Principles of Signals and Systems
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Lecture - 70
Group / Phase Delay - Part II

Hello, welcome to another module in this massive open online course. We are looking at the group and phase delay, let us continue our discussion.

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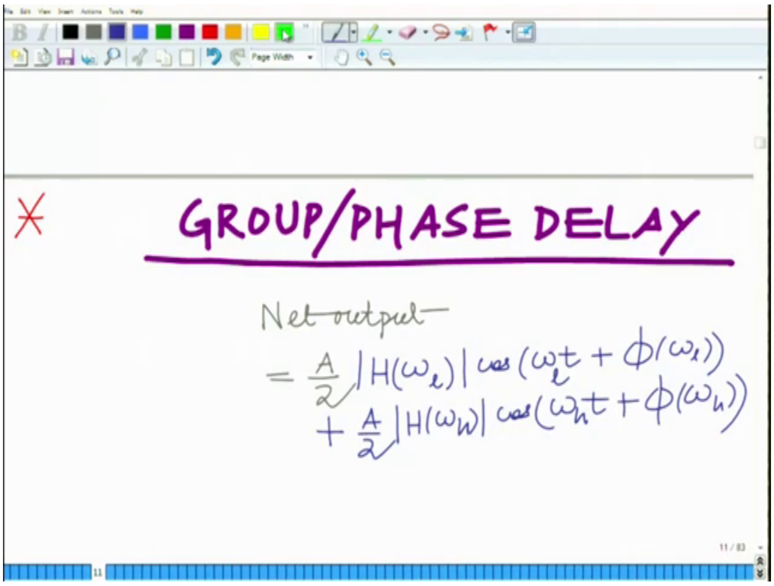

So, we have the group and phase delay all right.

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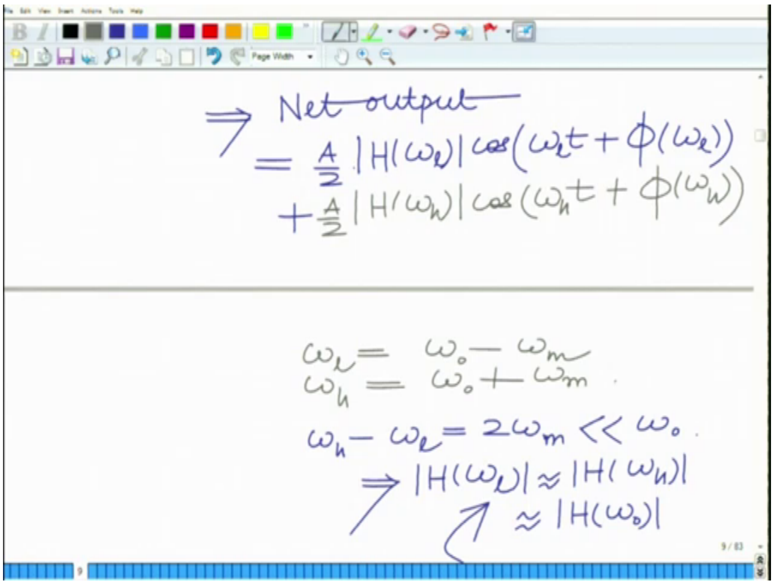

And before that let me just correct a minor error that is this should be H of omega h.

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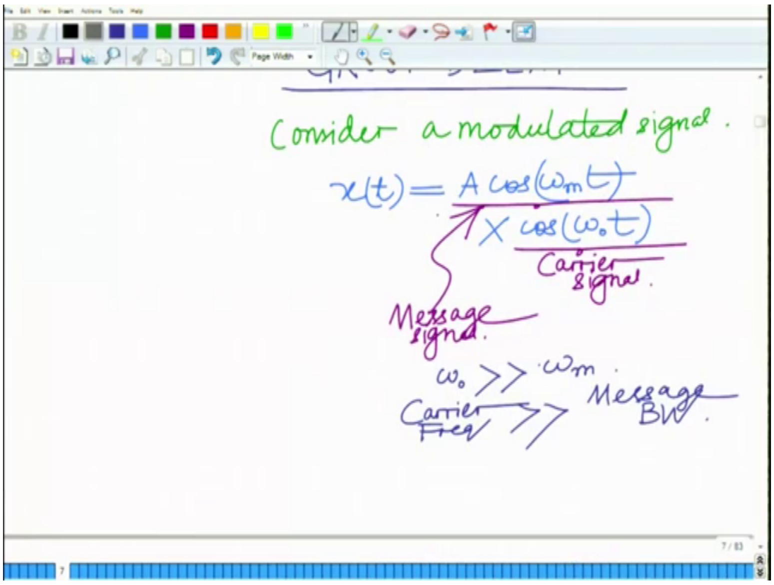

And therefore, the net output of the LTI system corresponding to this signal this modulated message signal remember that is what we are looking at A cosine omega m t into cosine omega naught t that is given as A by 2 net output equals A by 2 magnitude H of omega magnitude H of omega I think l times cosine omega l t plus phi omega l plus A by 2 magnitude H of omega h cosine omega h t plus phi of omega h.

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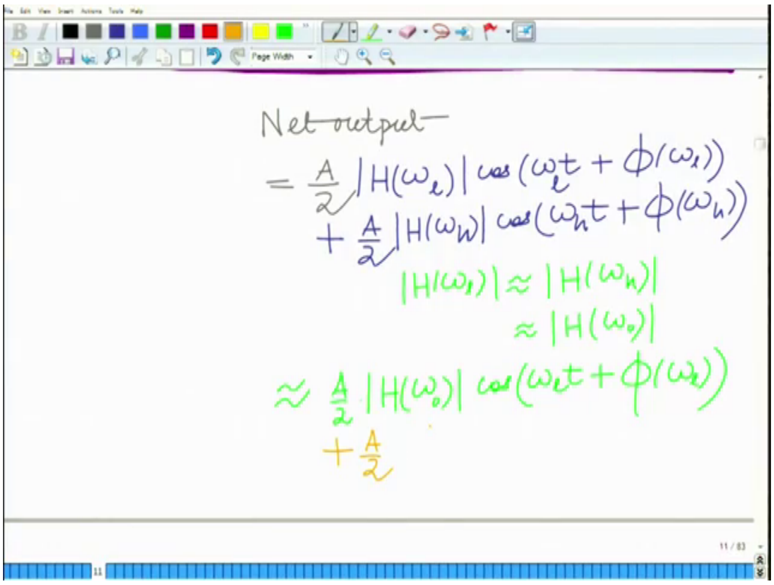

And now, we have said that these two quantities magnitude H of omega l is approximately equal to correct magnitude H of omega l approximately equal to magnitude H of omega h is approximately both of these are approximately equal to magnitude H of omega naught because omega m is very small correct. So, the spacing between omega l and omega h is very small that is 2 omega m which is much smaller compared to the carrier frequency omega naught, ok.

And therefore, this is approximately equal to A by 2 magnitude H of omega naught cosine omega l t plus phi omega l plus A by 2 magnitude H of omega h cosine omega h t plus phi omega h which is equal to; now, you can simplify this as A by 2 or A by 2 into 2.

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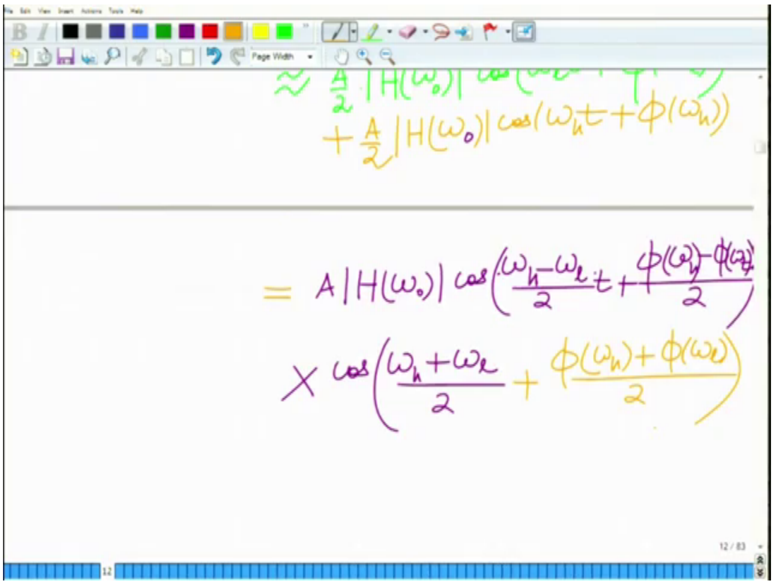

So, we have I am sorry this is cosine omega naught A by 2 this is 2 A 2, A by 2 that is a magnitude H of omega naught cosine omega h minus omega tau omega l by 2 into t plus phi of omega h minus phi of omega t divided by 2 times cosine of omega h plus omega l divided by 2 plus that is this is cosine a plus cosine b is twice cosine a minus b by 2, 2 cosine a plus b by 2. So, that is phi of omega h plus phi of omega l divided by 2, ok.

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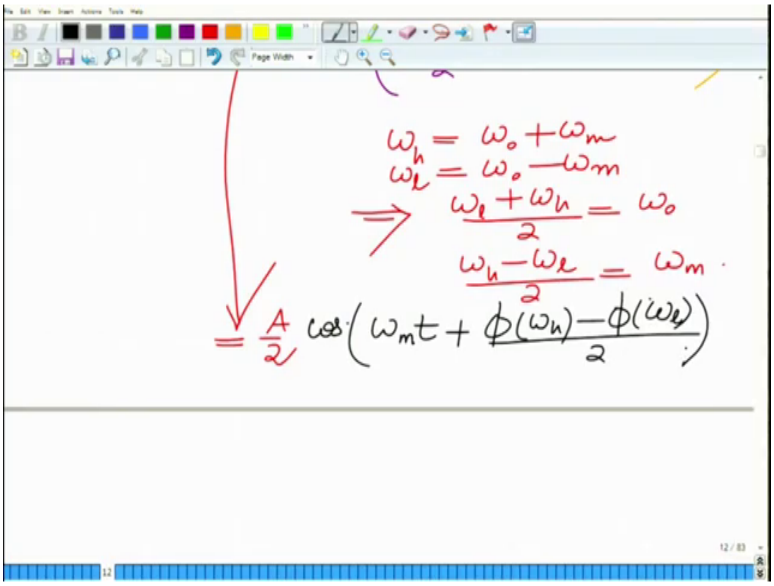

Now, if you see this if you check this you have omega h equals omega naught plus omega m, omega l equals omega naught minus omega m this implies omega h plus omega naught by omega h plus omega l by 2 plus omega h by 2 equals omega naught and omega h minus omega l by 2 equals omega m.

So, which means that this can be simplified that this quantity here can be simplified as A by 2 cosine omega l plus omega h by 2. Or what is the first quantity? Omega h minus omega l by 2 correct, so phi H of omega so that is cosine of omega m t plus phi of omega h minus phi omega l divided by 2 times cosine of omega h plus omega l by 2 that is omega naught t plus phi of omega h plus phi of omega l divided by 2, ok.

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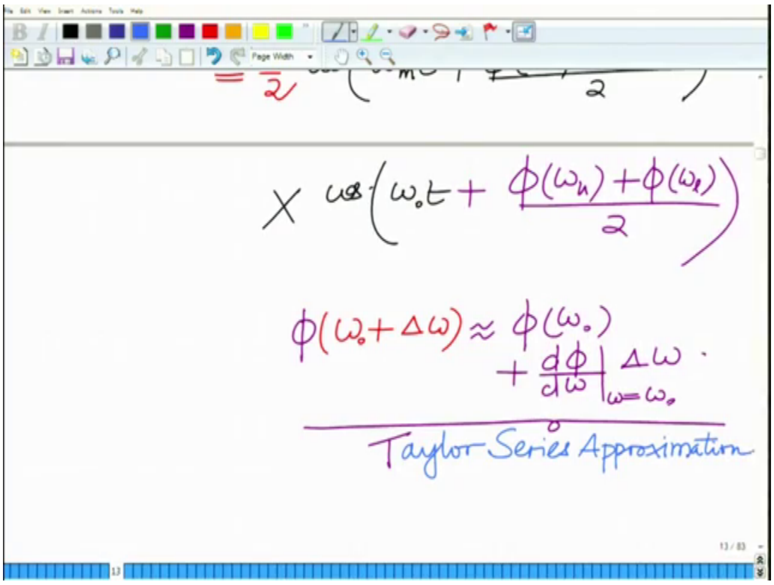

Now, what we are going to do. Because since omega m is small correct, so basically we are considering a small neighborhood around the carrier frequency omega naught. So, we can use Taylor series expansion to simplify this phase function phi all right, because the message frequency omega m is much smaller than the carrier frequency omega naught.

So, we have phi of, we have phi of omega plus delta omega phi of omega naught plus delta omega for small delta omega is approximately equal to phi of omega naught plus d phi by d omega evaluated at omega equal to omega naught into delta omega. This, we know is the well known Taylor series approximation, first order Taylor series approximation ok.

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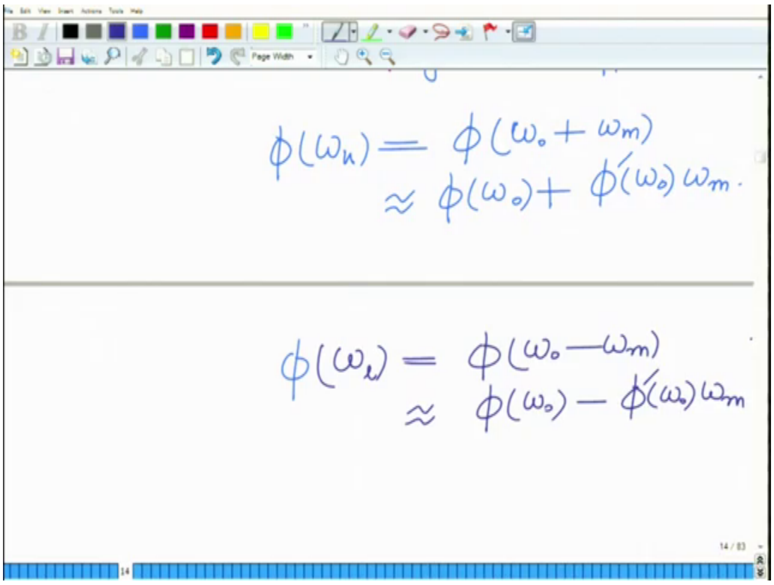

This is the Taylor series approximation which implies the net output can be simplified as let us look at. Now, let us look at phi of omega h. Now, phi of omega h you can see is phi of omega naught plus well omega m which is approximately equal to again using the Taylor series approximation this is approximately equal to phi of omega naught plus the derivative phi prime omega naught derivative at omega naught times omega m correct. And phi of omega l this is equal to phi of omega naught minus omega m which is approximately equal to phi of omega naught minus phi prime omega naught into omega m, ok. So, these two relations we are obtained using the Taylor series approximation.

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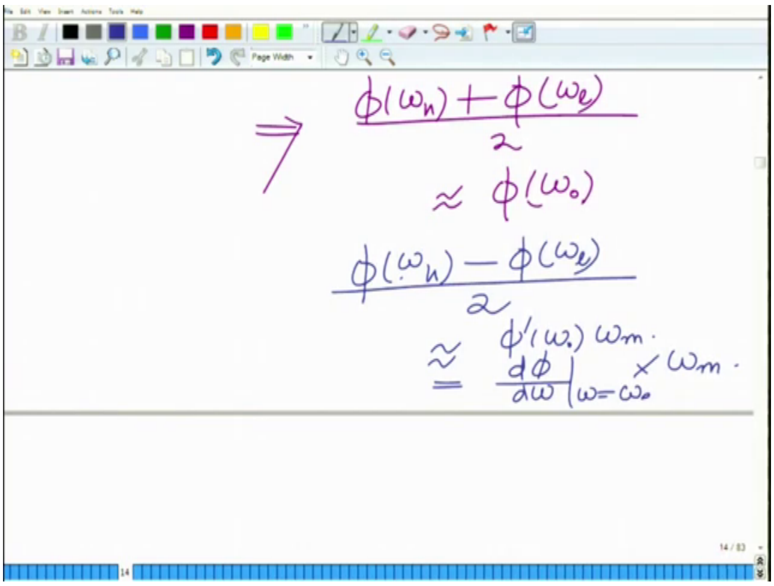

And therefore, this implies now, using these Taylor series approximations we can evaluate phi of omega h plus phi of omega l by 2 and phi of omega h minus phi of omega l divided by 2 approximately ok.

So, this implies phi of omega h well the first one is standard phi of omega h plus phi of omega l divided by 2 that you can see is approximately equal to phi of omega naught. And more interestingly phi of omega h minus phi of omega l divided by 2 this is approximately equal to phi prime omega naught into omega m that is basically your d phi by d omega evaluated at omega into equal to omega naught times omega m ok. So, this is your phi of omega h omega phi of omega h minus phi omega l by 2.

Now, substituting this now, we take these relations for approximate relations of phi of omega h plus phi of omega l by 2 and phi of omega h minus phi of omega l by 2 and substitute these in the expression for the net output ok, and that yields the net output.

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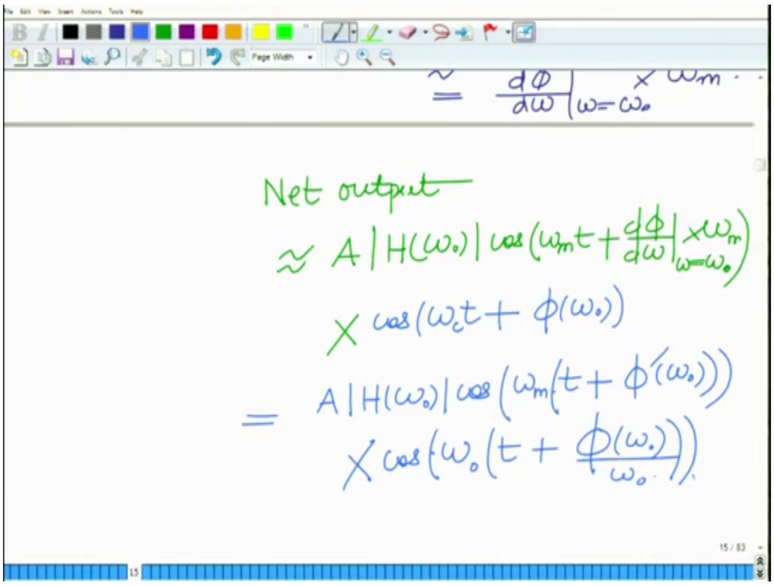

Is now, approximately a substituting these expressions we have it this is equal to a magnitude H of omega naught cosine omega m t correct cosine omega m t plus phi h minus phi l by 2 that is because we can you have seen d phi by d omega at omega equal to omega naught into omega m ok. Times you have something interesting that is cosine omega c t plus this is your simple phi of omega naught that is omega h plus omega l by 2 which is equal to.

Now, you can simplify this as A, magnitude H of omega naught cosine omega m t minus I am going to simply write this as phi prime omega naught correct, not minus or plus phi prime omega naught times cosine omega naught t plus phi of omega naught divided by omega naught.

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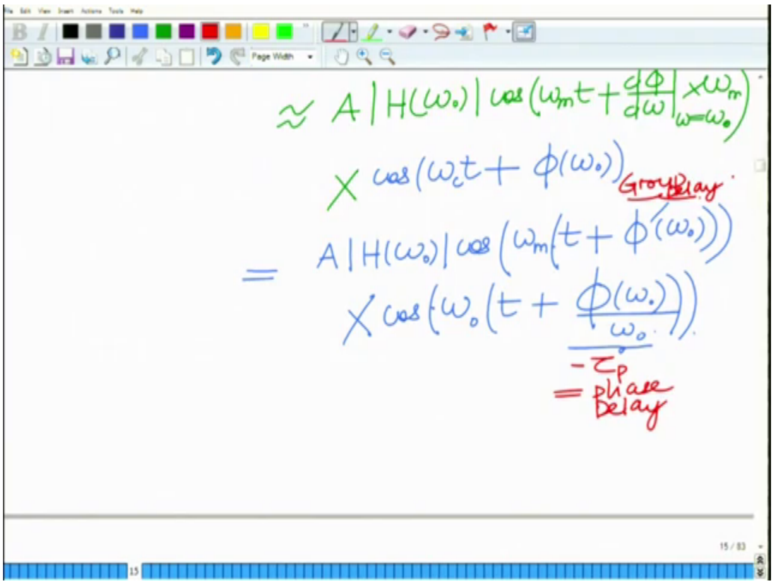

And we know that this quantity is basically this quantity is your minus tau p this is equal to the phase delay and this quantity that you see here that is the delay of the message signal this is now, your group delay, right. So, you have your phase delay and group delay.

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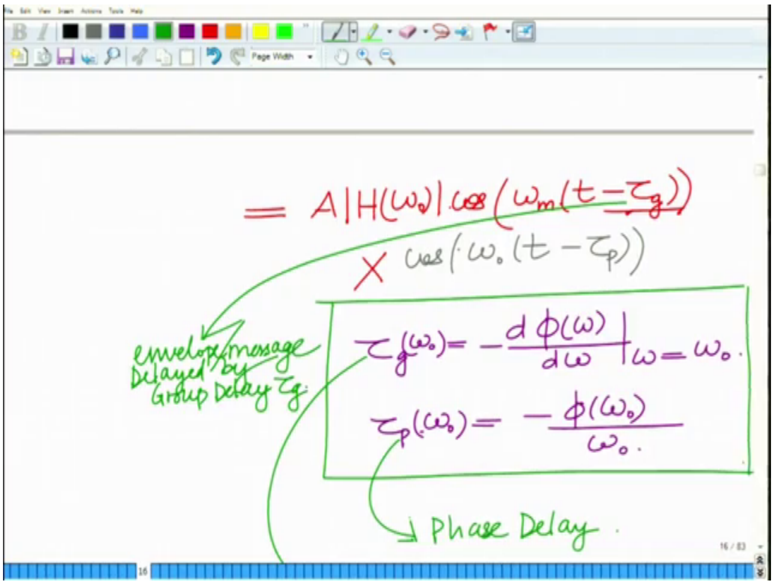

So, you can write this resulting output as A magnitude H of omega naught cosine omega m t minus tau g, where tau g is the group delay times cosine of omega naught t minus tau p. And therefore, tau g is the group delay minus d phi omega over d omega evaluated at omega equal to omega naught that is your group delay and tau p of omega naught equals minus phi of omega naught divided by omega naught ok, these are the group and phase delay expressions. So, these are the group and phase delay expressions and this is very important.

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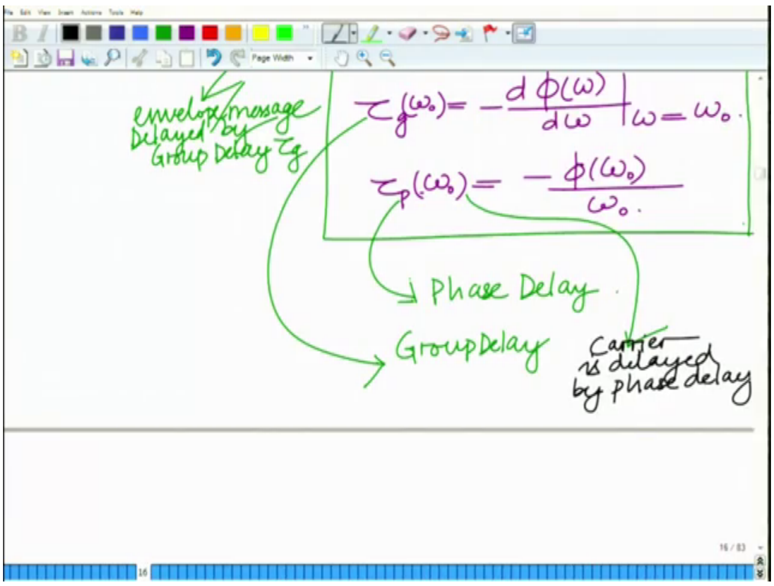

So, this is your group delay and this is your phase delay. And what is very interesting is that when you pass this LTI system with the impulse response ht all right and frequency response h omega phase response phi omega the envelope that is the message is delayed by the group delay correct that is tau g of omega that is minus d phi omega or d omega evaluated at omega equal to omega naught and the carrier is delayed by phi omega naught divided by omega naught. These are the group and phase delays ok.

So, what you can see is that the message is delayed by the group delay or the envelope message divided by the group delay tau g. And carrier as usual as we have seen before carrier is delayed by the the narrowband message is delayed by the group delay and the carrier is delayed by the phase delay.

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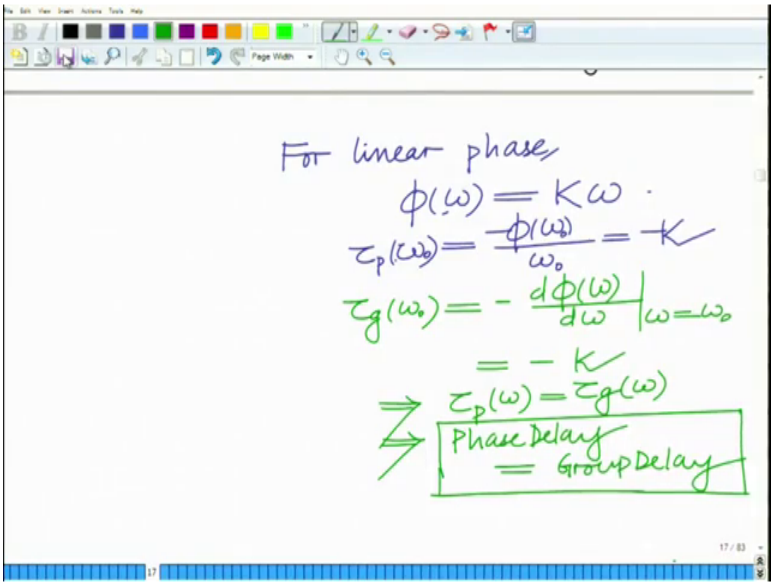

And further if the phase is linear that is which implies phi of omega equals k omega. Now, look at this the phase delay tau p of omega equals well phi of omega divided by or tau p of mega naught equals phi of omega divided by omega naught equals k or minus phi of omega naught divided by omega naught is square is minus k. And also the phase delay or the group delay tau g of omega naught is minus d phi omega by d omega evaluated at omega equals omega naught and this is also equal to minus k, which implies for a linear phase tau p of omega any omega for that matter is tau g of omega.

This implies that the phase delay equals the group delay ok. That is for a modulated narrowband message signal for a narrowband modulated message signal correct when you passes through a LTI system that is if the LTI system has a linear phase characteristic then the phase delay is equal to the group delay all right, ok.

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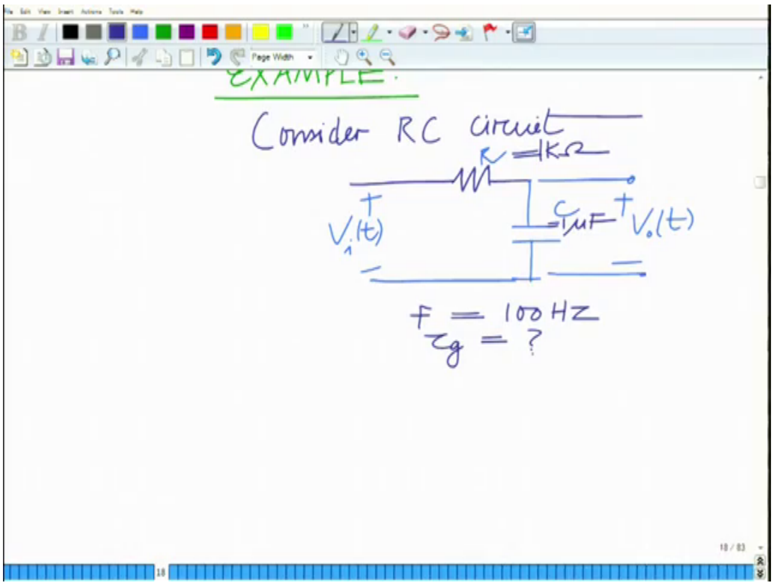

Let us do a simple example to understand this better example for the group and phase delay, ok. And consider an RC circuit with the standard serial RC circuit that we have. So, this is R, this is C and this is your output voltage, this is your input voltage that has V i t the output voltage V 0 t.

Now, we know that the transfer function ok. Now, what you want to do for this is at f equal to 100 hertz and we are given that R is equal to 1 kilo ohm and C is equal to 1 micro farad we want to derive what is the group delay, all right. So, what is the group delay for this serial RC circuit with output across the capacitor with given a 1 kilo ohm resistance and capacitance is 1 micro farad, ok.

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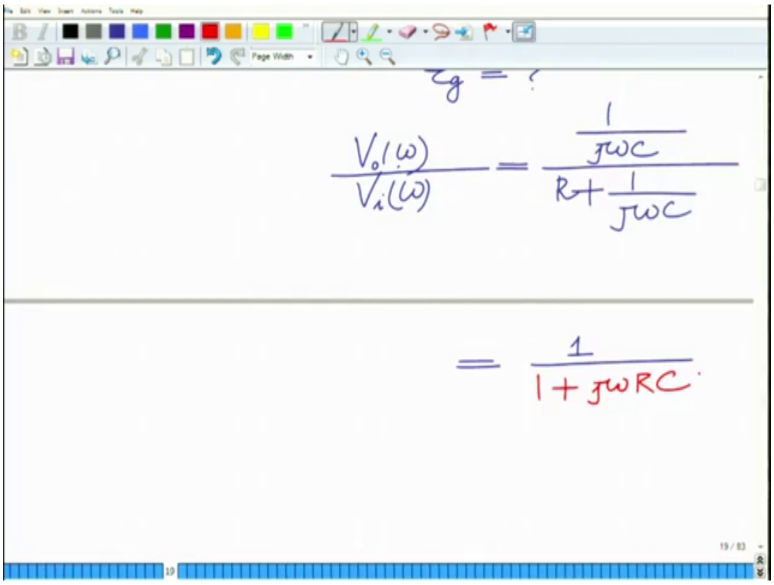

Now, we know that the transfer function of this is simply that is 1 over the impedance of the capacitor divided by 1 over divided by R plus 1 over impedance of the capacitor that is J omega C which is 1 over 1 plus J omega RC.

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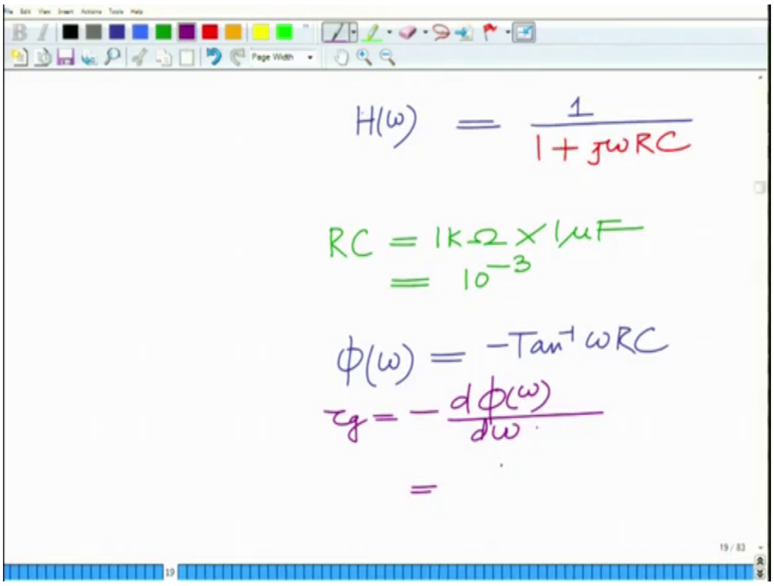

Now, you can see RC is equal to 10 to the power of RC is equal to 1 kilo ohm into 1 micro farad equals 10 to the power of minus 3 in fact, the units is seconds ok. And therefore, now, if you look at H of omega, now, if you see H of omega the transfer function this is your H of omega. Now, if you see ah, so H of omega equals the transfer function 1 over 1 plus J omega RC. So, if you look at the phase of omega this will be minus tan inverse omega RC because the phase of 1 plus J omega RC is tan inverse omega RC all right and the, but that is in the denominator. So, the net phases minus tan inverse omega RC.

And therefore, the group delay tau g the group delay equals minus d phi omega as a function of omega minus d phi omega over t omega that is tan inverse omega RC derivative of tan inverse omega RC which is basically. Well, RC divided by 1 plus omega square R square C square which is equal to now, substitute the values of RC 1 kilo ohm into 1 micro farad.

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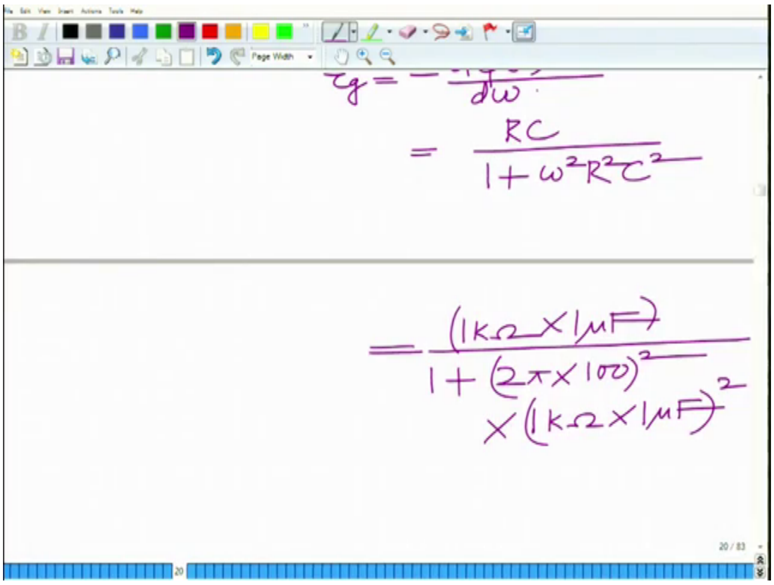

We know, well we know the value of RC, but anyway 1 kilo ohm into 1 micro farad divided by 1 plus frequency is given as frequency is given as 100 hertz. So, omega equals 2 pi into 100 square into 1 kilo ohm RC square into 1 micro farad square, ok. So, that is what you get net and that evaluates to basically 0.717 milliseconds, ok.

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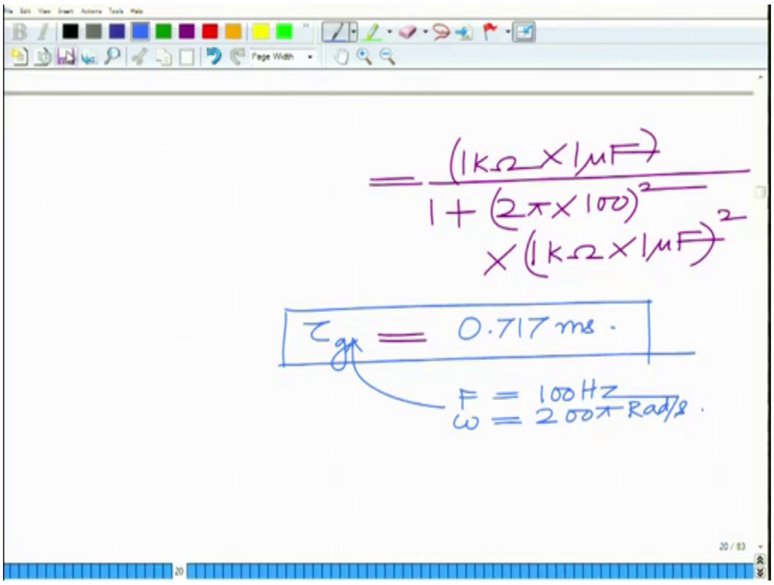

And this is remember at F equals 100 hertz or omega equals 2 pi into 100 that is 200 pi radians per second. So, this illustrates the group delay calibration for a simple example, all right.

So, in this module what you are done is basically we have completed or discussion on the group delay. Demonstrated a very interesting aspect that if it is when you pass a modulated message signal through an LTI system with the message signal correct narrowband modulation where the message signal frequency is much smaller than the carrier frequency, then one can approximate the output signal as the message being delayed by the group delay while the carrier being delayed by the phase delay. And we have calculated this group delay phase delay for a, for a linear system we have shown that these both are equal for a simple RC circuit which is basically a non ideal low pass filter we have evaluated this and demonstrated how to evaluate the group and phase delay at a given frequency, all right. So, we will stop here and continue in the subsequent module.

Thank you very much.