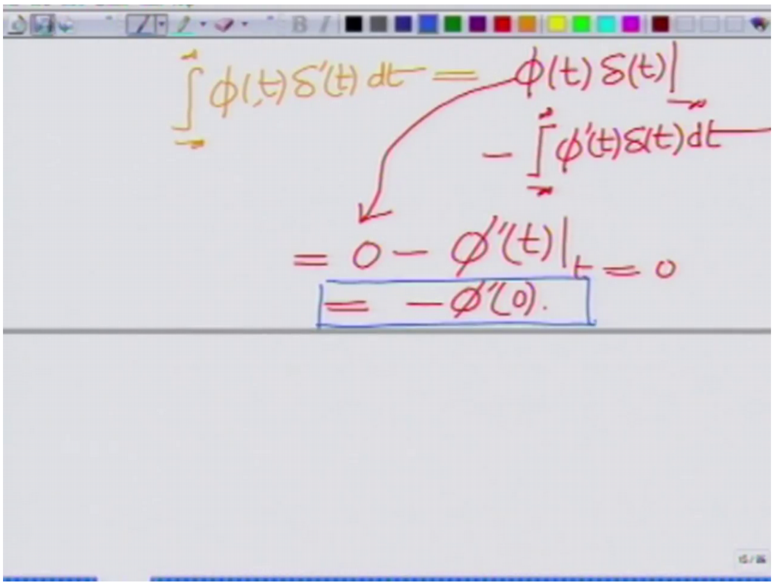

We know this quantity this quantity is nothing, but phi prime of t evaluated at t equal to 0. So, this is minus phi prime of 0 that is basically d phi by dt evaluated at t equal to 0 that is your answer; that is integral phi prime of phi of t delta prime t dt that is minus phi prime of 0. So, basically that is the solution to the problem that is integral minus infinity to infinity phi of t delta prime t d t.

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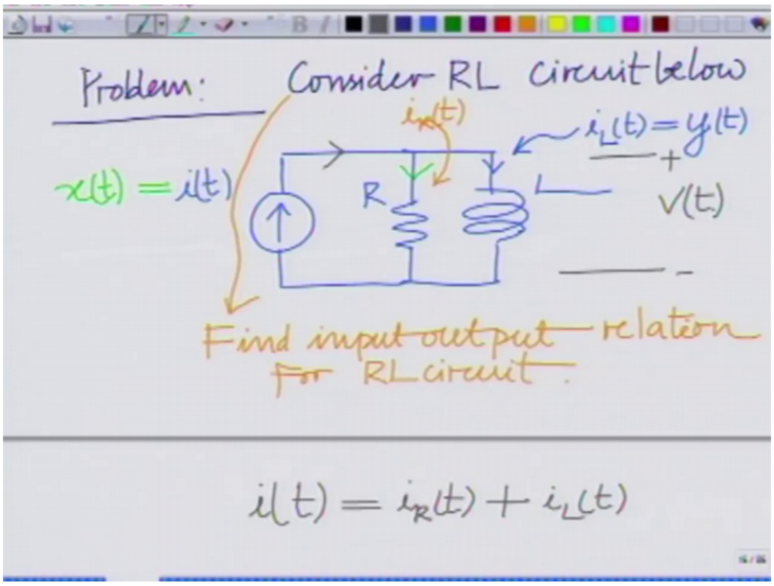

Let us now look at some examples related to the properties of systems and classification of systems. So, we will now look at examples related to properties; so, so far we have looked at systems, so now look at properties and classification. So, let us start with again yet another problem, let us consider a simple problem.

Consider the R L circuit below, this is a current source connected in parallel with a resistance and inductance. So, this is let us say the current is the input; let us say the current through the inductor this is the output, this is the input x of t; this is your current through the resistor, this is an R L circuit. So, we want to find an input; output relation for R L circuit below; for this R L circuit, find the input output relation for R L circuit above.

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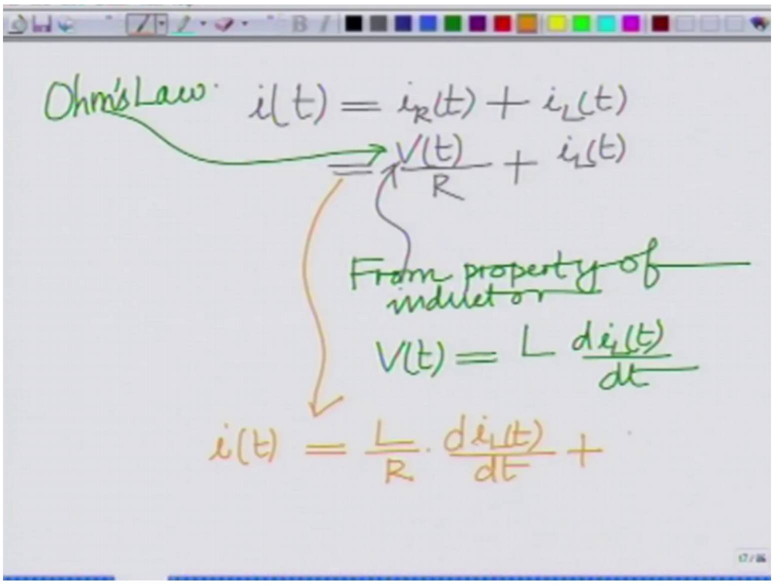

So, I have; well, firstly you can observe that by Kirchhoff's current law you have i of t; that is the current of the source i of t is the current in the resistance plus the current in the inductance. So, that is simply by KCL; Kirchhoff's current law. So, I can write i of t equals i of R t plus i of L of t.

Now, you can clearly see if this voltage, we denote the voltage of the circuit by V of t by Ohms law; i R of t equals V of t; i equals V by R. So, i R of t the current through the resistance is simply the voltage divided by resistance plus i L of t; this is the simply the corrector the inductor.

Now, observed by the property; so this is simply by follows from Ohms law that should be. Now observe from property of inductor, we have V of t; voltage across inductor is L d; i L by dt, which means I can write this equation as i L of t equals L by R d or i of t sorry not i L of t; i of t equals L by R, d i L t by dt plus i L of t that is the current through the inductor and we know this i of t, this is equal to x of t.

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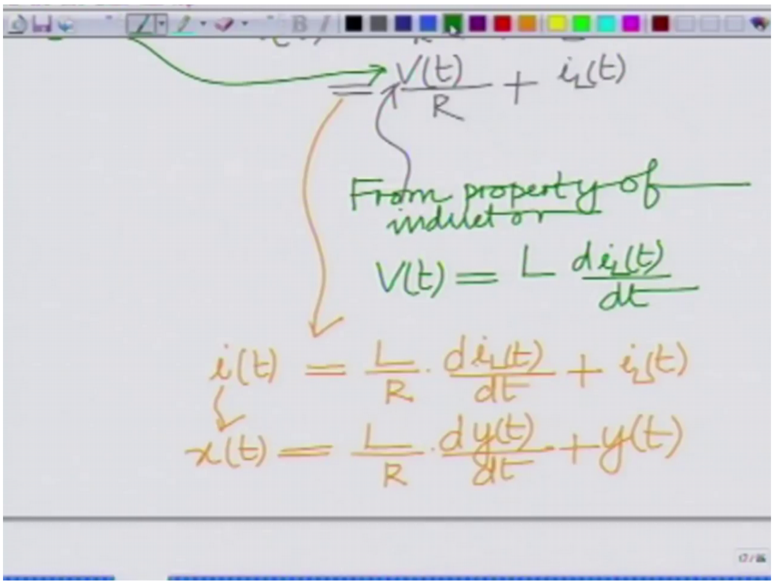

So, I finally, have x of t equals L by R; d of now i L of t is y of t that is output. So, this is d y t divided by the d y t with respect to d of t that the derivative is that is the, so L by R d y t over dt d y t or d y t dt plus y of t. And this is basically your input output relation or basically you can write it as L by R d y t; dt plus y of t equals x of t.

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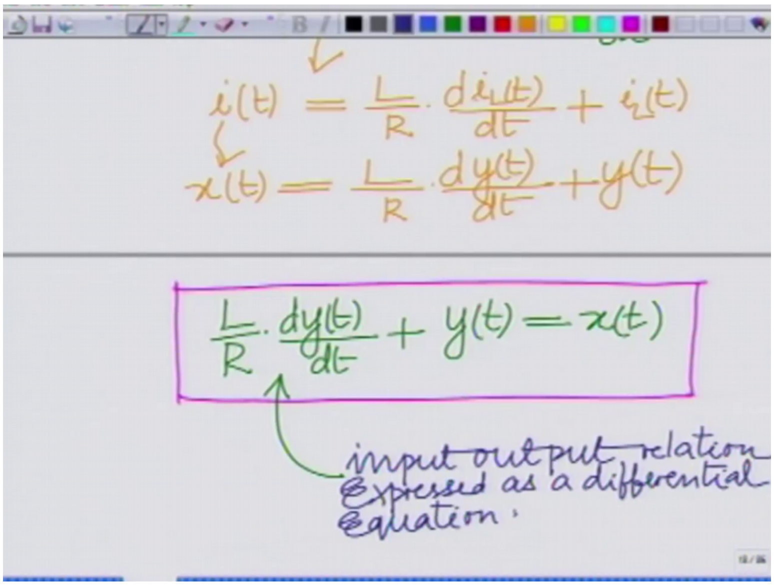

This is the input output relation which you can observe here is expressed as a differential equation, which is something very interesting. This is an input output relation that is expressed as a differential equation and we will see in future; how to solve this input output relation to explicitly derive an expression for the in output or explicitly derive explicitly derive an expression for the transfer function of the output in terms of the input; there are various ways to solve this.

So, right now this is represented; this input output relation between the input x t and output y t is represented as a differential equation in the time domain. Which is a valid way to represent an input output relation and it can be in act seen that this is a linear system. In fact, it can be seen that this is and verify that this is a linear system; this is a time invariant system, in fact satisfies all the properties of an LTI system.

So, it is for this example; I think I mean this gives the input output relation; this describes the input output relation.

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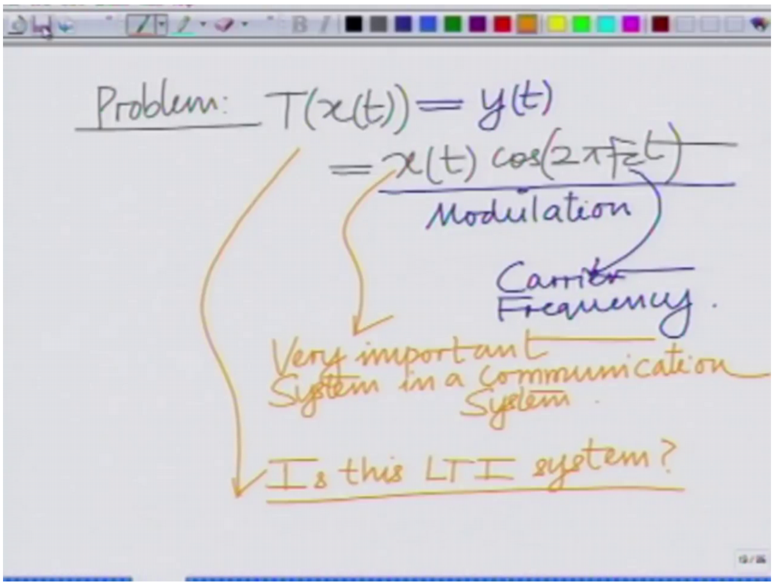

Let us continue with our problems; let us look at another interesting problem. This is a system which is termed as a modulator and which is another very important operation or another very important system in a communication system. T of x t equals x of t let us put it this way equals T of x t equals x of t cosine 2 pi; f c of t.

So, this is your T of x t, which is the output which is y of t equals x t cosine 2 pi f c t. This operation is known as modulation and this frequency f c; this is known as the carrier frequency. In fact, this is a very important property; this is a very important operation in a communication system. This is the very important system or you can call it as a subsystem in a larger communication system.

In fact, the process of modulation modulating the signal that is basically modulating it by a certain carrier forms a very integral component or in forms a key aspect of any communication system. For instance, if we look at any modern cellular communication system, for instance a system such as GSM has carrier frequencies around 800 to 900 megahertz; modern 4G wireless systems for instance have carrier frequencies around the gigahertz that is 2.1 to 2.3 Gigahertz range.

Even broadcast system such as FM systems have carrier frequencies around 90 megahertz correct? 90 megahertz to 100 megahertz amplitude modulation systems have basically frequencies in the range of carrier frequencies in the range of kilohertz and so on.

So, this is modulation; that is modulating it by a different frequency carrier frequency. So, that you can be transmitted in a certain frequency band so that the signal can be transmitted in a certain frequency band; specifically allocated for that kind of communication; for instances TV broadcast or AM radio or FM radio or 2G communication, 3G communication, 4G communication, this forms integral part of any wireless communication system.

Be it a personal communication system such as your cellular telephony or be it a broadcast; a common a broadcast communication system such as your radio or TV broadcast system. So, this is a very important class of subsystems that is because the modulation specifically; it is a system by itself, but it forms a subsystem in a larger system that is a communication system. So, understanding this it’s various properties essential and that is what its property about; that is what this problem aims to look at.

So, we want to see is this an LTI system? What we aim to examine is this modulation; is this an LTI system? This modulation system; is this an LTI system? So, that is what we want to explore in this problem; so, we have solved several problems in this module. So, we will stop here and look at continue looking at some other problems in the subsequent modules.

Thank you very much.