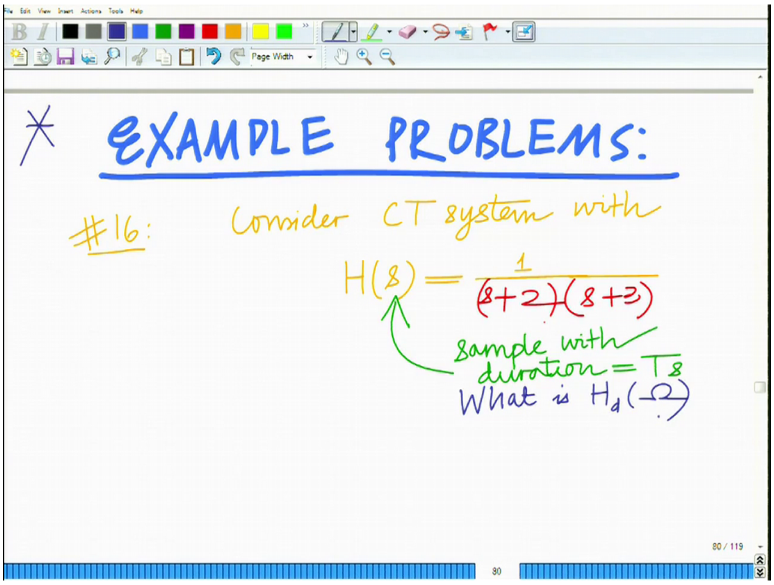
Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 66
Example Problems: DTFT – FIR, Discrete Fourier Transform

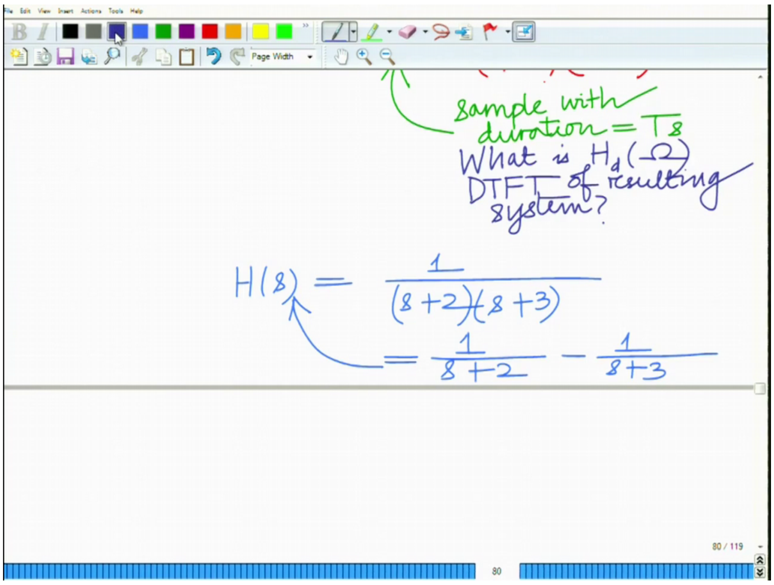
Hello welcome to another module in this massive open online course. So, we are looking at example problems for the DTFT. All right let us continue our discussion.

(Refer Slide Time: 00:22)



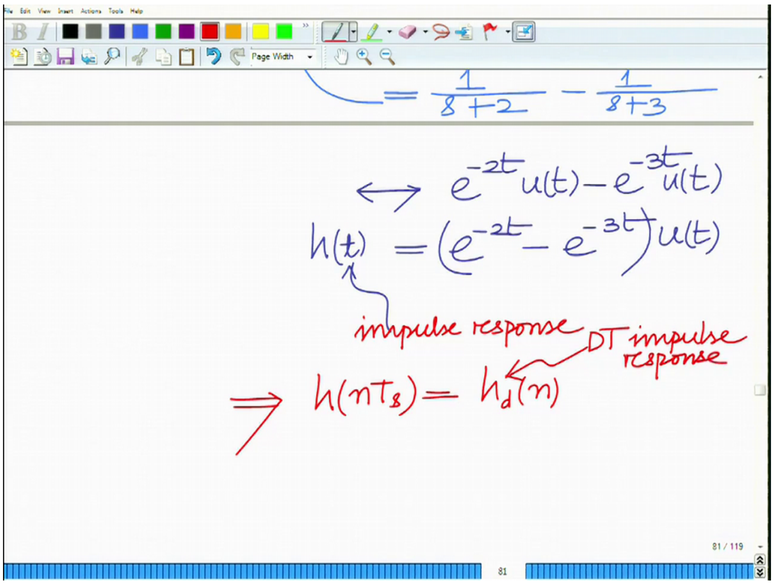
Now let us look at a Continuous Time system with the Laplace transform. So, remember we are looking focusing on this sampling problem number 16. Consider the continuous time system with the Laplace transform Hs equals 1 over s plus 2 into s plus 3. Now, if this is sampled at Ts; correct if we sample this impulse response, sample with duration equals Ts; then we want to find what is the corresponding what is the corresponding the discrete time transfer function the DTFT; DTFT of the resulting system correct ok.

(Refer Slide Time: 01:55)



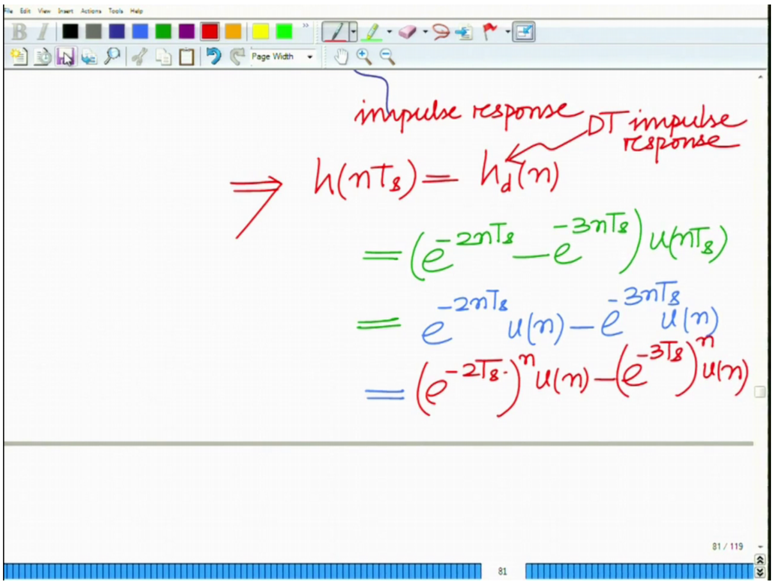
So, basically what we have here is you can see we have H of s equals 1 over s plus 2 into s plus 3 and we want to sample this correct. The sampling duration Ts; that is we want to take samples at intervals of multiples of Ts; that is 0, Ts, twice Ts, minus Ts, minus twice Ts and so on. All right that is a sampling process all right we know that ok. And therefore, now if you if you express this in partial fractions. So, I will have 1 over s plus 2 minus 1 over s plus 3 and considering a causal system ok.

(Refer Slide Time: 02:58)



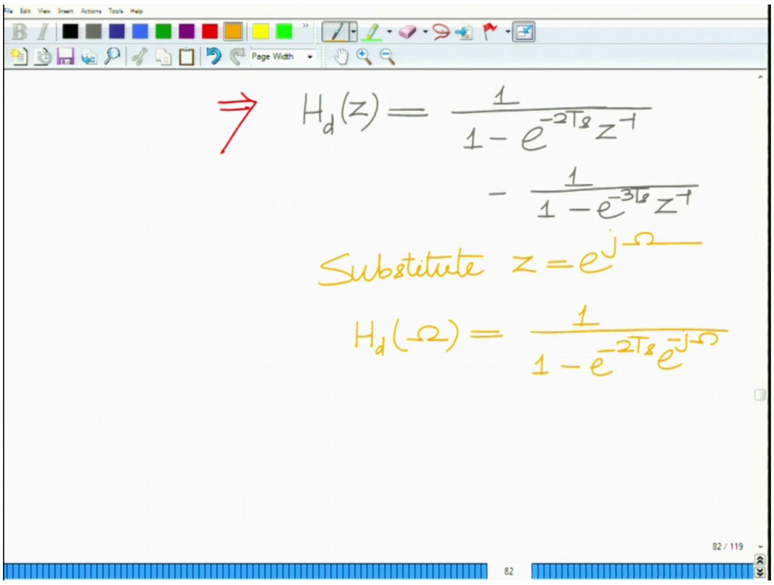
We have the corresponding inverse Laplace transform; e raised to minus twice T ut minus e raised to minus 3 T ut; that is this is a causal system; that is a right handed signal corresponding to this Laplace transform which is e raised to minus 3 T minus e raised to minus 3 T times ut and that is basically your ht. This is the impulse response ok, which implies if you sample this at Ts that is you sample it, at n times Ts; that is your nth sample of the discrete time impulse response that is the DT discrete time impulse response.

(Refer Slide Time: 04:17)



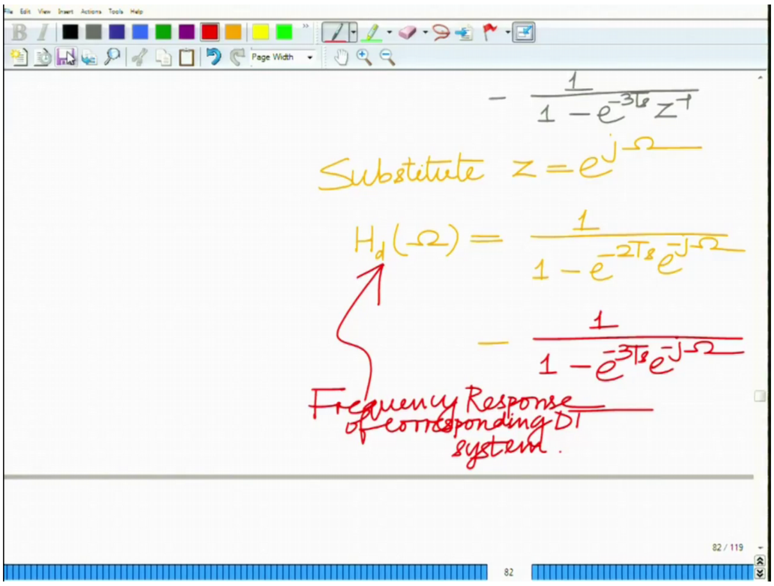
That will be equal to e raised to minus twice nTs minus e raised to minus 3 nTs u of nTs which is basically u of n ok. So, this is basically e raised to minus twice nTs u of n minus e raised to minus 3 nTs u of n. So, you can write this as e raised to minus twice ts, e raised to n un or a 1 raised to n un minus e raised to minus 3 times Ts raised to a 2 raised to n a 2 raised to the power of n un ok. And therefore, all right, so we have sampled the impulse response of the continuous time system and derived the corresponding discrete time impulse response. Now we can derive the DTFT of the system.

(Refer Slide Time: 05:31)



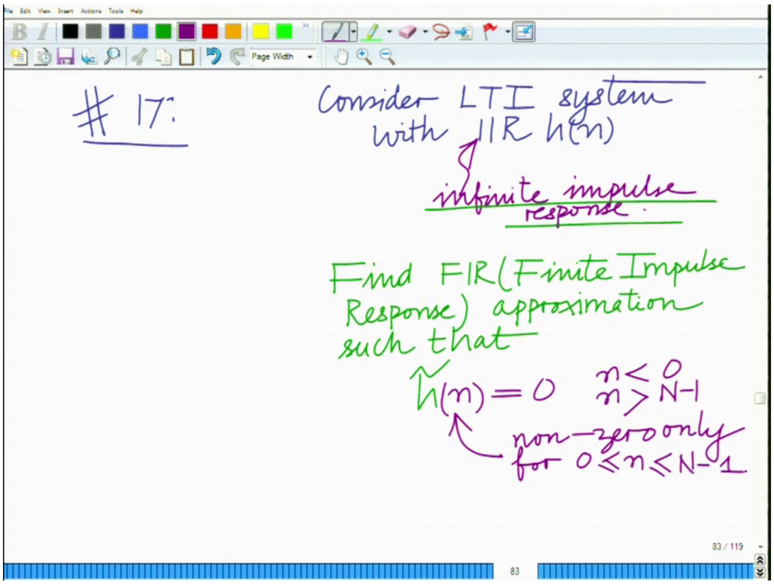
So, now substituting, so now Hdz taking the z transform what we have is we have an un for this causal system, the z transform is 1 over 1 minus e raised to minus twice nTs minus or 1 over 1 minus a that is; e raised to minus [music] twice Tsz inverse plus 1 over or minus 1 over 1 minus e raised to minus 3 Ts z inverse. Now, substitute z equal to to derive the DTFT, substitute z equals e raised to j omega and that gives us H d of omega e raised to minus j omega minus 1 over 1 minus e raised to minus e raised to minus 3 Ts e raised to minus j omega ok.

(Refer Slide Time: 06:49)



So, this is your DTFT; this is the DTFT of the, this is the DTFT of the transfer function correct. This is the DTFT of the frequency response of the corresponding ok. So, this is the frequency response of the corresponding discrete time system; frequency response of the corresponding discrete time system all right. So, that basically completes this problem where we are sampling it at sampling, sampling duration Ts or sampling frequency 1 over Ts and were finding the corresponding DTFT or the frequency response of the discrete time system ok. .

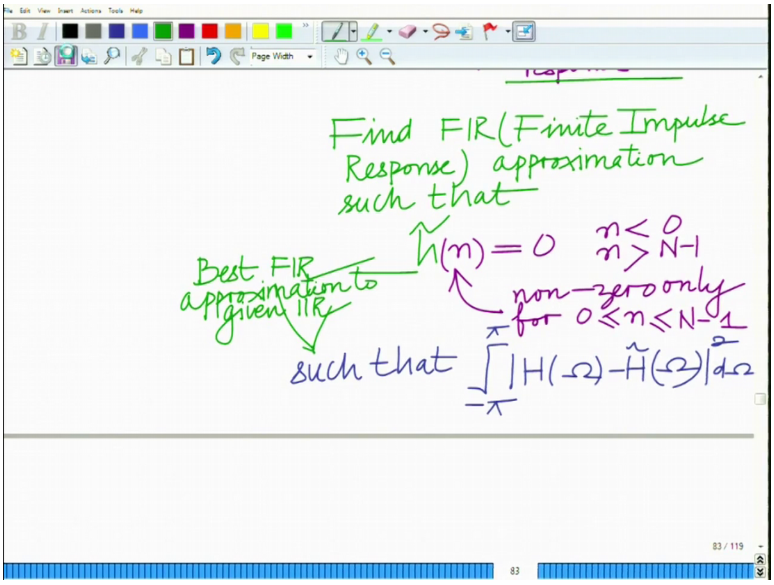
(Refer Slide Time: 08:14)



Let us look at the next problem for this we will continue a consider again the LTI system. Consider an LTI system with infinite impulse response hn ok. So, this is an infinite impulse response; which means hn is nonzero from minus infinity to infinity; it is infinite ok. So, it is nonzero over an infinite duration ok. Now what we want to do here, so this is basically your infinite impulse response ok. This is your IIR ok.

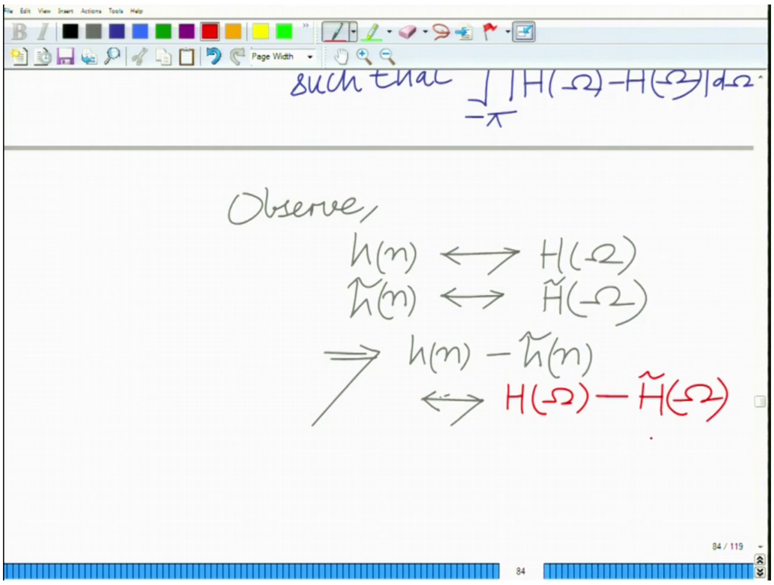
Now what we want to do is we want to find an approximation. Find finite impulse FIR approximation; find an FIR approximation such that find an FIR approximation such that h tilde n. So, this is a finite impulse response filter; H tilde n equals 0 for n less than 0 or n greater than n minus 1; nonzero only for 0 less than equal to n less than equal to.

(Refer Slide Time: 10:32)



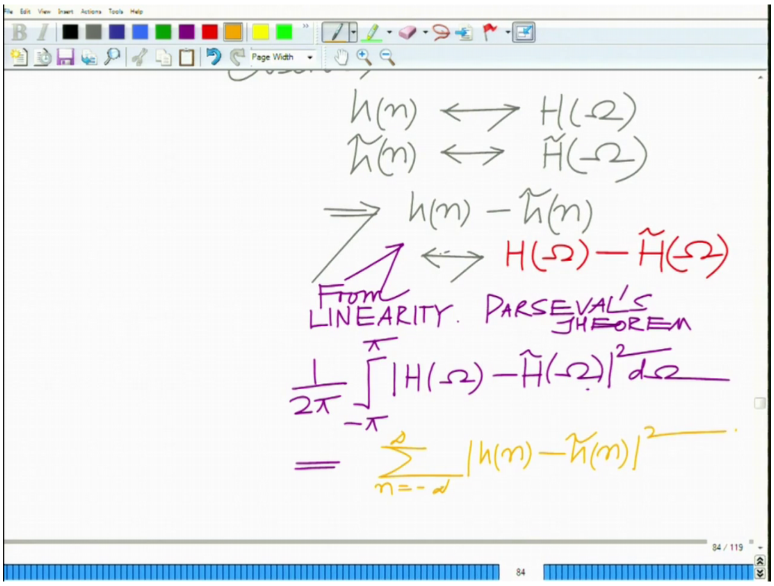
So, what we want to find is we want to find the best FIR approximation such that the following quantity; that is if you look at this quantity minus over minus pi 2 pi because the DTFT is periodic. We want this quantity that is H of omega minus H tilde of omega that is we want to minimize the error between the DTFT of the original filter. In fact, this is the squared error correct? This is the cumulative squared error between the DTFT of the original DTFT H omega of the original filter hn and the DTFT H tilde omega of the new filter H tilde ok. So, you want to find the best FIR approximation. So, what this gives us is the best FIR approximation to the given IIR that is the idea of this problem ok. So, that is best FIR approximation; this is the best FIR approximation to the given IIR filter ok.

(Refer Slide Time: 11:48)



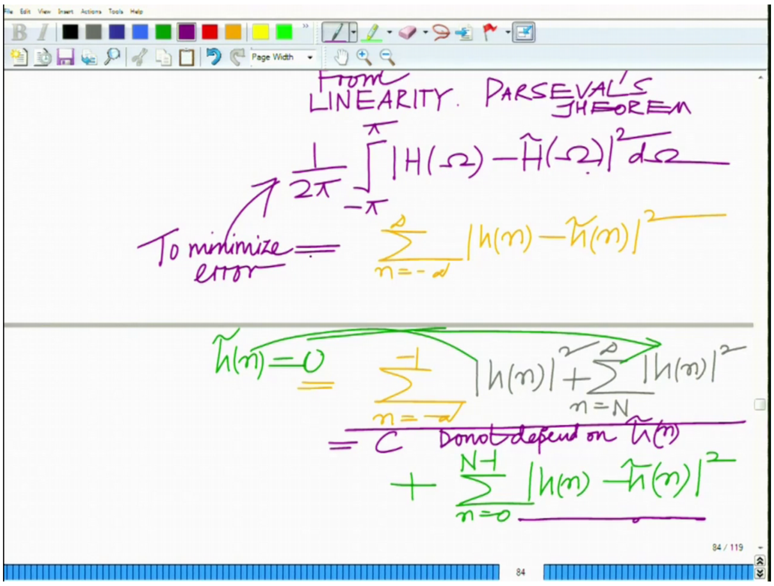
Now firstly, to solve this first observe that we have hn which has the DTFT or frequency response; H of omega and h tilde n has the dt of T; H tilde of omega this implies h of n minus h tilde n has the DTFT; H of omega minus H tilde of omega.

(Refer Slide Time: 12:31)



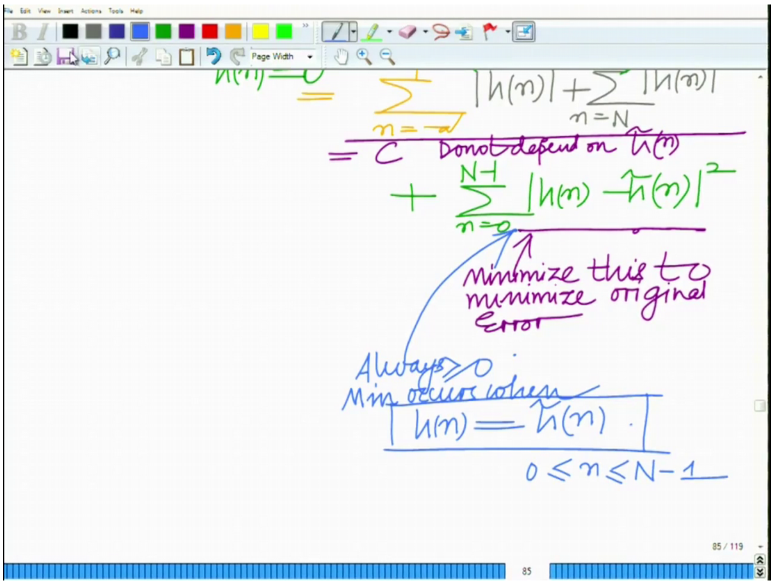
And therefore, now this follows from linearity remember ok. Ok that is h of n; that is h of n minus h tilde n that is the DTFT H of omega minus H tilde of omega. Now, therefore, if you look at this quantity minus 1 over 2 pi magnitude H of omega minus H tilde omega square from the Parseval's theorem from Parseval's theorem this is equal to summation n equals minus infinity to infinity magnitude hn minus magnitude h tilde n square.

(Refer Slide Time: 13:35)



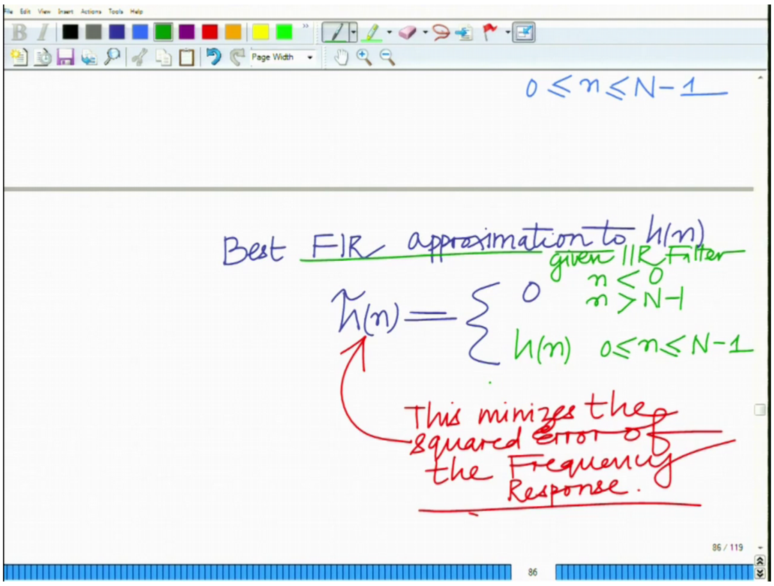
Now you can split this into three components; one from n equals minus infinity to minus 1. Now n equal to minus infinity to minus 1, h tilde n is zero. So, hn minus h tilde n is only hn. So, this reduces to magnitude h of n square. Plus now summation let us look at summation n equal to capital N to infinity. Again in this range h tilde n is 0. So, this reduces to simply; now this is because in this range h of n or h tilde n is equal to 0. Therefore, the only remaining component is summation n equal to 0 to N minus 1 hn minus h tilde n square. Now, remember these two quantities are therefore a constant. These do not depend on h tilde and therefore, this equal to constant c and this now to minimize this therefore, to minimize to minimize error one needs to minimize this quantity.

(Refer Slide Time: 15:09)



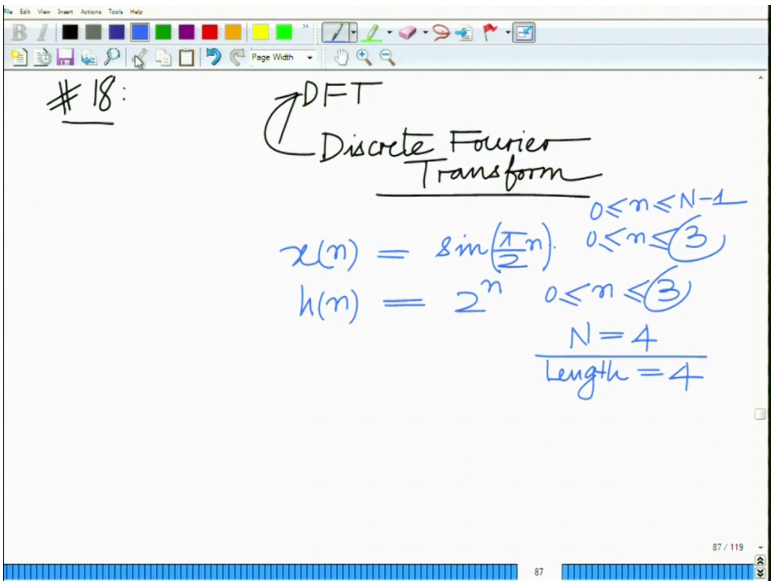
Minimize this to minimize the original error and this is minimum because look at this is square of a quantity; magnitude square is always greater than equal to 0. The minimum value is 0 and that occurs when h tilde n equals hn; minimum of this occurs is always greater than or equal to 0. The minimum occurs when h of n equals h tilde n for 0 less than equal to n; less than or equal to n minus 1 ok.

(Refer Slide Time: 16:14)



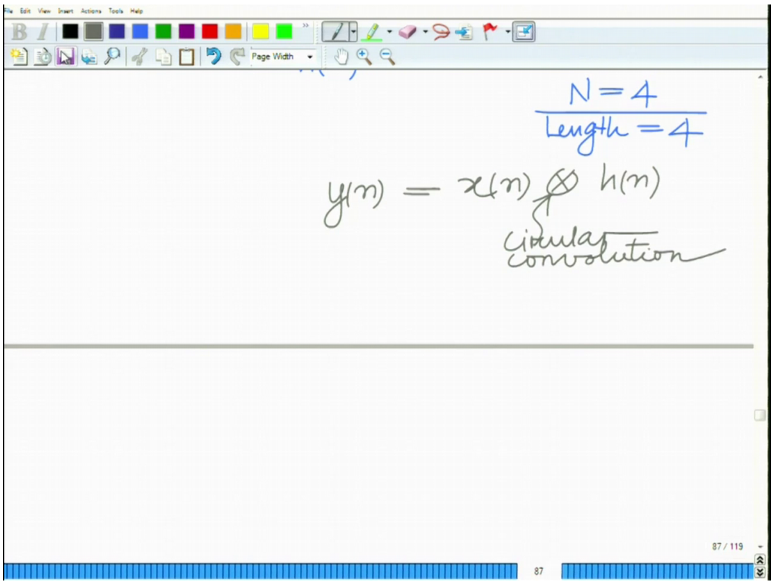
So, the best FIR approximation capital N tap FIR approximation; is h tilde n equal to 0 n less than 0 or n greater than n minus 1 and equal to h of n for 0 less than equal to n less than equal to N 1 ok. This minimizes the error; this minimizes the error in the frequency response; this minimizes the squared error minimizes the squared error of the frequency response all right. This gives the best FIR approximation to h of n which is the given best FIR approximation to the given IIR that is a given infinite impulse response filter h of n ok. So, this gives the best FIR approximation to the given infinite impulse response filter hn all right. So, let us continue to the next problem.

(Refer Slide Time: 18:00)



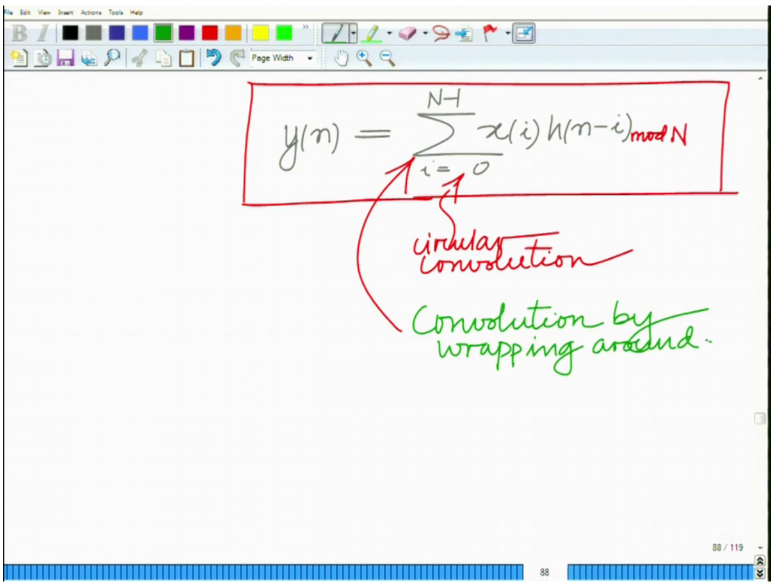
We have now we are going to start problems on the DFT. Remember this is for a finite length sequence; this is for the discrete Fourier transform and what we want is we were given two sequences; xn equals sin pi by 2 n and this is for 0 less than equal to n less than or equal to 3 and hn; the impulse response is basically 2 raised to n 0 less than equal to n less than equal to 3.

(Refer Slide Time: 19:49)



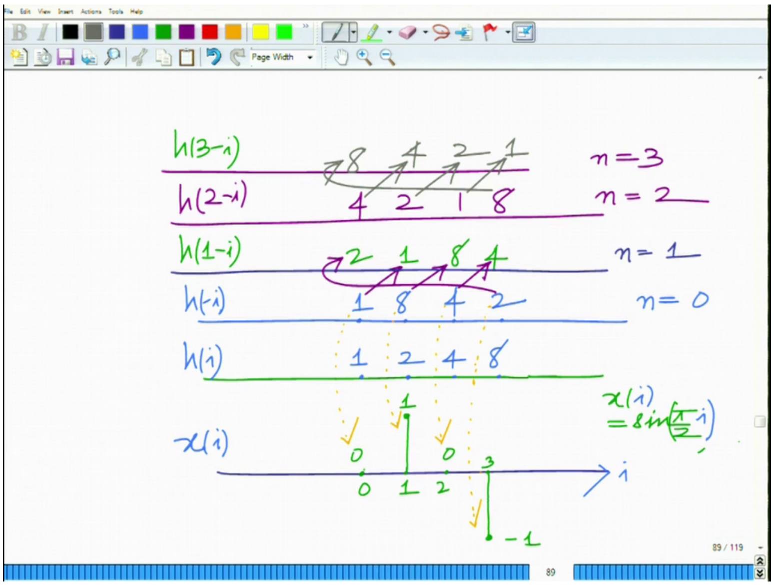
Now, what we want to do is we want to find. Now clearly you can see here for this finite length sequences N is equal to 4. So, we have 0 less than equal to n less than equal to N minus 1 ok; N minus one. So, capital N; that is length of the sequence is 4. Now what we want to do is we want to find yn which is the circular convolution of xn with hn ok. So, you want to find circular convolution of xn with hn ok. Remember for finite length sequences one can define a circular convolution which is nothing but a wraparound convolution and that is defined as the following with the following expression.

(Refer Slide Time: 20:21)



That is you have y of n equals summation i equals 1 to or i equals 0 to N minus 1 x of i h of n minus i. Now this is your normal convolution. In circular convolution this N minus i will be modulo capital N ok. So, this is N minus i modulo N in which case in this case this modulo of 4. That is so, you are basically wrapping the sequence around and you are performing the circular convolution, wrapping the sequence around and basically. So, this for performs convolution by wrapping with the sequence around ok

(Refer Slide Time: 21:37)

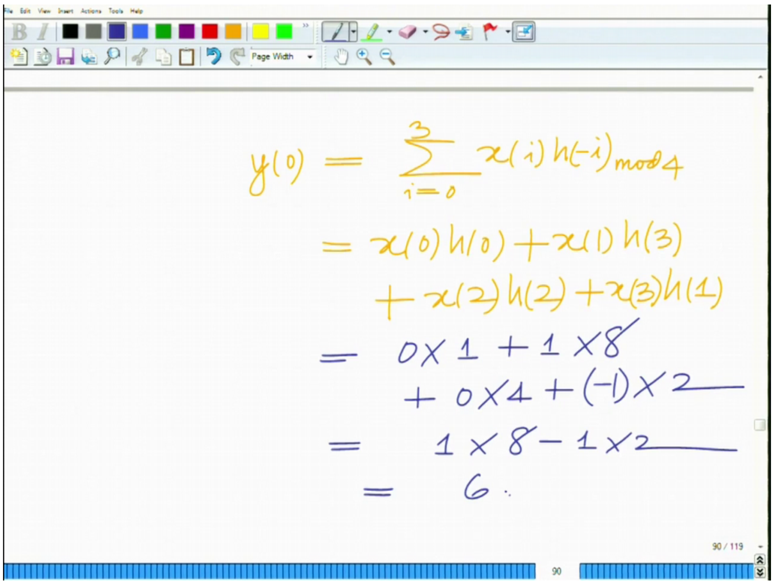


So, let us take a look at this. What we are given is we are given this sequence xn. Let me just draw it in a different page probably that will help. So, we are given a sequence so, this is 0 and we have well x of n is sin n pi by 2 n. So, sin x of 0 is 0; this is 0; x of 1 is sin pi by 2 which will be 1. So, this is 0; this is 1; x of 2 is sin pi by 2 into 2 which is sin pi which is again 0. I am talking I am sorry I am talking about h of I am talking about x of n here and x of 3 sin pi by 2 into 3 is that is sin 3 pi by 2 that is minus 1. So, this is x of n which is equal to sin pi by 2 into n.

And h of n; remember. Now let us plot h of n and this will be a little is an interesting thing. So, what will happen here is h of n, h of 0 is 1. So, i am just not going to draw the stems here but i am just going to mark the values. This is h of n; h of 0 is 1; h of 1 is 2 to the power of 1 is 2; h of 3 is 2 to the power of 3; h of 2 is 2 to the power of 2 which is 4 and h of for this thing h of 3 is 2 to the power of 3 which is 8 ok. Now when we draw, so this is h of n; now at n equal to 0 remember I have to consider h of n minus i. For each x of i have to consider so, this is corresponding to i equal to 0. So, this is h of n minus i correct or this is remember a term time n equal to 0; I am sorry this is n equal to 0. Let us make this as i; index as i because the summation is with respect to the index i. So, this is x of i sin pi by 2 i ok. So, this is basically your index i. So, this is h of i. Now n equal to 0; I have to consider n minus i will be h of minus i and h of minus i remember this will be modulo 4.

So, h of 0, h of minus 0 is 0; this will be 1; h of minus 1, h of 0 that will be h of minus 1, but minus 1 modulo 4 is 3. So, this will be h of 3 which will be 2 to the power of 3 that is 8. So, this will be 8, h of minus 2 will be h of 2; which is 4 and h of minus 3 modulo 4 will be h of 1 which is 2. So, this is your h of minus i. This is corresponding to h. So, now here I have x of i; corresponding to n equal to 0, I have x of i into now h of minus i modulo 4. So, that will be now you can see 0 into 1 plus 1 into 8 if you multiply the corresponding term plus 4 into 0. So, that will be and that will be if you look at this so, H of.

(Refer Slide Time: 25:50)

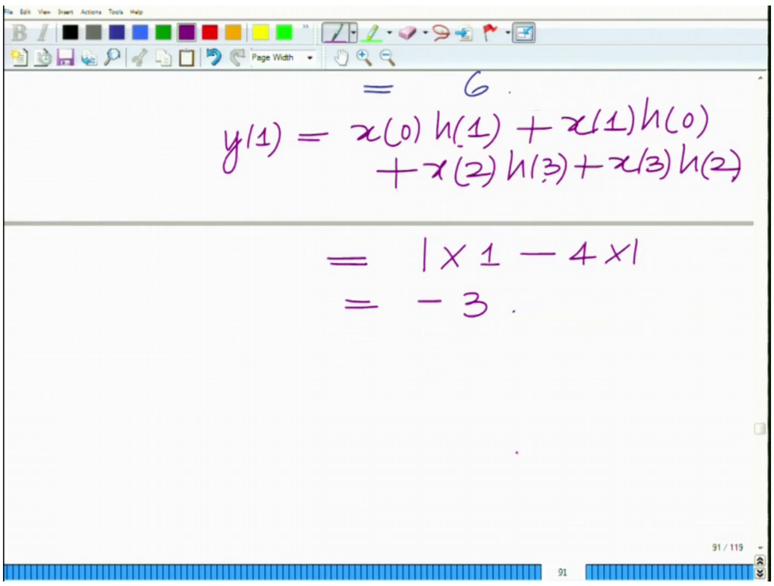


So, so we have resulting y of 0 equals summation i equals 0 to 3 x of i into h of n minus i; which is h of minus i modulo 4; which will be x of 0 into h of 0 plus x of 1 into h of minus 1, but h of minus 1 modulo 4 is nothing, but h of 3 plus x of 2 into h of minus 2 which is modulo 4 which is 2 x of 3 into h of minus 3 which is h of 1 ok. And that will be now if you look at this that will be 1 into 8 minus 1 into 2 ok. So, now let me just write it a little bit more clearly; x of x of 0 is 0. So, 0 into h of 0 which will be 1 plus x of 1 is 1 into h of 3 which will be 8 plus x of 2 which is 0 times h of 2 which is 4 plus x of 3 which is minus 1 into h of 1 which is 2. So, that is 1 into 8 minus 1 into 2; which is equal to 6 and that is what you get from the figure also. That is 1 into 0 plus 8 into 1 plus 4 into 0 minus 2 into plus 2 into minus 1 ok.

Now, similarly now what happens in the next time instant, look at the next time instant. Next time instant n equal to 1 you will have h of 1 minus i. So, h of 1 minus 0 that will be h of 1. So, h of 1 minus so that will be h of 1 that is 2; h of 1 minus 1 that is 0 that will be 1; h of 1 minus 2 that will be h of 1 minus 2 that will be h of minus 1 modulo 4 is h of 3. So, that will be 8 and h of 1 minus h of minus 2 modulo 4 is 2; so, that will be 4. So, now what you can see here? At n equal to 1, so this 2 is moving to the left and the rest all are rotating to the right.

So, this is basically what is happening is in the next time instant the 2. If you can take a look at this something very interesting that is happening here. So, that 2 is basically rotating to the left and the rest all are moving one step to the right and therefore what you have now is basically; this is a circular convolution as you can see at each time instant the sequence is basically wrapping around itself. So, the rightmost quantity is moving to the left and all the others are shifting one place to the right. So, it is basically with each time step it is wrapping around. So, this is a circular convolution ok.

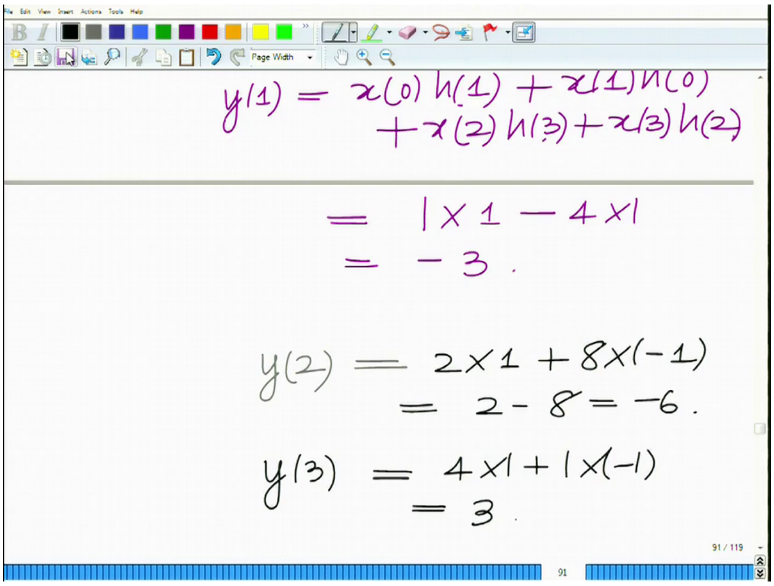
(Refer Slide Time: 29:22)



So, y of 1 will therefore be x of 0 into h of 1 plus x of 1 into h of x of i; x of 1 into h of 1 minus 1 that is h of 0 plus x of 2 into h of 1 minus 2 h of minus 1 modulo 4 is h of 3 plus x of 3 into h of 2 ok. And you can simplify this that would be 1 into 1 minus 4 into 1 which is equal to minus 3 ok. And that you can see from the figure also. You can see that this will be 2 into 0 plus 1 into 1; that is 1 plus 8 into 0 minus 4 into 1; so that is 1 minus 4 that is minus 3 ok.

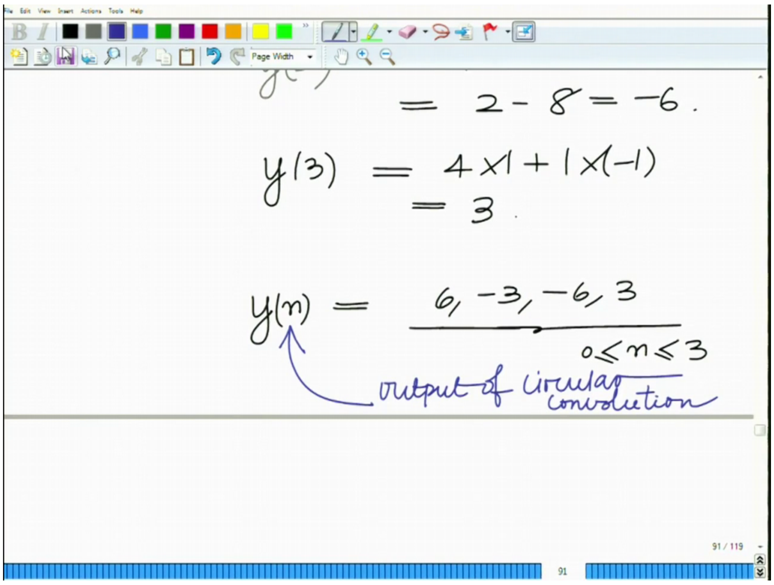
And the next time instant again n equal to 2; you have h of 2 minus i. So, therefore 4 will move to the right. So, we will have 2, 1, 8 and the next time instant you have n equal to 3. So, you will have h of 3 minus i and what you will have here is the 8 will move to the left and each will move one step to the right; 8, 4, 2, 1 and you can calculate again the circular convolution.

(Refer Slide Time: 31:06)



What you will have is y of 2 equals 2 into 1 plus 8 into minus 1 that is 2 minus 8 equals minus 6 and y of 3 equals 4 into 1 plus 1 into minus 1 that is 4 minus 1 equals 3.

(Refer Slide Time: 31:37)



So, y of so, the y sequence; this will be 6 minus 3 minus 6 comma 3, for 0 and defined obviously, for 0 less than equal to n less than equal to 3. This is the output of the circular convolution. So, this is the output of the circular convolution of the given sequence and now realize that we have to be used a time domain interpretation for the circular convolution. In the subsequent module what we are going to do is we are going to carry it out in the frequency domain.

By using the DFT of this finite length sequences we can use the DFT and remember in the DFT domain the circular convolution becomes a multiplication of the corresponding DFT. So, we will use that principle to evaluate the output of the corresponding circular convolution using the DFT in the frequency domain all right. So, we will stop here and continue in the subsequent module.

Thank you very much.