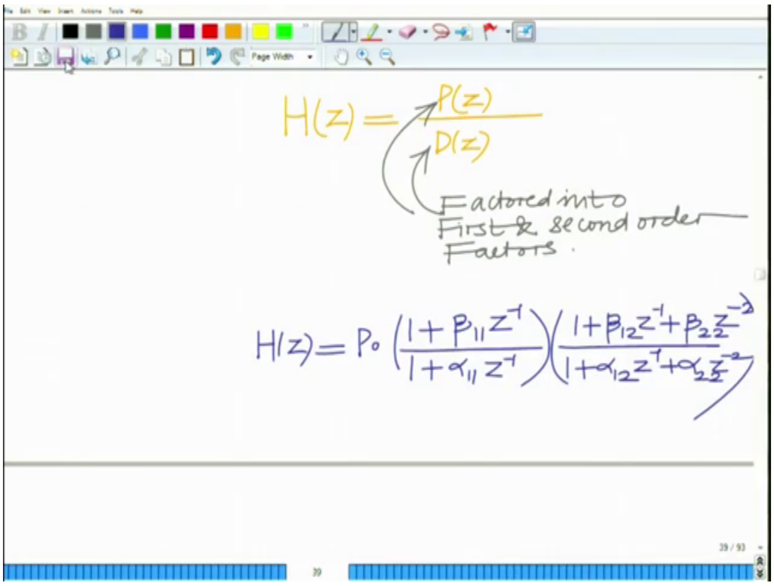
Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 74
IIR Filter Structures: Cascade Form

Hello welcome to another module in this massive open online course. So, we are looking at IIR filter structures let us continue our discussion by looking at another form that is the cascade form ok.

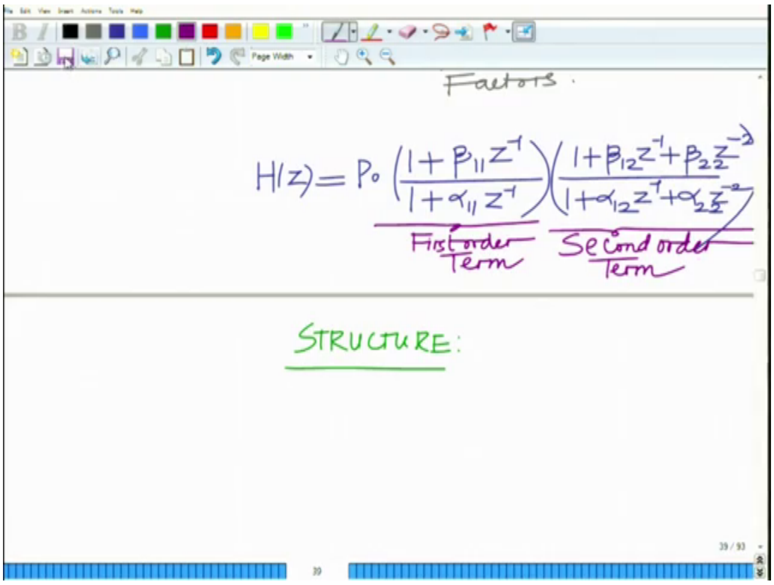
(Refer Slide Time: 00:27)



So, we want to start looking at the cascade form for IIR filter representation ok. And what happens in the cascade form is that we have our transfer function H z which is the rational function ok; z that is P z over D z and these polynomials are factored into first and second order factors. These P z and D z these are factored into first and second order factors.

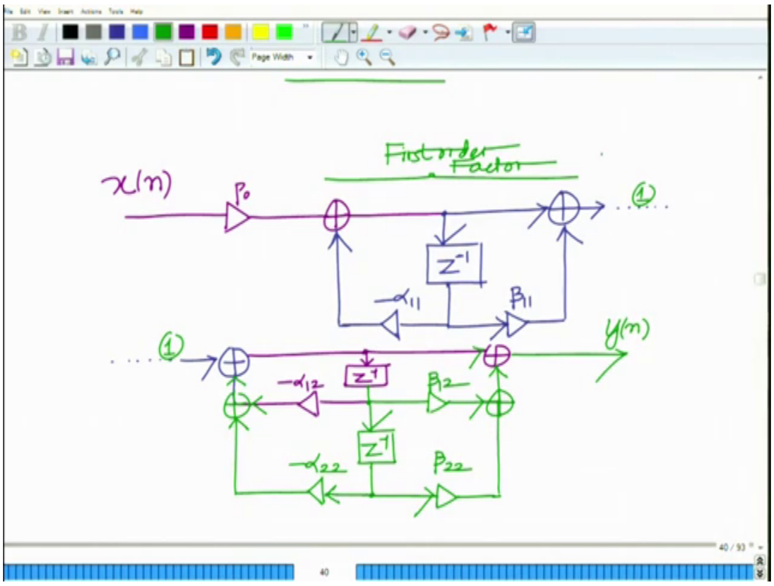
And for instance let us say this is given as H of z equals well P naught into 1 plus beta 11 z inverse divided by 1 plus alpha 11 z inverse; all right is a canonical form, 1 plus beta 12 z inverse plus beta 22 z raise to minus 2 divided by 1 plus alpha 12 z inverse plus alpha 22 z minus 2. So, we do this and this can be realized as follows the structure for this.

(Refer Slide Time: 02:19)



So, you can see this is the cascade of several terms all right; several first order and second order terms all right. So, we have P naught times a first order term times a second order term all right. So, this is the constant gain P naught. This is your first order term and this is your second order term.

(Refer Slide Time: 03:02)

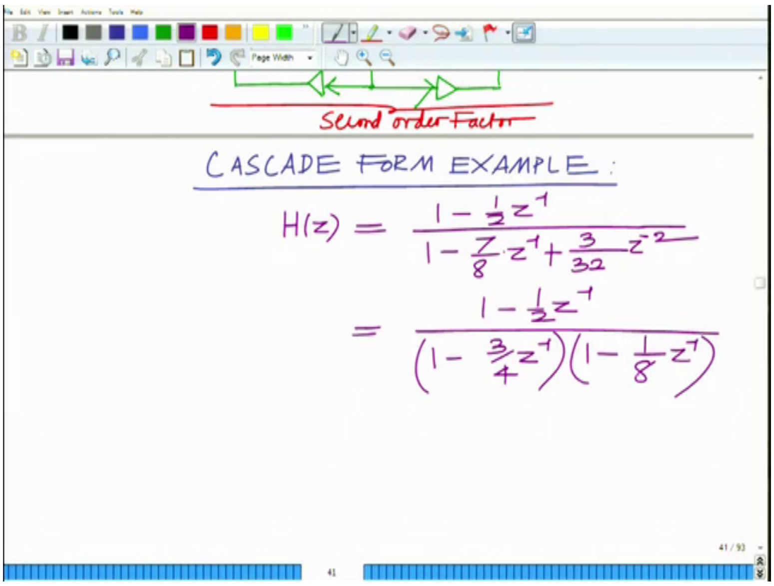


And this can be represented as follows: I have x n, cascade of one system after the other. So, x n let us put it this way goes through gained P naught correct and then we have z inverse and we have the adder over here and this will go to another gain this gain will be beta 1 corresponding to the numerator. So, we will go to the adder. We have another gain minus alpha 11 where alpha 11 is a coefficient in the denominator all right.

And remember this is a cascade. So, this will continue all right and let us continue this below and so, this cascade will continue and you will have here another adder and this will be minus alpha 12, z inverse. Similarly you will have here and you will have another adder here and this gain is beta 12 and this is the adder, this is the output y n and you will have another term that is z inverse this is beta 22 and this is minus alpha 22 and you have well of course, here you have adders. So, you have the adders and well this is minus alpha 12, beta 12, minus alpha 22, beta 22 and here so, let us denote this point as 1. So, this point is basically what just to indicate that it is continuing all right.

So, basically this is the first order term remember. So, this is the cascade this is your first order factors. So, you have to factorize this is the first order factor and this represents your second order factor and this represents your second order factor all right. So, that basically shows the cascade form that basically shows the cascade form representation of the IR filter structure all right.

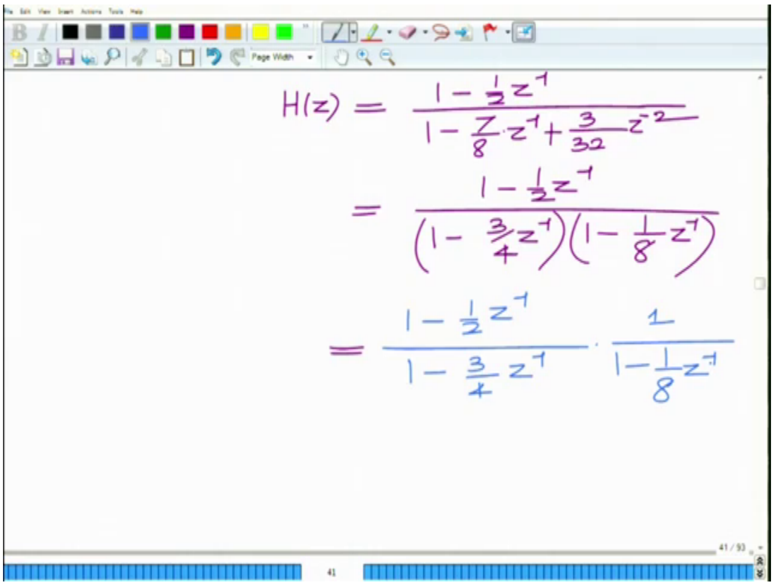
(Refer Slide Time: 07:06)



Let us do an example to understand this. Let us go back to the example that we have considered previously. So, this is our cascade for example and what we have is we have H z equals 1 minus half z inverse over 1 minus 7 by 8 z inverse plus 3 by 32 z minus 2.

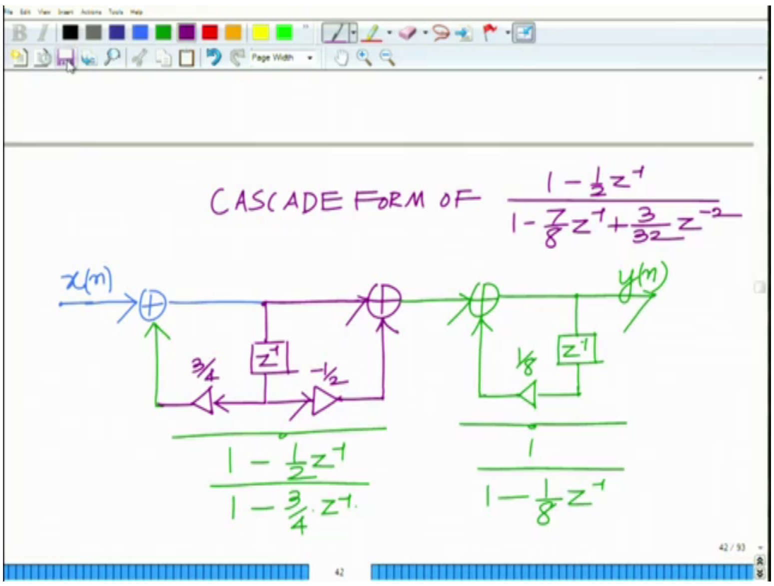
This is equal to 1 minus half z inverse over 1 minus 3 by 4 z inverse into 1 minus 1 by 8 z inverse correct.

(Refer Slide Time: 08:38)



And therefore, I can write this as 2 factors; one is a first order factor. This is basically your separate this into 2 factors; 1 minus half z inverse. In fact, both of them will be first order factors times 1 over 1 minus 8 z inverse. So, both of these are first order factors and the representation is as follows.

(Refer Slide Time: 09:02)



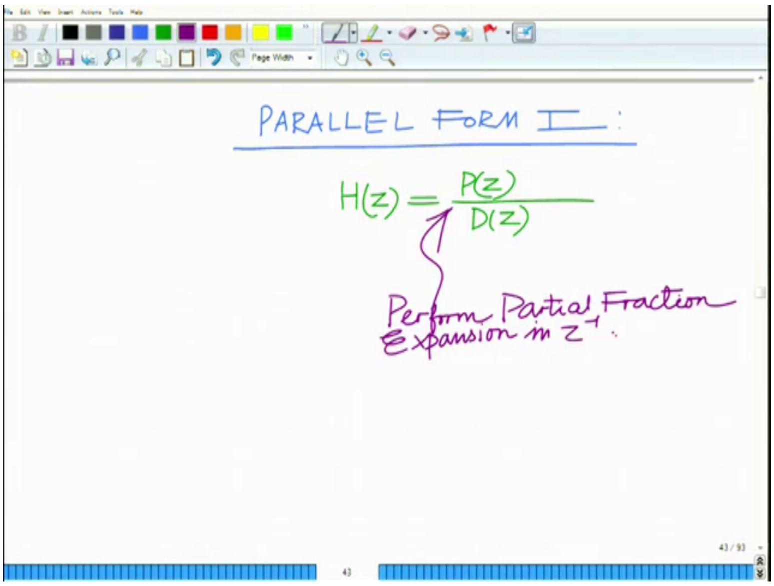
So, we have this, your x of n all right and that will go here you have a delay. You have a delay z inverse and here you have the gains that are basically minus half and here you have the gain that is 3 by 4 and that is going inside the adder correct ok.

And this is your first factor and then the output is basically again, this goes into z inverse and this is 1 over 8 and the output of this is basically nothing other than your y n all right. So, this is the first factor all right. This corresponds to your 1 minus half z inverse correct. You can see each of this is the DF 2 representation; 1 minus half inverse over 1 minus 3 by 4 z inverse and this factor is your 1 over 1 minus 1 by 8. This is your 1 over 1 minus 1 by 8 z inverse all right.

So, that is what we have this is the example of the cascade form. So, cascade form; this is your cascade of the example that is basically which is 1 minus 1 minus half z inverse over 1 minus 7 by 8 z inverse plus 3 by 32 z raised to minus 2 ok. So, this is the example of the so, this basically is the example for the cascade form.

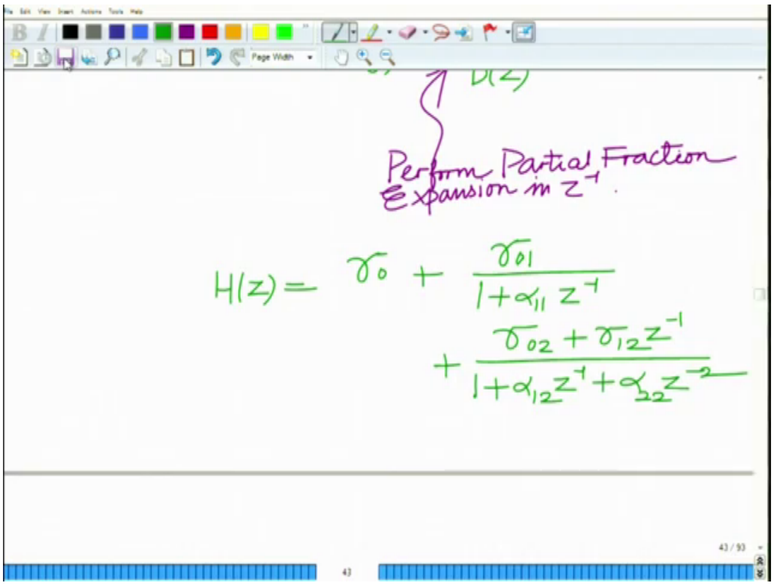
And now let us look at another form for IR implementation. This is the parallel form 1 all right; in which we decompose it into a sum all right; a partial fraction expansion of various factors ok.

(Refer Slide Time: 12:13)



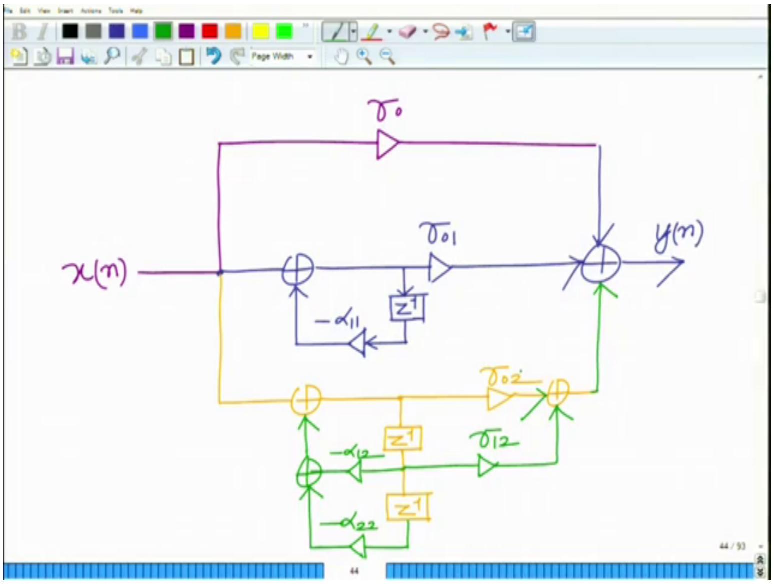
So, we have the parallel form 1 and let us again consider H z equals P z over D z. And now we perform a parallel fraction expansion in z inverse, perform a partial fraction expansion in z inverse.

(Refer Slide Time: 13:08)



So, we can write or rather express H z as H z equal to gamma naught over or this is gamma naught plus gamma naught 1 over 1 plus alpha 11 z inverse plus; this is the second order factor gamma 12, gamma 1 2 1 gamma 12 z inverse by 1 plus alpha 12 z inverse plus alpha 22 z minus 2 ok. And therefore, what you can see is that we have split it into a sum a partial fraction expansion of the first order and second order factors ok.

(Refer Slide Time: 14:17)

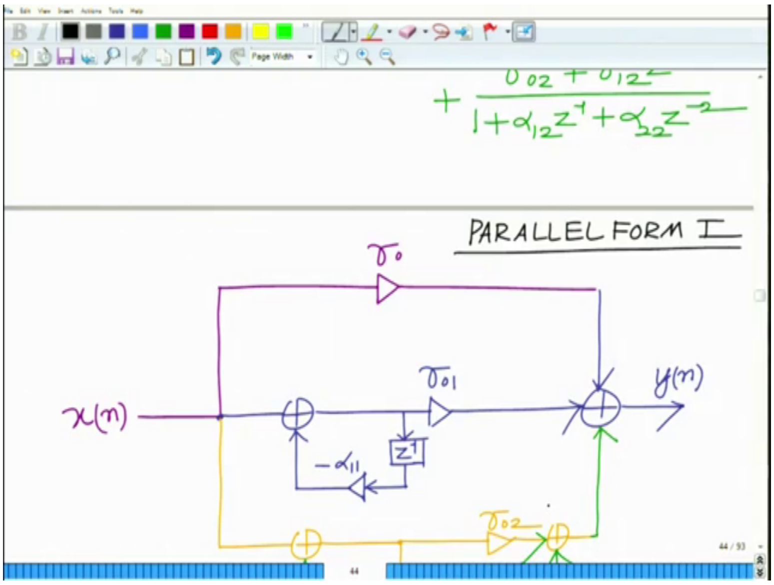


And the structure of this parallel form this can be given as follows. So, you have x n. So, now we will have these parallel branches. So, you have the first one that is the gain gamma naught then you have the gain gamma naught 1 and this is y n and here you have the delay z inverse and you have minus alpha 11, this is the DF 1 representation of the first term and then you will have the second order factor.

And the corresponding representation will be you will have this; z inverse all right and you will have this gain gamma naught 2 you have the adder and another adder over here and finally, you will have this joining this branch and this is naturally given as this gain is you can say this is; gamma 12 all right and this is going to be you have 2 this is minus alpha 12 and this is and this last gain here is minus alpha 22 all right.

And therefore, you have this various and this is this is basically your parallel form you can see this is basically your parallel form 1 ok.

(Refer Slide Time: 17:34)



Just making sure that we have no errors here; plus alpha 12 minus alpha 22, gamma 12, gamma 02 and this is basically your parallel form 1 all right.

So, basically what we have done in this module is we have essentially looked at these several examples we have introduced the cascade form, looked at an example, also looked at the parallel form and we will continue this discussion in the subsequent modules.

Thank you very much.