**Principles of Signals and Systems**
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**Lecture - 75**
**IIR Filter Structures: Parallel Form - I, Parallel Form - II, Examples**

Hello welcome to another module in this massive open online course. So, we are looking at the parallel form 1 for IR filter implementation all right. Let us continue looking at this with the aid of an example all right.

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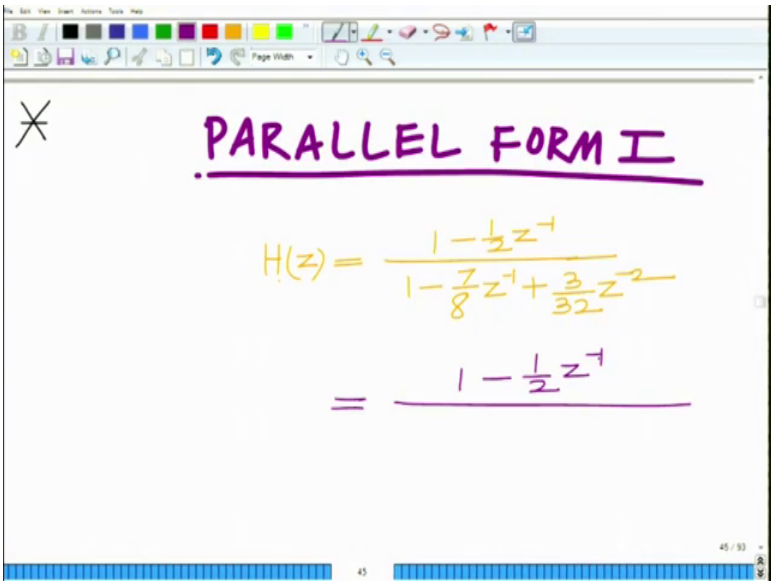


So, we are looking at parallel form 1 and let us consider the following example a very simple example we have H of z equals 1 minus half z inverse divided by 1 minus 7 by 8 z inverse plus 3 by 32 z minus 2; equals 1 minus half z raised to minus 1 divided by 1 minus 3 by 4 z inverse into 1 minus 1 by 8 z inverse which is equal to 2 by 5 1 minus; remember you have to split it into the partial fraction expansion. So, this is basically the partial fraction expansion plus 3 by 5 divided by 1 minus 1 by 8 z inverse ok.

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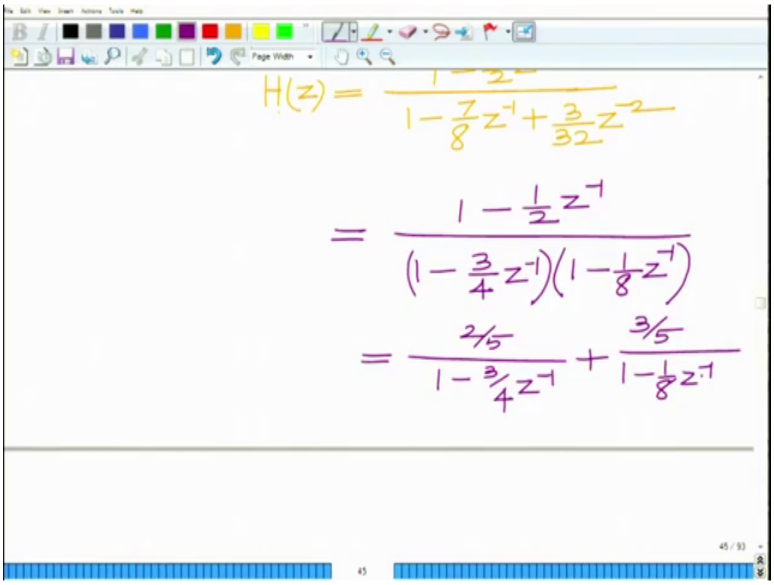


So, this is the partial fraction. So, these are the factors in the partial fraction expansion. And you can see each of these is a first order fraction; first order factor in the partial fraction expansion all right and now we are going to illustrate the PF 1; that is a the parallel form 1 structure for this IR filter ok. And therefore, it is very simple now you can see it has 2 factors. So, we will have 2 branches.

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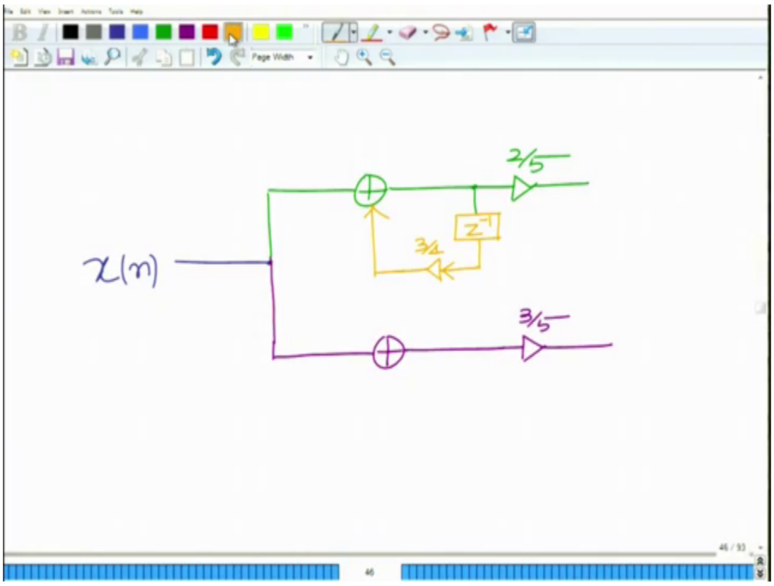


So, this will have 2 branches the top branch will be corresponding to the first factor that will be 2 by 5 by 1 minus 3 by 4 z inverse. So, that will have a gain that is 2 by 5 and then z inverse 3 by 4 and this is the first factor ok. And the other one is naturally going to be this is the bottom branch 3 by 5.

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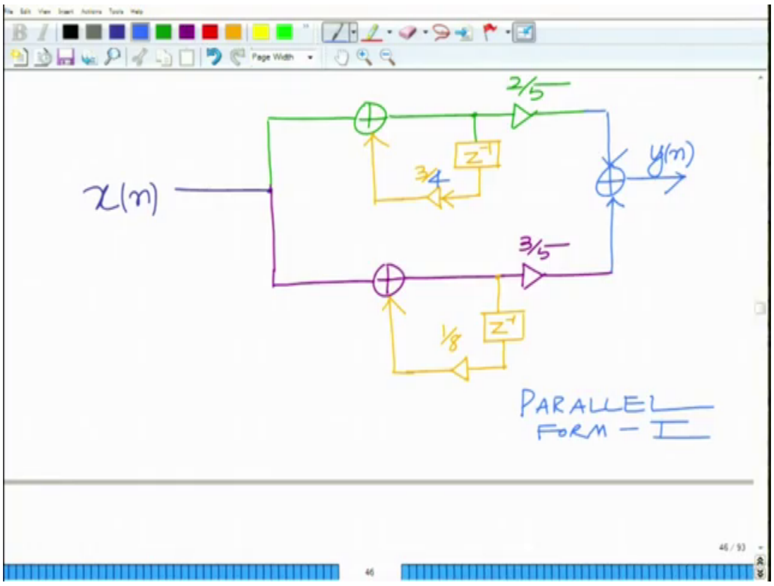


And this is your delay z inverse and this is the gain 1 over 8 ok. And therefore, now what we have is we have to add the outputs of both the branches. So, we have an adder and the output will be y of n ok. So, this is basically your parallel form 1 ok. So, this is the parallel form 1 ok. So, this is a parallel form 1 structure for the IR filter. So, this is 3 by 4 correct.

And similarly now you have the parallel form 2. Another structure is a parallel form 2 where given a transfer function a rational function in z. You use a partial fraction expansion in z. In parallel for form 1 you use a partial fraction expansion in z inverse, in this you use a partial fraction expansion in z ok.

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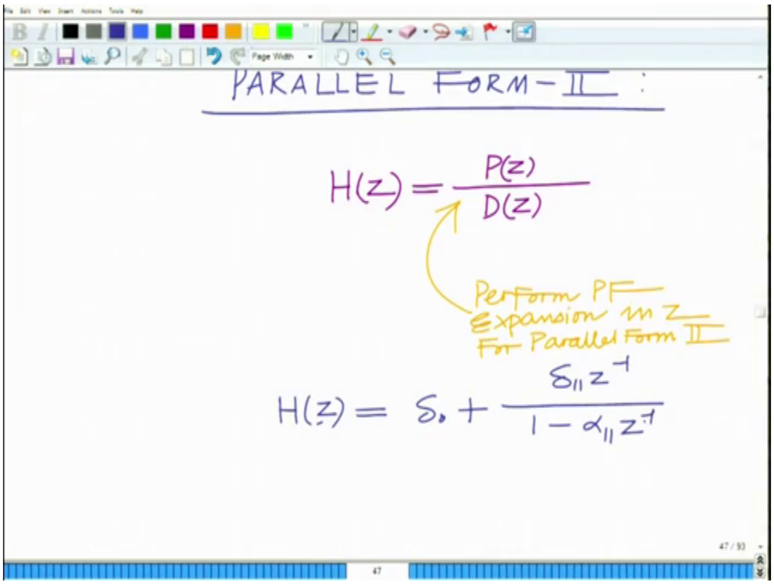


So, this is your parallel form 2 and in parallel from 2 you have H z equals P z over D z this is the partial fraction expansion and perform on this. So, you perform partial fraction expansion; PF expansion, PF expansion in z for parallel form 2. And therefore, the H z the transfer function can be represented as H z equals delta 0 plus you have the first order fraction a factor delta 11 z inverse by 1 minus alpha 11 z inverse. plus a typical second order factor of the form delta 12 z inverse plus delta 22 z minus 2 by 1 plus alpha 1 2 z inverse plus alpha 22 z minus z raised to minus 2. And the parallel form to structure is given as follows again the general structure

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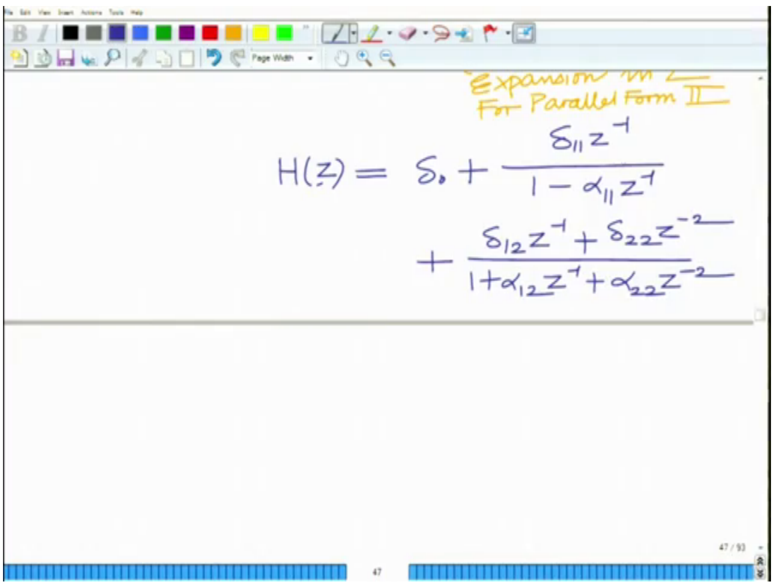

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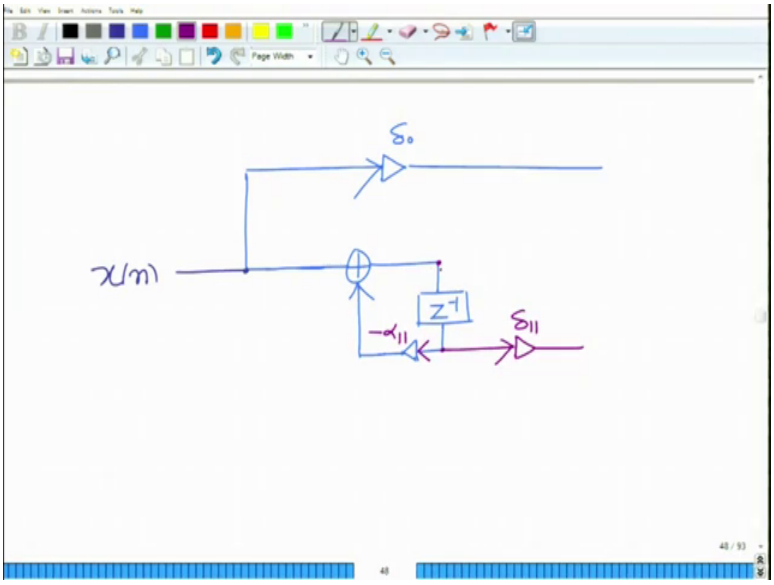


So, this can be given as follows. So, you have your x n and you have the top branch which is simply the gain. Once again that is your factor delta naught, then you have a middle branch that is the delays z inverse and then you have your term minus alpha 11; this is minus alpha 11 and here you have the gain delta 11 ok, because it is delta 11 in z into z inverse that is the factor ok.

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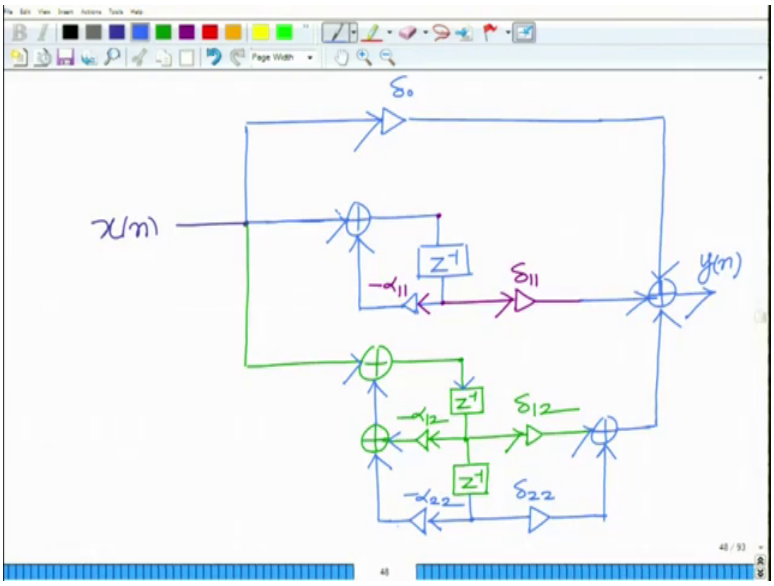


So, this is this and then you have another of the form; there is another delay this is a second order factor. So, you have 2 delays and then you have this delta 12, then you have the term minus alpha 12 and another adder here and the output and therefore, here you have another minus alpha 22 and delta 12 and this is delta 22. And this is basically then now you can see all these 3 outputs can be combined can be added together to yield the final output ok.

So, this is the second order factor and if you combine as you add them, so this is the top gain is dealt naught, then here the first adder factor z inverse minus alpha 1 and delta 11 gain. Then you have the second order factor that will be z inverse and z inverse minus alpha 12, delta 12 minus alpha 22 delta 22 all right and well that is the that is the net parallel form. So, this is your parallel form 2.

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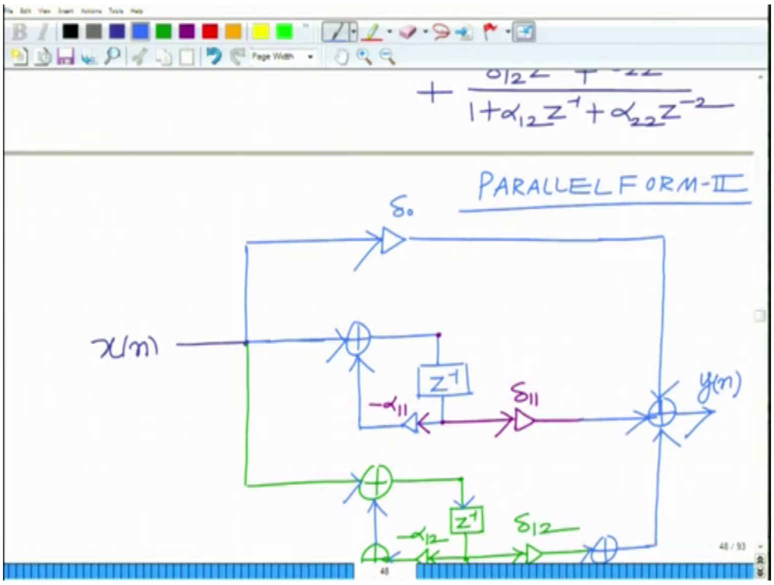


And now let us do a simple example to understand this better. Once again let us consider the same transfer function H z that we have been considering, but now do partial fraction expansion in z all right and that gives us the and the resulting factors all right gives us the partial fraction the parallel form 2 structure ok.

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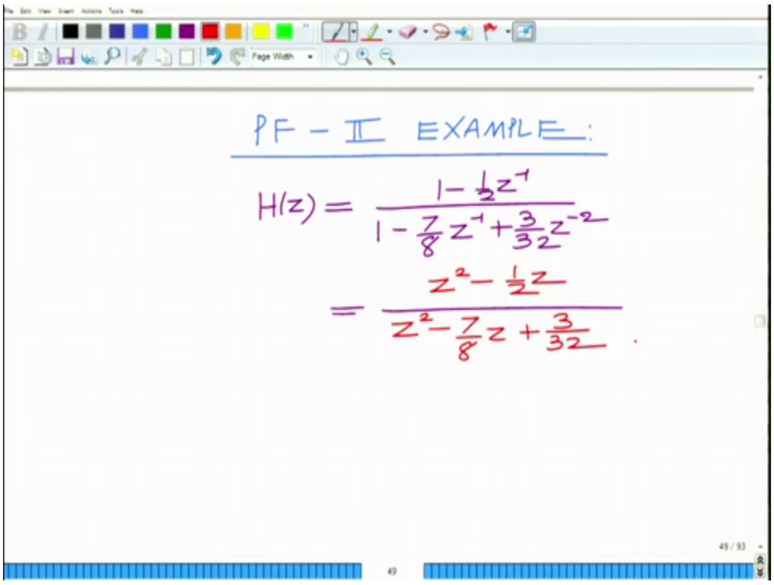


So, let us go back to our example the PF 2 example. And you have your H z; 1 minus half z inverse by 1 minus 7 by 8 c inverse plus 3 by 32 z minus 2 equals z square minus half z divided by z square minus 7 by 8 z plus 3 by 32. Remember because we have to do partial fraction expansion in z.

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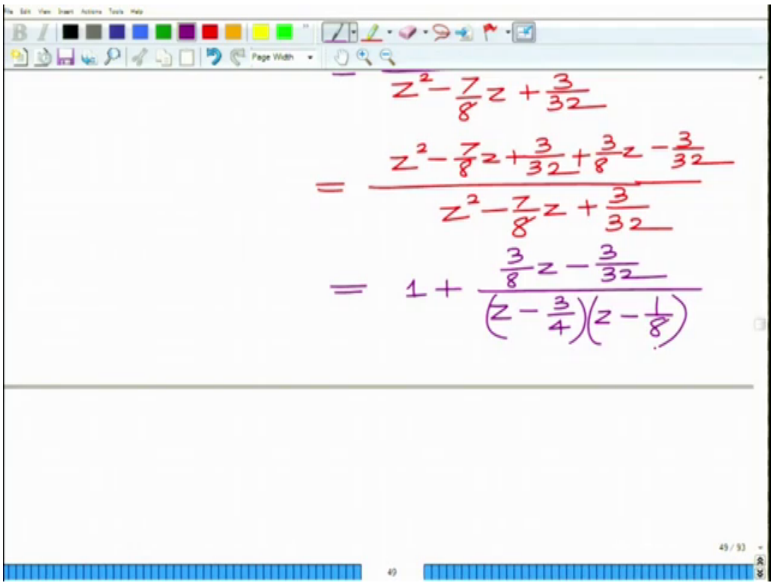

So, now I can simplify this as z square minus 7 by 8 z plus 3 by 32 plus 3 by 8 z minus 3 by 32 divided by z square minus 7 by 8 z plus 3 by 32; which is equal to 1 plus 3 by 8 z minus 3 by 32 divided by z minus 3 by 4 into z minus 1 by 8 ok.

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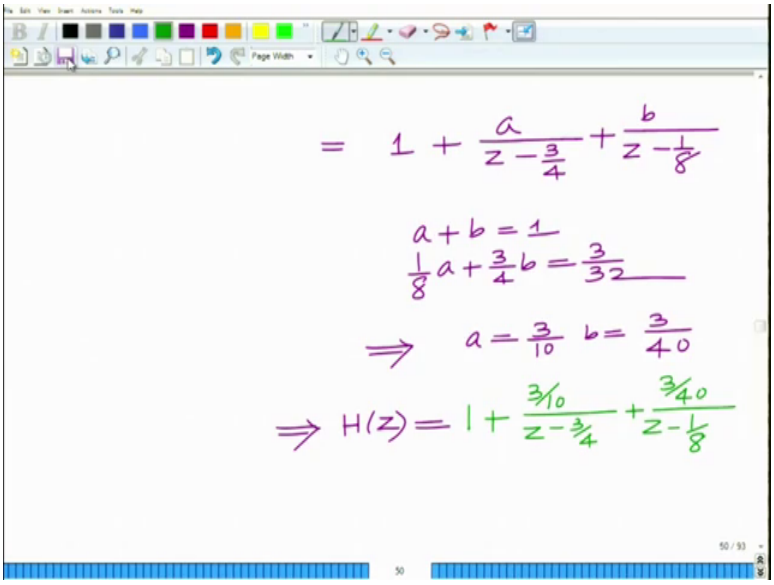


And now let us expand this into partial fractions, we can quickly do this. Let us say this is equal to 1 over z minus 3 by a over z minus 3 by 4 plus b over z minus 1 by 8. You can see that the coefficient of z; a plus b equals 1; further coefficient of z or negative a coefficient of the constant; 1 by 8 plus 1 by 8 a plus 3 by 4 must be equal to. So, minus 1 by 8 a minus 3 by 4 b must be equal to minus 3 by 32 which implies 1 by 8 a plus 3 by 4 b must be equal to 3 by 32.

And solving this yields a equals well 3 by 10, b equals 3 by 40; which implies that now, H z equals 1 plus 3 by 10 by z minus 3 by 4 plus 3 by 40 divided by z minus 1 by divided by z minus 1 by 8 ok.

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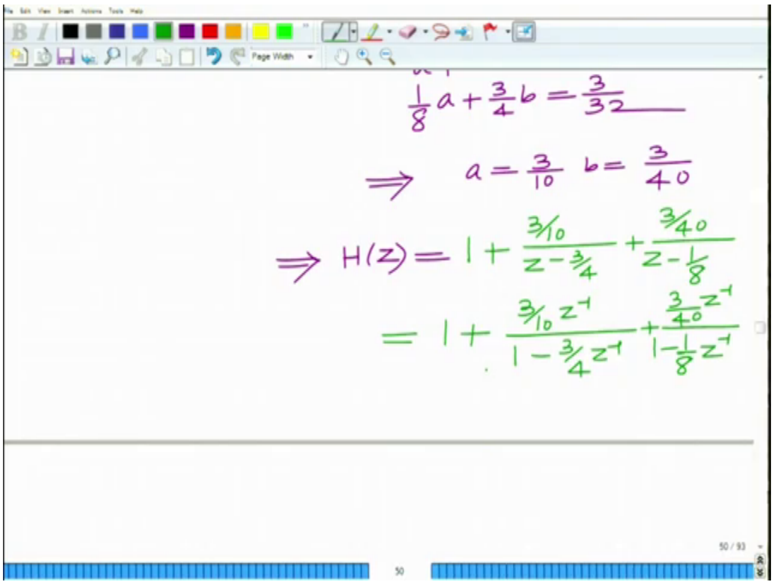


And now finally, multiplying numerator and divided by denominator by z inverse yields 1 plus 3 by 10 z inverse divided by 1 minus 3 by 4 z inverse plus 3 by 40 divided by 1 minus 1 by 8 z inverse and 3 by forty z inverse ok. And this is the partial fraction expansion in terms of z ok. And then we are converting it into z inverse because remember we can implement only delays all right.

So, now we can implement this PF 2; the parallel filter 2, the parallel form 2 realization and that is given as follows: it is very simple.

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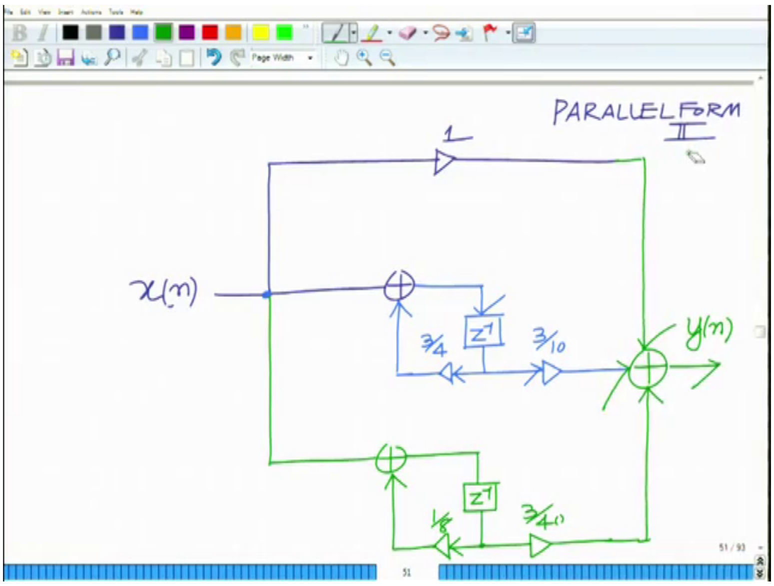

So, I have x n; the constant gain is 1. So, this top branch is 1, this as well. This is z inverse 3 by 4 and the give related to this is 3 by 10 and the final branch z inverse and there is a gain of 1 by 8 and there is a gain of 3 by 40 and this will be added; all these branches are added and then you get the output y of n. So, you have x n the top branch is 1. Then you have z inverse again 3 by 4 3 by 10 plus the bottom branch which is 1 by 8 and the gain is 3 by 40 and all these are added and this is your parallel form 2 all right.

So, basically what we have done is we have seen we have seen several structures for IR filter implementation; starting from the direct form: direct form 1, direct form direct form 2, direct form 1 transpose, direct form 2 transpose; that is transpose of both the direct forms and finally, the cascade form and also the parallel form 1 and the parallel form 2 all right.

So, these are the several forms for IR filters correct. So, you have the several different forms as I had outlined before and that is your direct form 1, direct form 2. The transpose of each of these direct forms and the cascade form and the parallel forms 1 and 2 all right. And each of them results in a certain number of adders and delays and you can check that from the examples as well as the structures that we have listed, all right. So, we will stop here and we will continue in the subsequent modules.

Thank you.