Principles of Signals and Systems
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Lecture – 07
Example Problems in Signals and Systems – Plot, Odd/ Even Components, Periodicity

Hello, welcome to another module in this massive open online course. So, we have looked at a basic introduction to the Principles of Signals and Systems, we have looked at various kinds of signals and systems or a classification of different kinds of signals and systems. So, let us do some problems to understand these concepts better.

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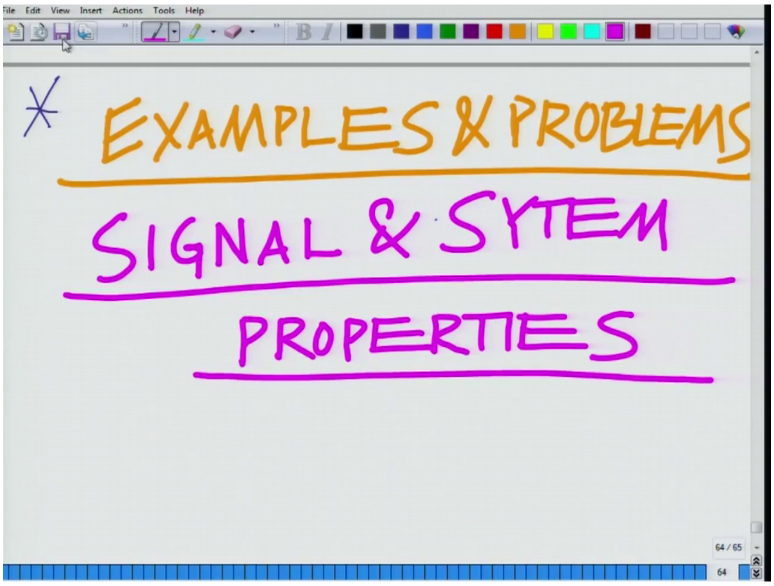

So, what we are going to do now is focus on problem solving or let us do some examples. Let us do this as examples and problems in signal and system classification or let us put it as signal and system properties; based on the concepts that we have studied so far.

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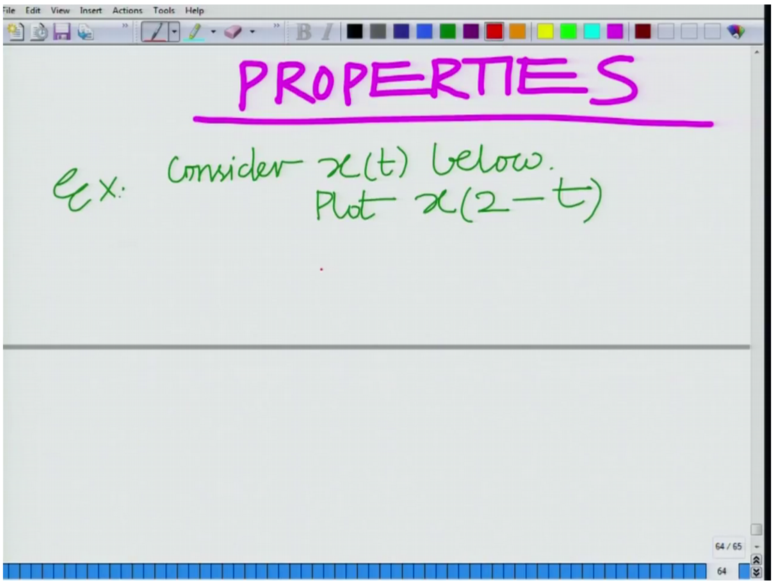

Now, the first example consider x of t given below and plot x of 2 minus t and the x of the signal x of t that is given is the following thing that is from minus 2 to 2; this is going to be at minus 2, it is minus 1, at 2 it is 1.

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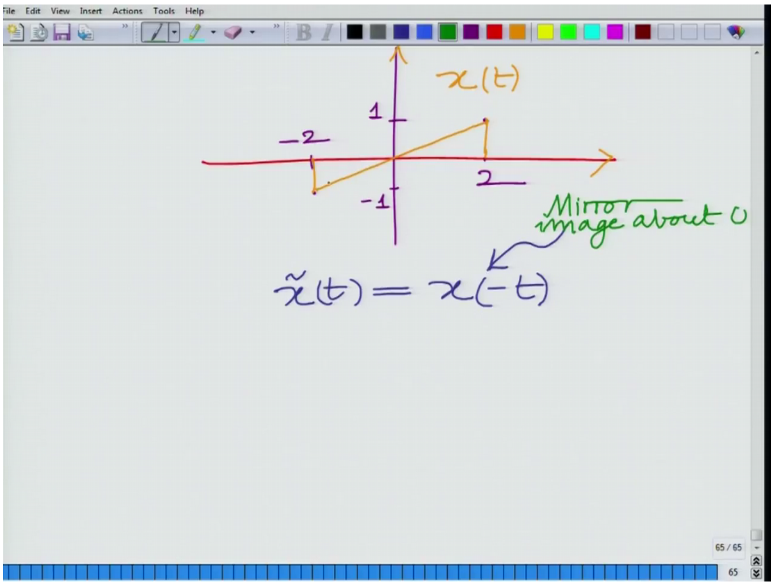

And the signal is given as the following; this is the signal. So, this is your x of t; so we are given signal x of t which is nonzero in the time over the time minus 2 seconds to 2 seconds and we are required to plot what is x of 2 minus t.

Now we can approach this as follows in a very simplified fashion; the first thing that we will try to do is we will define a new signal x tilde of t; which is equal to x of minus t that is x of minus t. And as you know x of minus t simply corresponds to flipping the signal about the y axis; that is taking a mirror image about 0. This is mirror image about 0; so, the flipping about mirror image about 0 which is basically we already have seen f of; so, which is basically if you look at this, so we have to consider the mirror image of this signal.

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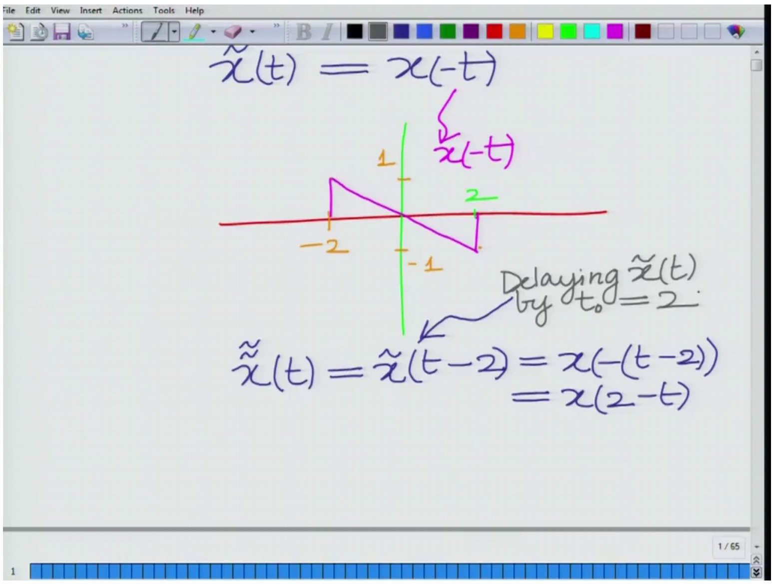

And therefore, let us now plot x tilde t equals x of minus t which is a mirror image of this; mirror image of this signal about 0 and that looks like this. So, if you look at that that is from minus 2 again it is nonzero only from minus 2 to 2 and at minus 2; it is going to be 1, correct? And at 2; it is going to be minus 1. So, this is 1, this is minus 1; so, this is your x of minus t and now you can see; so we have x of x tilde t which is x of minus t. And now I am going to define another signal that is x double tilde t; which is x of x tilde of t minus 2.

So, now I am going to define a new signal x double tilde t; which is x tilde t minus 1, but x tilde t is x of minus t. So, this will be x of minus of; t minus 2 which is equal to x of 2 minus t. But x tilde t minus 2 you can see is simply delaying x tilde by 2 seconds; t naught equal to 2, which means I take x tilde t that is x of minus t take x tilde t and shift it to the right; delay it. Delaying a signal means basically shifting it to the right.

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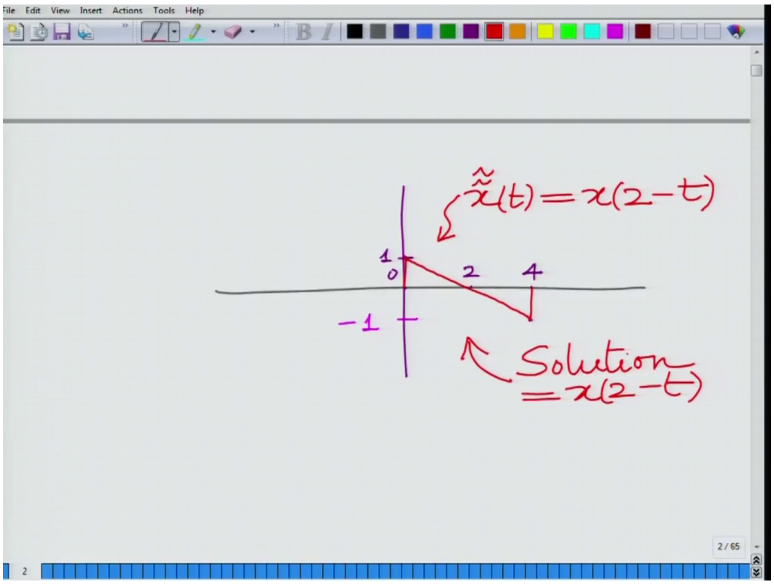

So, that is going to be now x tilde t is going to be; if you shift it to the right what is going to happen is; its going to now be nonzero. Previously it was nonzero from 2; minus 2 to 2, now it is going to be nonzero from 0 to 4 and at 0; it is going to be equal to 1 and at 4; it is going to be equal to minus 1.

And therefore, the net signal is going to look something like this; this is your x double tilde t which is equal to x of 2 minus; so, this is basically the final answer or the solution; that is equal to basically x of 2 minus t, this is a plot of the signal x of 2 minus t. So, first what we did to approach this is basically problem; we flipped it about 0 that is grad users x of minus t, shifted it to the right by 2; that is delayed it by 2; that gives us x of 2; so that is the first problem.

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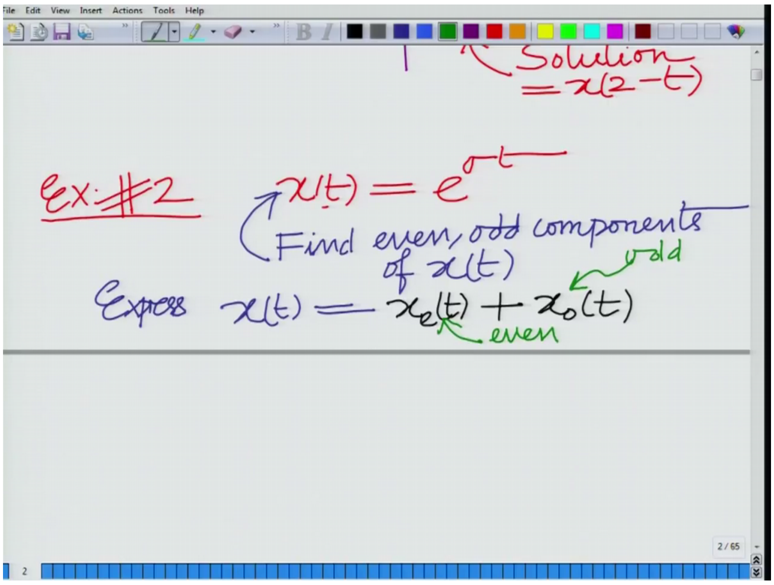

Now, let us do another example; now let us say we are given a signal x t equals e power sigma t. This is as you know this is the exponential signal, we are required to find the even and odd components of x of t; that is express x of t as; x of even t, plus x of odd t.

So, this is an even signal; this is an odd signal and this is an even signal. So, we are required to express x of t as a sum often even an odd signal; that is find the even and odd components of the signal x of t. This can be found as follows using a simple trick; one can use the simple trick to find the even and odd components of the signal x of t.

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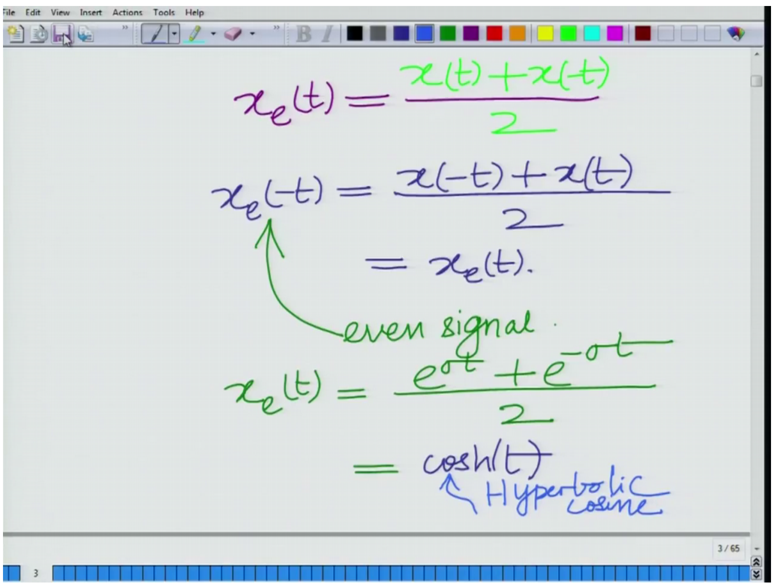

I can take x of t; the even component of t x; e t can be obtained as follows. This is x of t plus x of minus t divided by 2; you can see this is an even signal because x even of minus t is; x of minus t plus x of minus of minus t that is x of t divided by 2; which is equal to x e t.

So, this is an even signal; so that can you can see. So, x even of t in this case will be e to the power of sigma t; plus x of minus t, e to the power of minus sigma t divided by 2 which is nothing, but you can see this is the cosine hyperbolic; this is the cosine hyperbolic of t.

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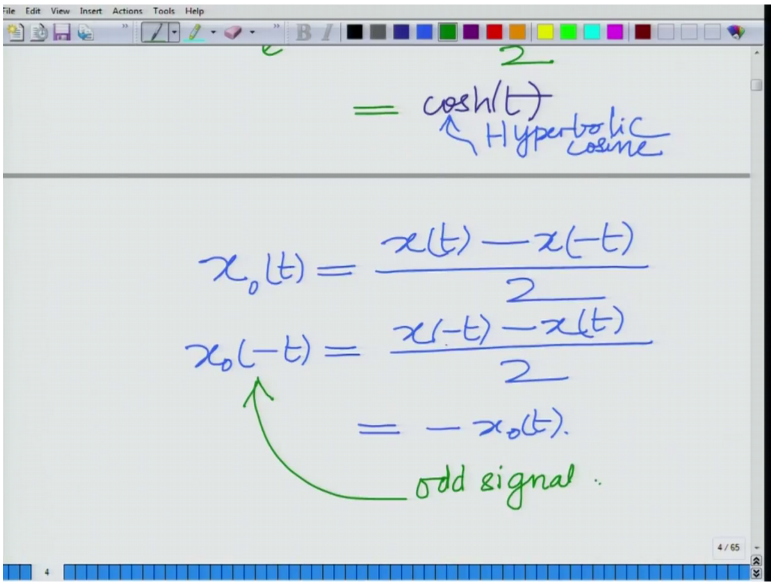

And similarly, the odd component of t is given as x of t minus x of minus t divided by 2 and you can see x of odd of minus t is x of minus t minus x of minus of minus t divided by 2; which is nothing, but minus of x of odd of t; so, this is indeed an odd signal.

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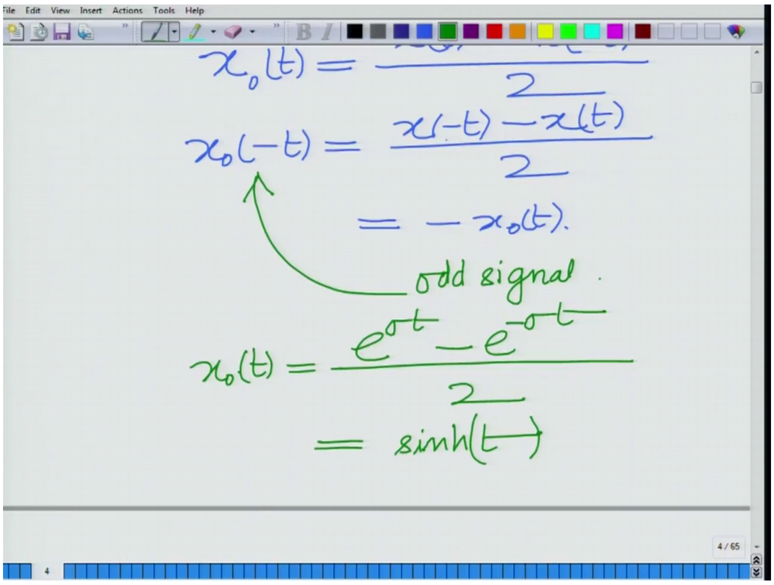

And for this particular this thing; we have x odd of t equals; e power sigma t minus x of minus t power minus sigma t divided by 2; that is sine hyperbolic of t; this is the hyperbolic sine function. This is we have seen hyperbolic cosine, this is a hyperbolic sine function and now you can see x even t.

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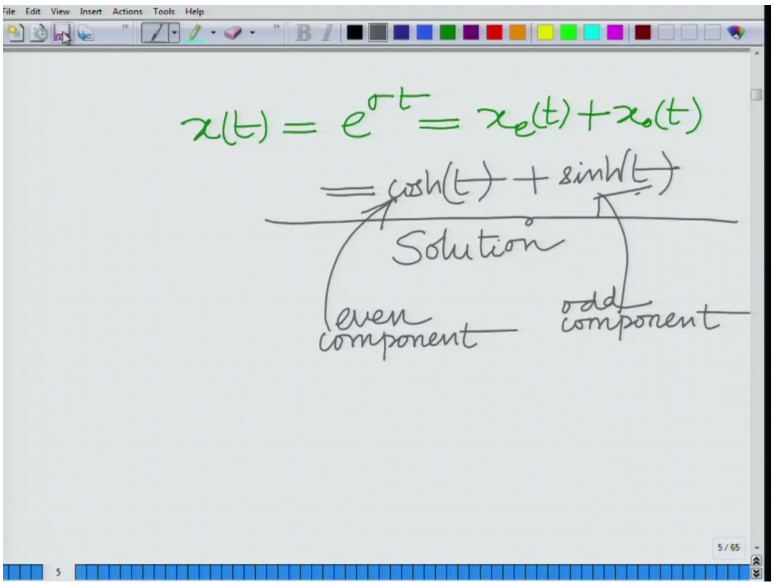

You can verify that x t equals e to the power sigma t; equals x even t plus x odd t which is equal to the cosine hyperbolic function; cosine hyperbolic cosine of t, sine hyperbolic function of and t and this is the final solution. So, this is the even component of x t and this is the odd component of x t. So, we are given a signal x of t and we have decomposed into its even and odd component.

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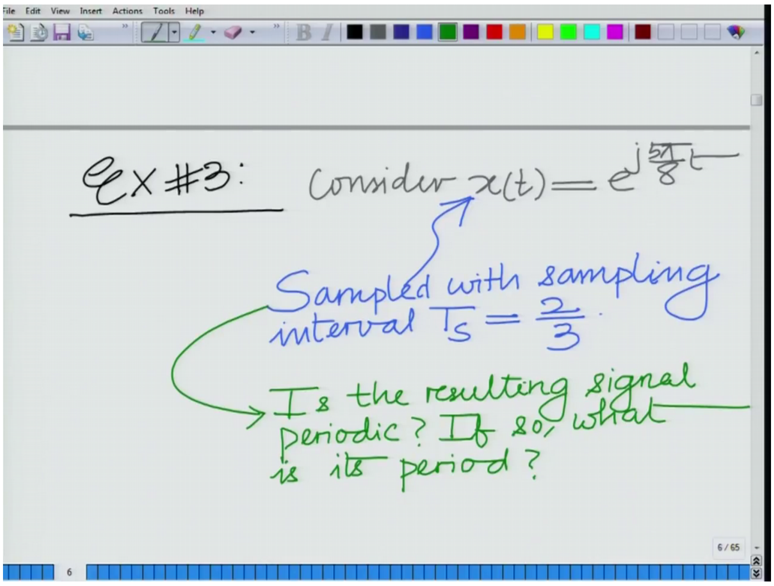

Let us do another example; consider the signal x t equals e to the power of j; 5 pi by 8 t this is x t. This is sampled with sampling interval t s is equal to 2 by 3; so, we have a signal that is sampled with sampling interval equal to 2 by 3. What we want to find out is if the resulting signal is periodic; if so what is the period? So, the question that we want to answer is; is the resulting signal periodic? If so, what is the period of this resulting signal?

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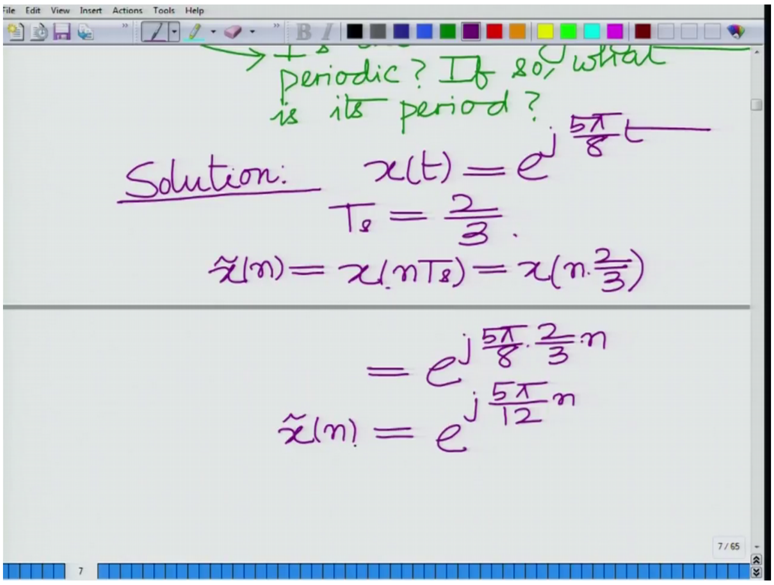

Now, we are given that the solution is as follows; we are given that x t equals e to the power of j; 5 pi by a t and t s is equal to 2 by 3; which means the nth sample. If we denote the sample discrete type signal as x tilde n; that is x at n times the sampling. So, x tilde n that is the nth sample of the discrete time signal is at n times t s; where t s is a sampling interval and sampling interval is given as 2 by 3.

So, this is going to be at n times 2 by 3; so, this is going to be a x of n times 2 by 3 which is therefore, equal to e to the power of j 5 pi by 8 into 2 by 3 over n, which is equal to e to the power of j 5 pi divided by 12 n; this is your resulting discrete time signal.

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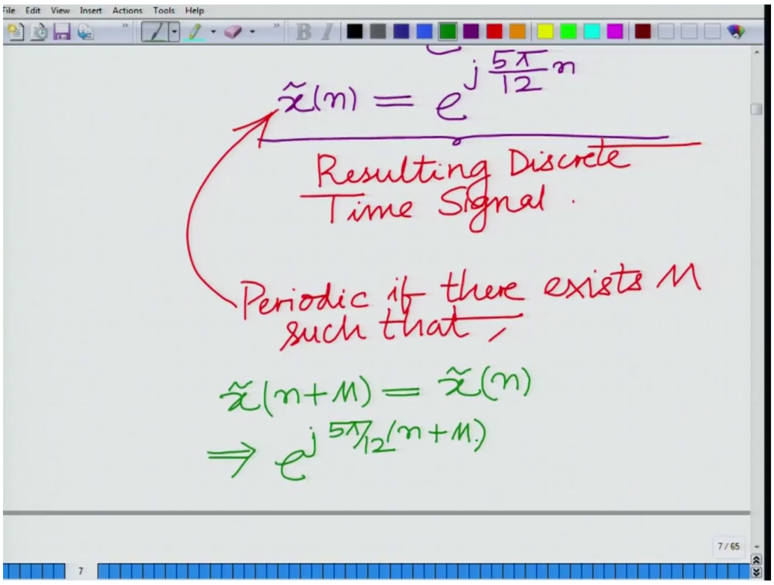

Now, this is periodic if there exists M; such that x tilde of n plus M equals, x tilde of n which implies e to the power of j 5 pi by 12 into n plus M.

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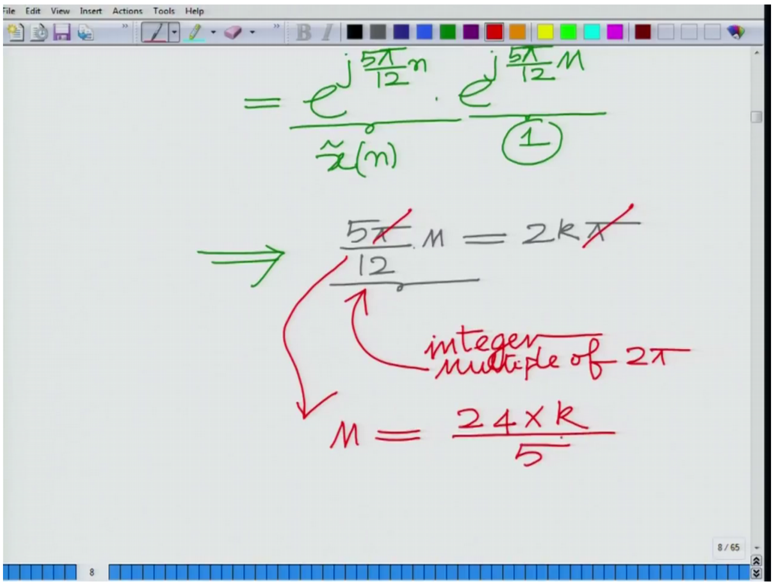

This can be simplified as e to the power of j 5 pi by 12 n into e to the power of j 5 pi by 12 M. So, this is equal to your x tilde n; if this should be equal to x tilde n, it implies that this should be equal to 1; this quantity e to the power of j 5 pi over 12 M should be equal to 1; only then will (Refer Time: 18:57) equal to x tilde n, which implies 5 pi by 12 times M should be a multiple of 2 pi integer. So, this quantity should be an integer multiple of

2 pi; which implies that M equals; well we cancel the pi, so m equals 24 times k divided by 5.

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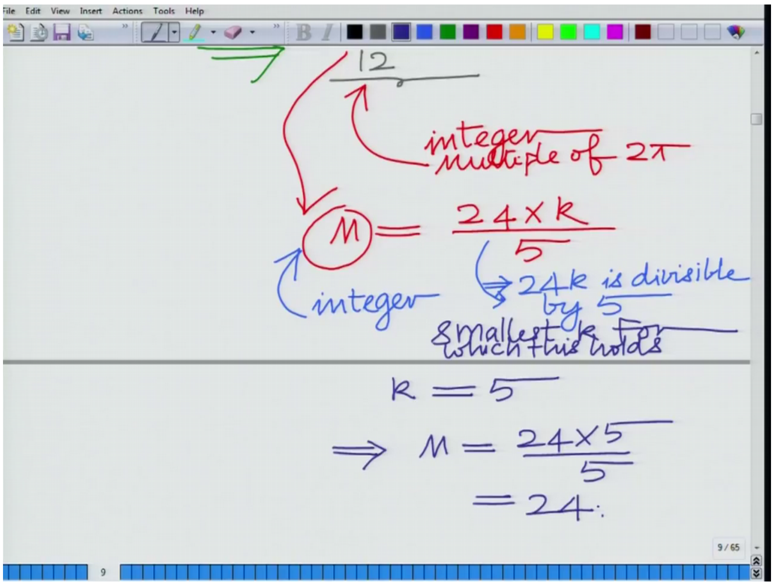

Now, if you observe this; you will notice that m is an integer which means 24 k should be divisible by 5 which means implies 24 k is divisible by 5.

Now, smallest k for which this holds is equal to; is k is equal to 5 which implies you can see because 24 plus and 5 do not have any common factors. You can see that the smallest k for which 24 k is divisible by 5 is; k equal to 5, I have to set k equal to 5, which implies M is equal to 24 into 5 divided by 5 equals 24 and in fact, this is the fundamental period M equal to 24.

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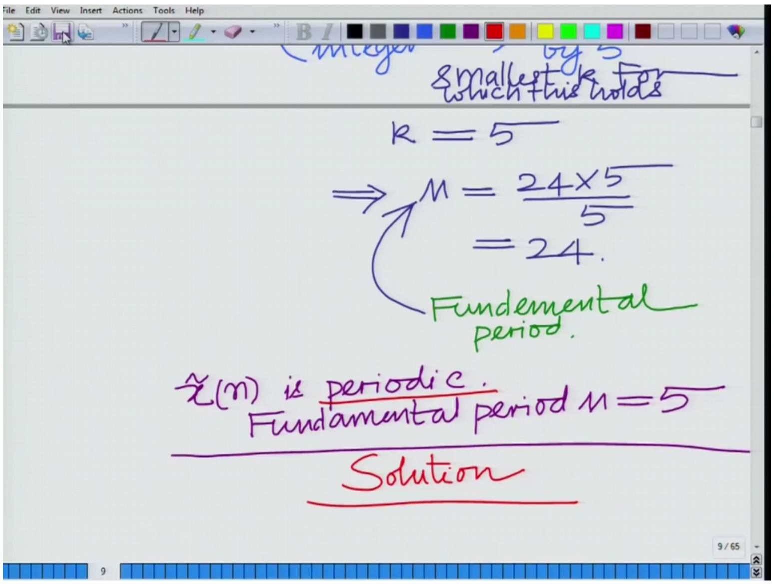

So in fact, it is periodic and fundamental period M is equal to 5 and this is the solution to the problem. So, x tilde n is periodic and the fundamental period is M equal to 5; the resulting discrete time signal x tilde n is periodic and the fundamental period is 5. So this practical; so these examples have probably helped you better understand solving these problems and appreciate the various properties of signals and systems. So, we will continue this in the subsequent modules.

Thank you very much.