Principles of Signals and Systems
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Lecture – 64
Example Problems: DTFT – Impulse Response

Hello, welcome to another module in this massive open online course. So, we are looking at example problems in DTFT, there is a discrete time Fourier transform. So, let us continue our discussion all right.

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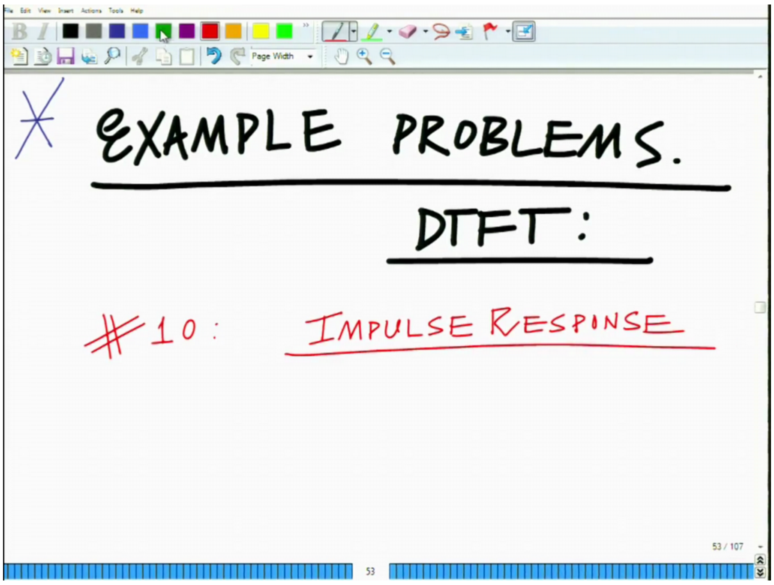

So, we are looking at, all right example problems in DTFT. So, let us start this problem number ten, if I remember correctly. Let us start with you know this problem deals with the frequency response, not frequency response actually the difference equation or impulse response in fact the impulse response of the given discrete time system. So, we have the difference equation.

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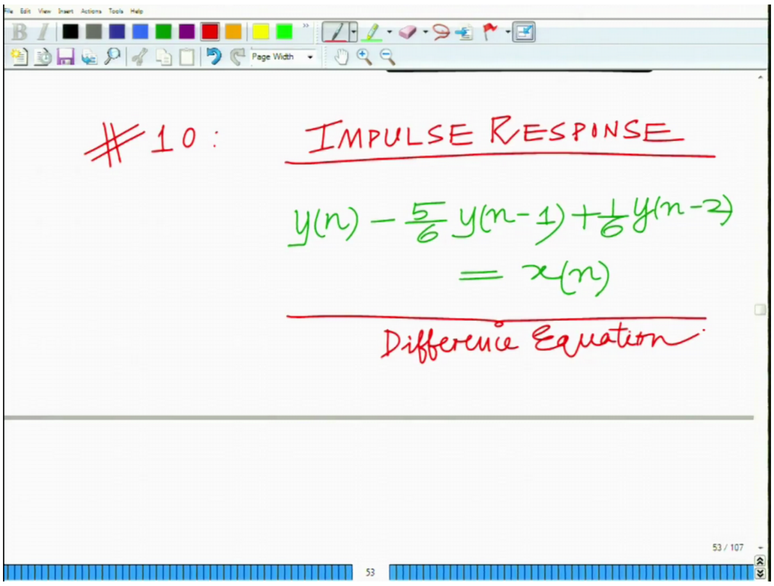

That is yn equals or yn minus 5 by 6 yn minus 1 equals 5 n minus 5 by 6 yn minus 1 plus 1 by 6 yn minus 2, this equals xn ok, and basically this is the given difference equation ok. So, this is the discrete time difference equation, something that similar is very similar to a differential equation for the continuous time.

So, this is also known as a difference equation. In fact, this is a constant coefficient difference equation all right, we have seen how to solve this or we have seen how to derive the impulse response from the constant coefficient difference equation using the discrete time Fourier transform ok.

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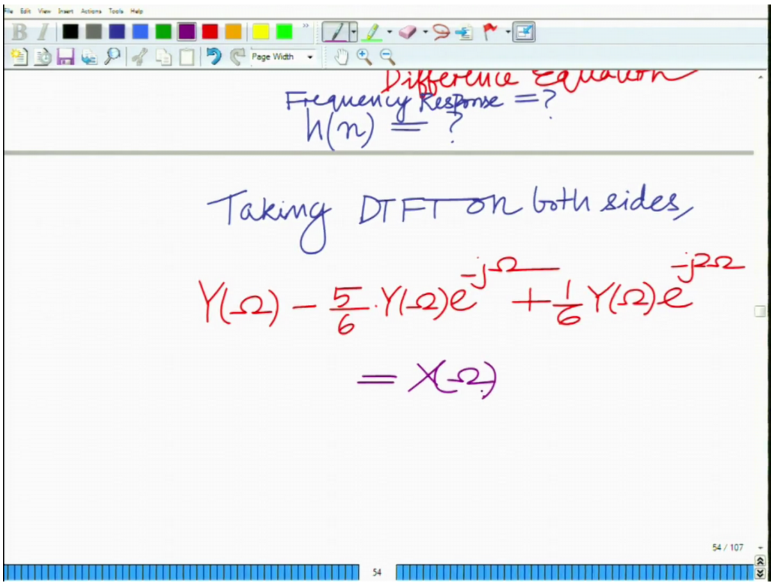

Now, once again; so what we want to find for this problem is, what is the impulse response, that is for this system first, what is the impulse what is the frequency response, which should be very easy to find, what is the frequency response and what is the impulse response. Now frequency response is simply obtained by taking the DTFT on both sides, taking the DTFT on both sides what we have is that DTFT of yn is Y omega minus DT 5 by 6 DTFT of yn minus 1 is Y omega into e raised to minus g omega, because time shift becomes modulation in frequency plus 1 over 6 DTFT of Y 2 omega yn minus 2 is Y omega e raised to minus j 2 omega.

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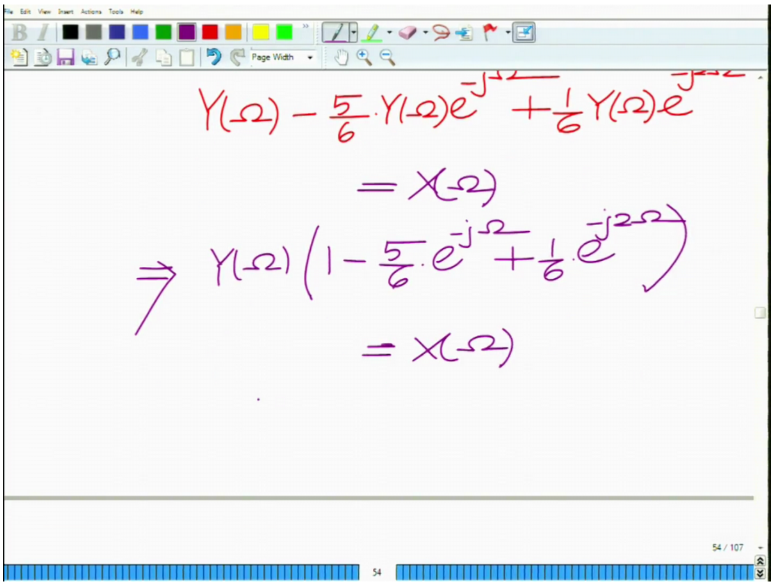

This should be equal to X of omega, which implies that is H of omega which is, which implies just to write it a little bit more clearly Y of omega or Y of omega into 1 minus 5 over 6 e power minus j omega plus 1 over 6 e power minus j 2 omega equals X of omega.

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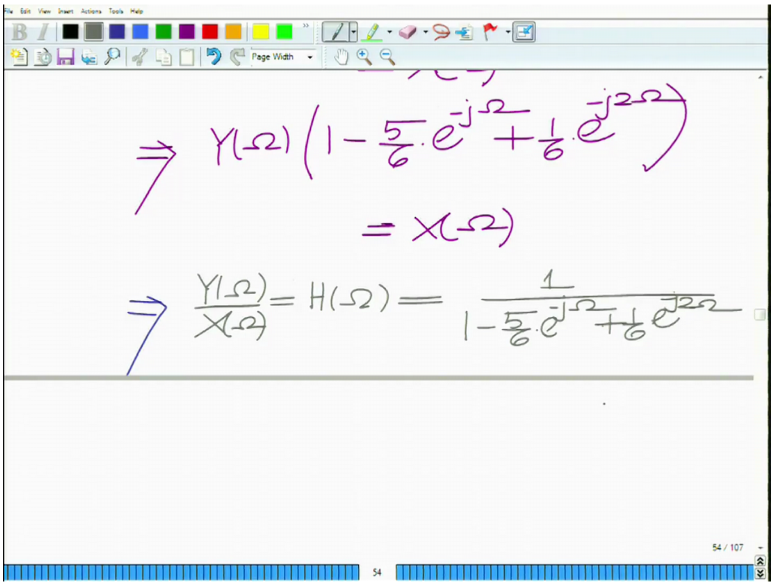

It implies Y of omega over h of X of omega equals H of omega divided by 1 minus 5 over 6 e power minus j omega plus 1 over 6 e raised to minus j 2 omega.

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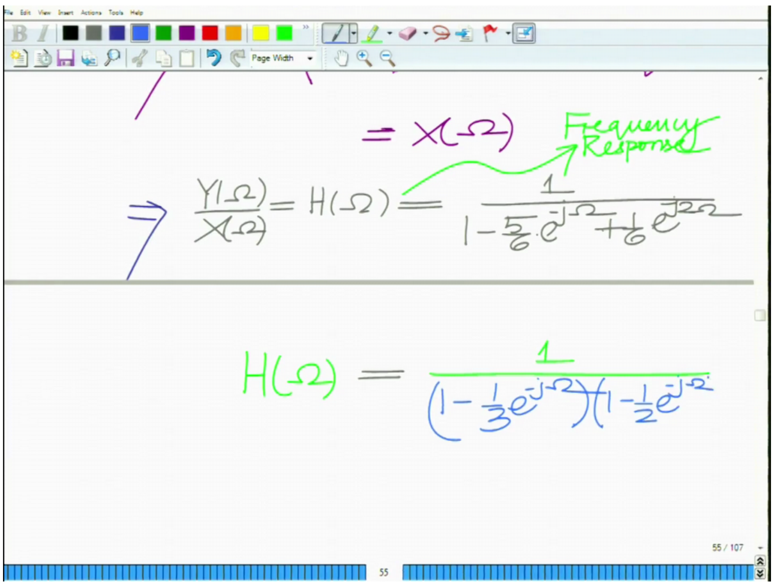

Now what I am going to do is. I am going to first start by factorizing this all right, factorizing this all right. And basically from that the partial fraction expansion and from that derive the impulse also. So, this is the frequency response h omega ok. So, this answers the first part of the question. No remember this is basically, already your frequency response of the discrete time LTI system ok. Now we have to derive the impulse response ok.

So, if you take the inverse DTFT of the frequency response you get the impulse response. So, I have H of omega equals 1 over and you can easily factorize this. This you can see is 1 minus 1 1 over 1 minus 1 over 3 raised to minus j omega into 1 minus 1 over 2 e raised to minus j omega ok. And now split this into partial fractions

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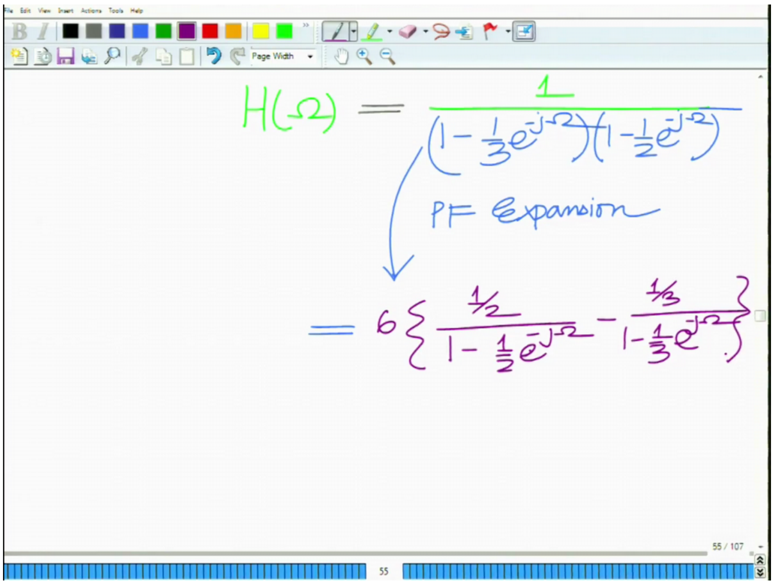

The partial fraction expansion PF expansion use the partial fraction expansion and this gives you something very simple, this is 6 times half over 1 minus half e raised to minus e raised to minus j omega minus 1 over 3 1 minus 1 over 3 e raised to minus j omega and now you take the inverse DTFT of each component

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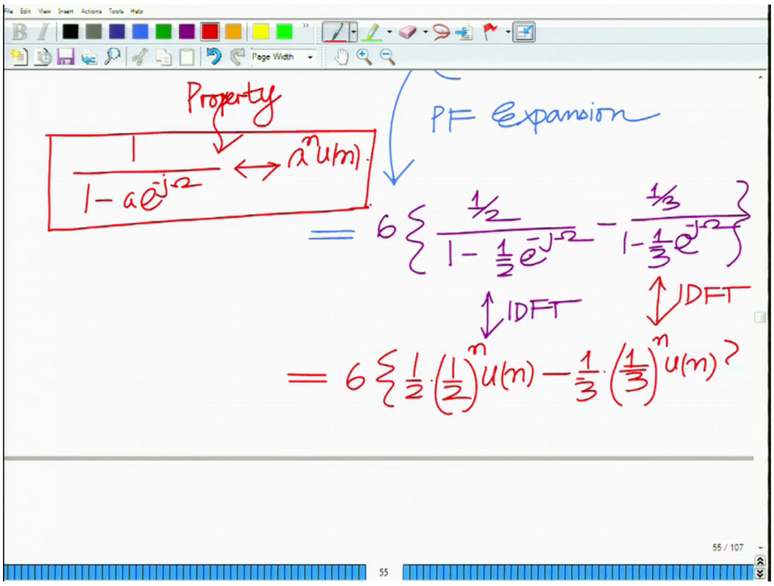

You take the IDFT, take the IDFT and we will use the property, basically 1 over 1 minus a e raised to minus j omega has the IDFT a raised to n un. So, this is the property that we are going to use ok.

So, this is the and using this property we can see taking the IDFT we have 6 half one over 1 minus half e raised to minus j omega is half raised to the power of n un minus 1 over 3 times 1 over 1 minus 3 e raised to minus j omega is 1 over 3 raised to n un, which is basically.

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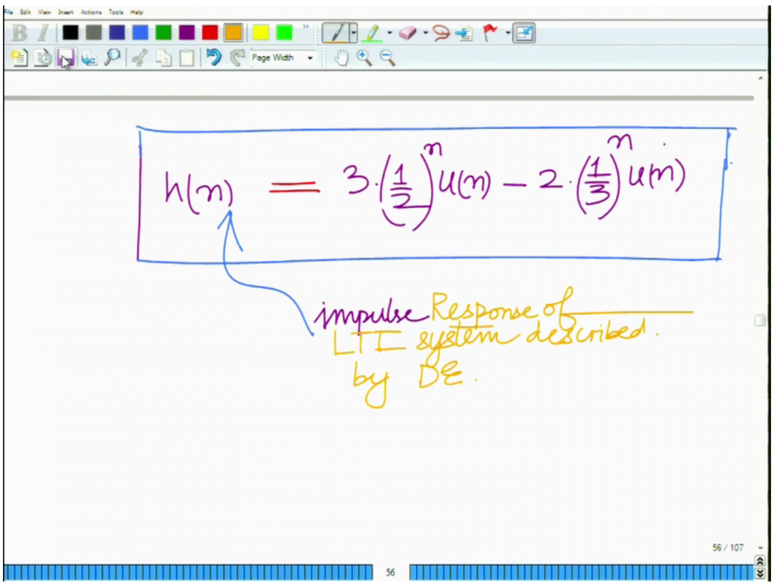

Now if you simplify this, this will give you, well 3 times half raised to the n un minus 2 times 1 over 3 raised to the power raised to the power n un. So, this is your h of n, this is the impulse response of the, this is the h of n ok. This is the impulse response of the LTI system described by the difference equation described by the given difference equation all right.

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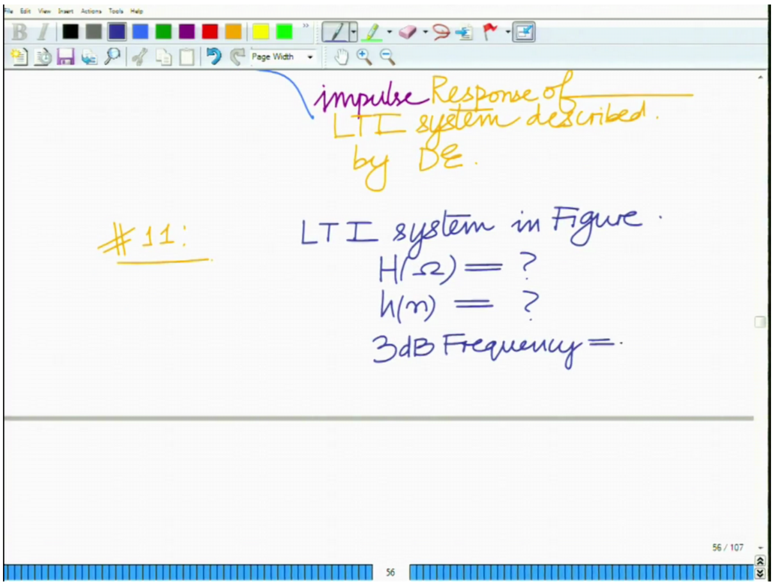

So, now let us proceed to problem number eleven which is the following all right. So, we consider the LTI system given below and we have to find what is the frequency response. You have to find a couple of thing what is the frequency response H of omega, what is the impulse response H of omega and also for this filter, what is the 3 dB frequency. Remember 3 dB frequency you look at a low pass filter, the 3 dB frequency is defined as that point on frequency at which the amplitude is basically 1 over square root 2 that of the maximum.

So, basically the power of that point corresponds to half of the maximum half at the maximum frequency all right. So, we require to find also the 3 dB frequency of this filter ok, in that sense. So, let us start with the impulse response. Now first let me describe the figure the figure is very simple, it is a very simple system.

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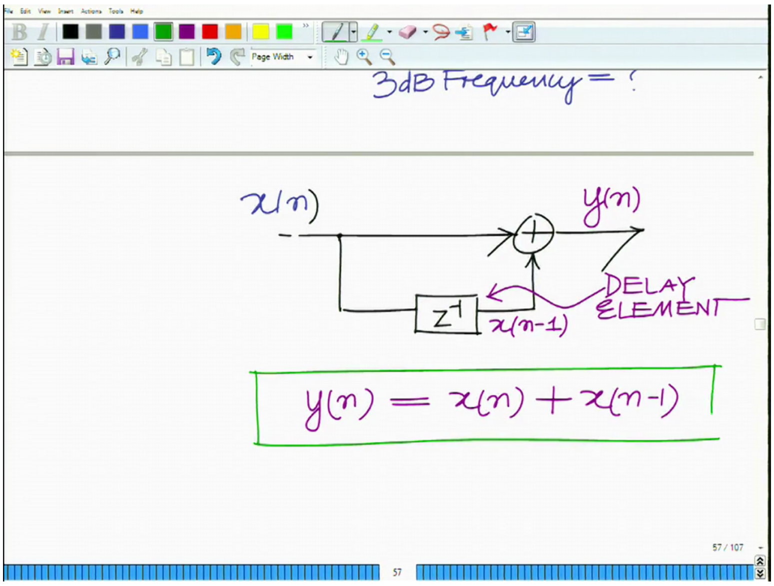

So, I have the input x of n ok. So, I have x of n, x of n going through the summer and also x of n which is being going to this book z inverse which we already know corresponds to a delay all right. So, this is basically your delay element or delay block and this is your y ok.

And you can see if xn is the input, the output here is xn minus 1 and you are summing xn and xn minus 1. So, the difference equation that describes, this is basically yn, its very simple equals xn plus xn minus 1. So, this is a difference equation that describes the above system

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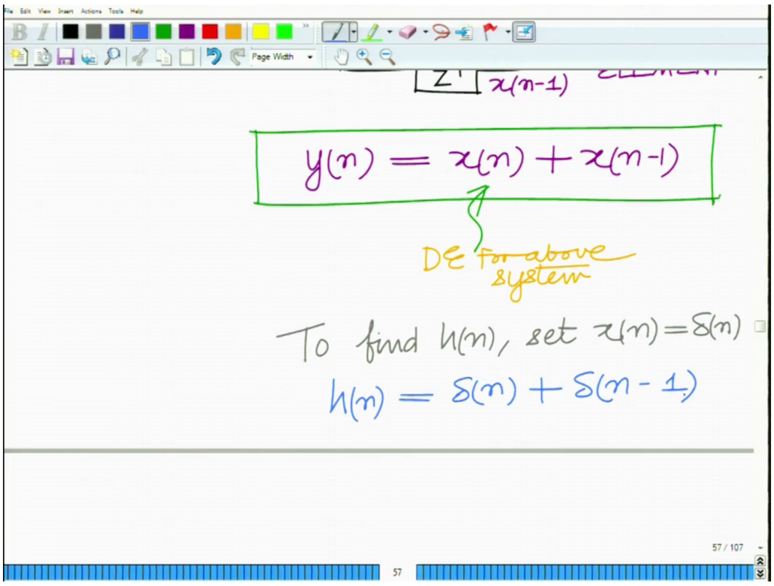

So, this is the difference equation that describes the difference equation for the above system. Now to find impulse response, we said impulse response is basically the response of the system to an impulse all right. So, we set xn to be the impulse delta n and then we find the response or the output of the given LTI system and that is very simple.

So, now to find, to find hn set xn equals delta n and what we get is, hn equals delta n plus delta n minus 1. Now you can see h of 0 equals delta 0 plus delta 1 equals 1.

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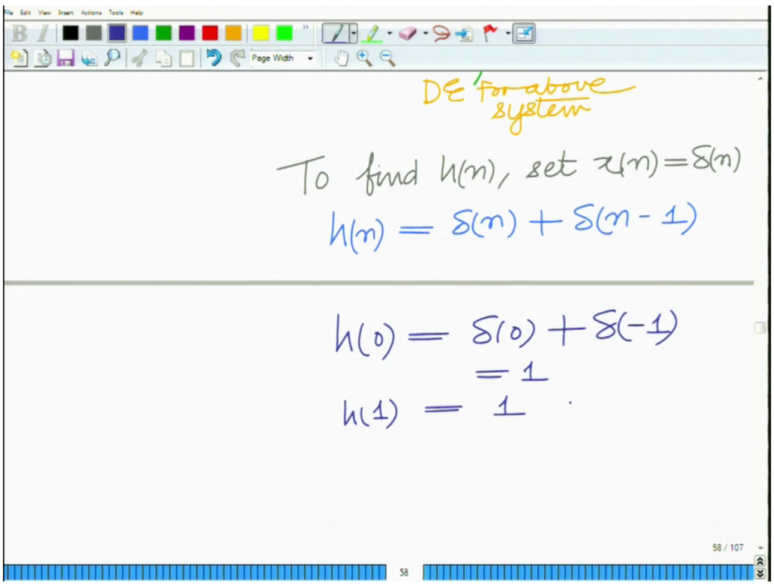

Similarly you can see h of 1 equals 1 and you can see that h of n for all other n equals 0, for n not equal to 0 or 1.

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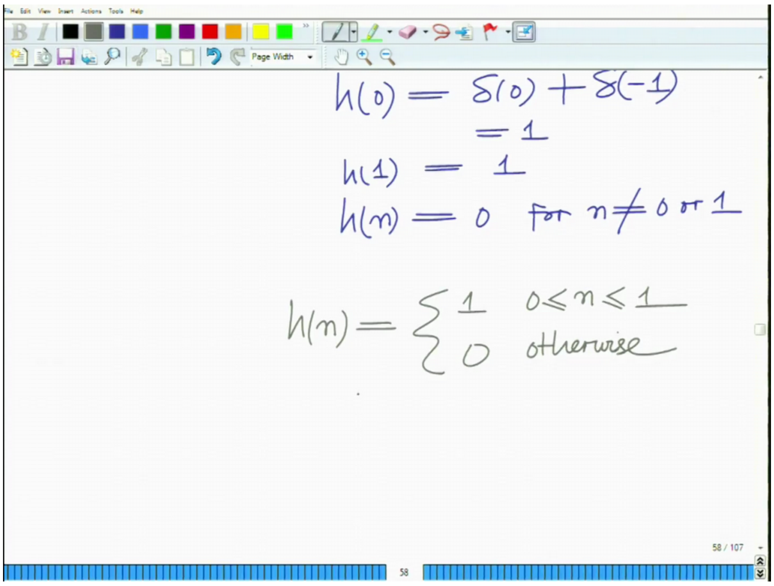

So, the impulse response is very simple; that is for n equal to 0, h of 0 is 1 for n equal to 1 h of 1 is 1 and for all other n h of n is 0.

So, now the impulse response is basically can be characterized as h of n equals 1 0 less than equal to n less than or equal to 1 and this is 0; otherwise the impulse response is 0 otherwise, and this is basically your impulse response of the system is basically your impulse response and now what we also get is X of omega.

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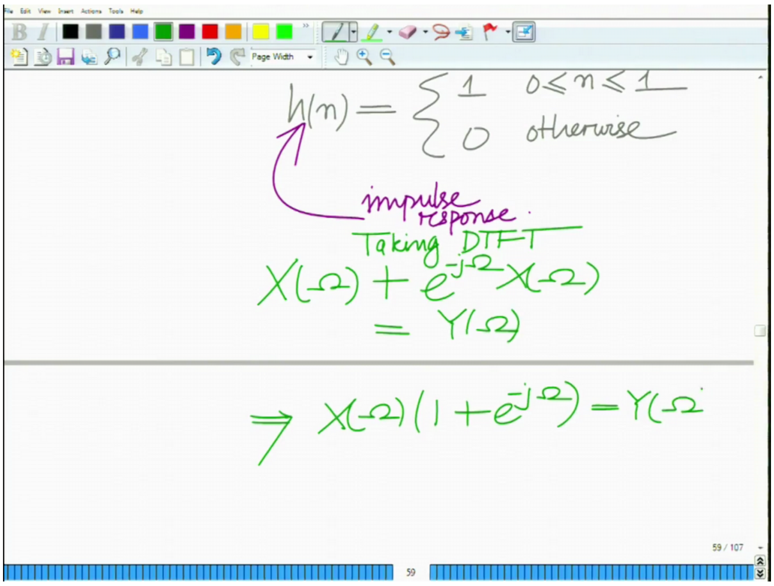

Now taking the DTFT to find the frequency response we have X of omega plus e raised to minus j omega xo omega, this is equal to. Well this will be equal to Y of omega taking the. So, this is basically e raised to minus j omega X of omega; that is a DTFT of xn minus 1 all right, if X of omega is the DTFT of xn ok.

And on the right hand side taking the DTFT; that means, Y of omega; so this implies X of omega into 1 plus e raised to minus j omega equals Y of omega implies H of omega which is Y of omega by X of omega; that is equal to 1 plus e raised to minus j omega which is equal to.

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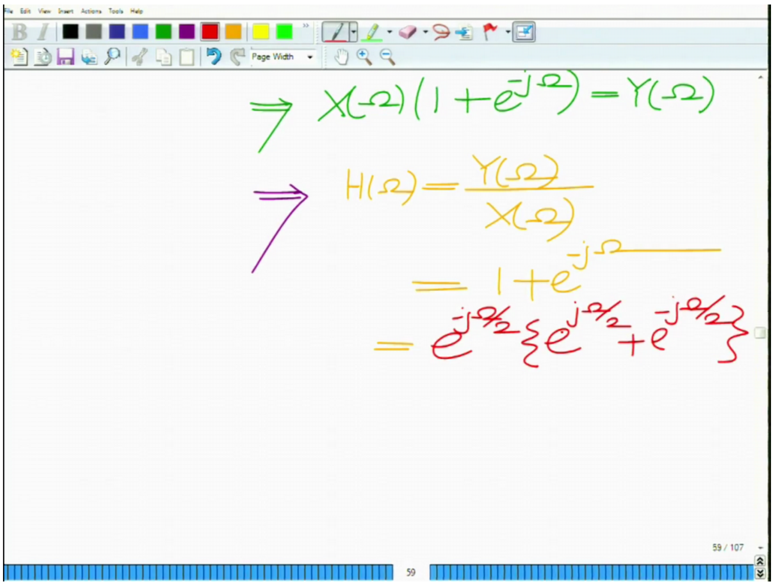

You can further simplify this as take e raised to minus j omega over 2 are common. This will be e raised to j omega over 2 plus e raised to minus j omega over 2, but this e raised to j omega by 2 plus e raised to minus j omega by 2 is cosine omega by 2.

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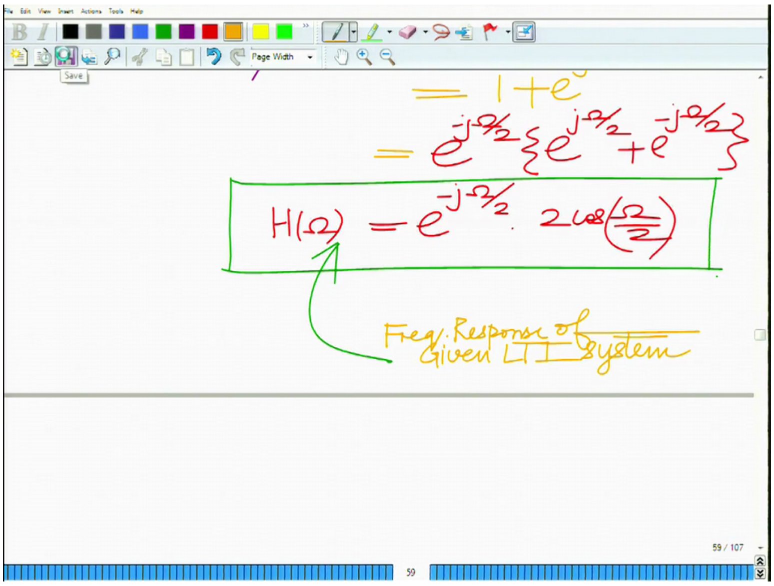

So, this is simply e raised to. In fact, this is 2 cosine omega by 2. So, this is e raised to minus j omega by 2 2 cosine omega by 2 and this is H of omega ok. So, this is your a frequency response of the given LTI system. In fact, you can treat it as a filter. In fact, it will be a low pass filter, we will see that. So, this is the frequency response, this is a frequency response of the given LTI system and if you look at the magnitude response, magnitude of H of omega you can see, this is clearly 2 magnitude cosine omega over 2.

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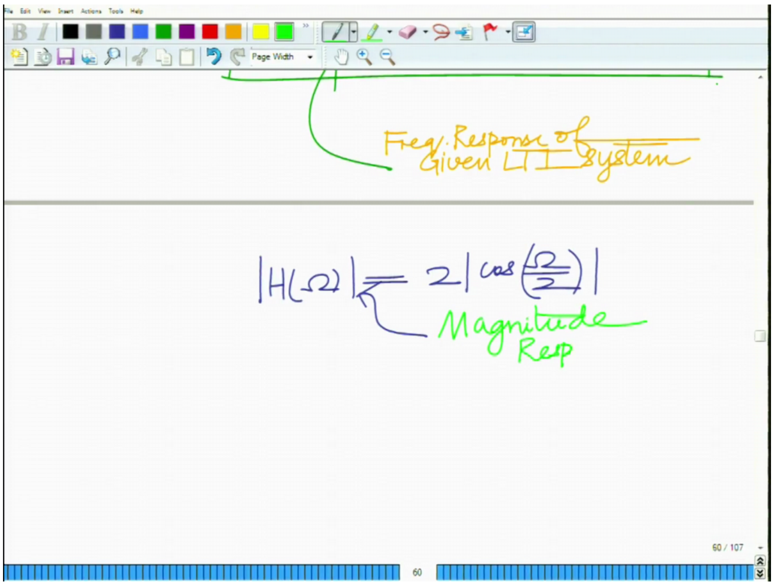

This is basically your magnitude response. The magnitude response is 2 magnitude cosine omega by 2.

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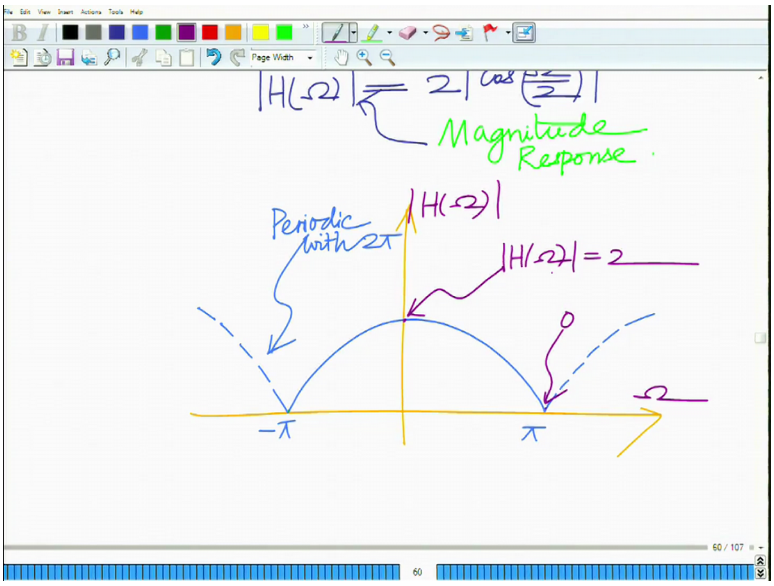

So, if you draw this it will looks something like this right, its very interesting and of course, you can see its symmetric, its periodic. Remember every DTFT has to satisfy the property that it has to be periodic with period 2 pi. So, this is periodic with, this is periodic with period equals 2 pi correct. So, if you look at cosine omega that will be, it will be periodic with period equals 2 pi ok. And now if you look at this um, if you look at this quantity here, I can draw this as follows. So, this is 2 cosine, magnitude 2 cosine omega by 2, when omega equals pi, this will be twice. So, at omega equals 0, this is twice cosine 0 which is 2 all right.

And when omega equals pi at the ends all right, will be two magnitude cosine omega pi by 2, but cosine; that is two magnitude cosine pi by 2 cosine pi by 2 is 0. So, at pi and minus pi as well it will be 0 and if you look at the frequency response it will be something like this, which looks basically, and of course, it is periodic with, periodic with ok. This is periodic with period equals 2 pi and this is the peak you can see magnitude H of omega equals 2. So, this is a plot for magnitude H of omega and this is omega ok. Here it is 2, here it is 0.

So, it starts at the maximum at 0 and taper zone all right decreases towards minus pi and pi. So, clearly this is a low pass filter, but of course, its not an ideal low pass filter, because its not suppressing that is the attenuation, it not is not 0 outside of a cutoff frequency. So, clearly its not an ideal low pass filter all right,and therefore, we know that for a non ideal low pass filter we can characterize the effective bandwidth, one of the ways one of the metrics to characterize the bandwidth is using the 3 dB frequency ok. So, first you can see that this is a non ideal low pass filter in fact there is a non ideal discrete time low pass filter.

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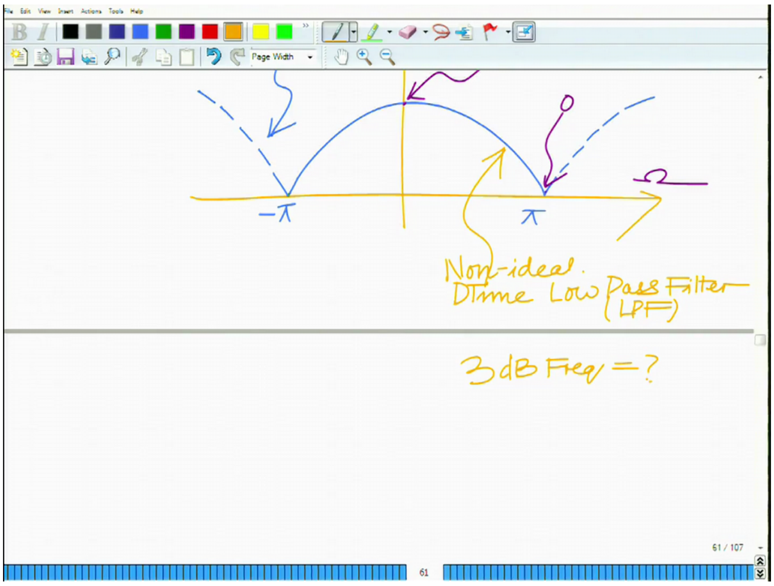

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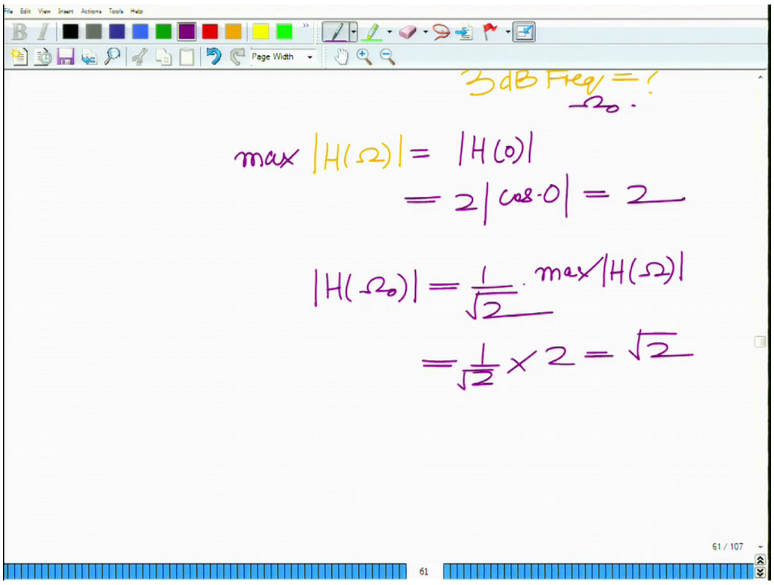

And therefore, what do we need to, what we want to find is what is the 3 dB frequency of this non ideal low pass filter and that can be magnitude H of omega and if you look at the max of this, this occurs at H of 0 that occurs at 0 which is 2, magnitude cosine 0 which is equal to 2; now the 3 dB frequency omega naught. So, let us call this omega naught. Now this will have to be such that magnitude H of omega naught is 1 over square root of 2 times maximum of the magnitude of H of omega which is 1 over square root of 2 times 2, which is square root of 2.

Remember at the 3 dB frequency the amplitude has to be 1 over square root of 2 times that of the maximum amplitude all right, times that of the maximum gain that is the amplitude gain of the low pass filter.

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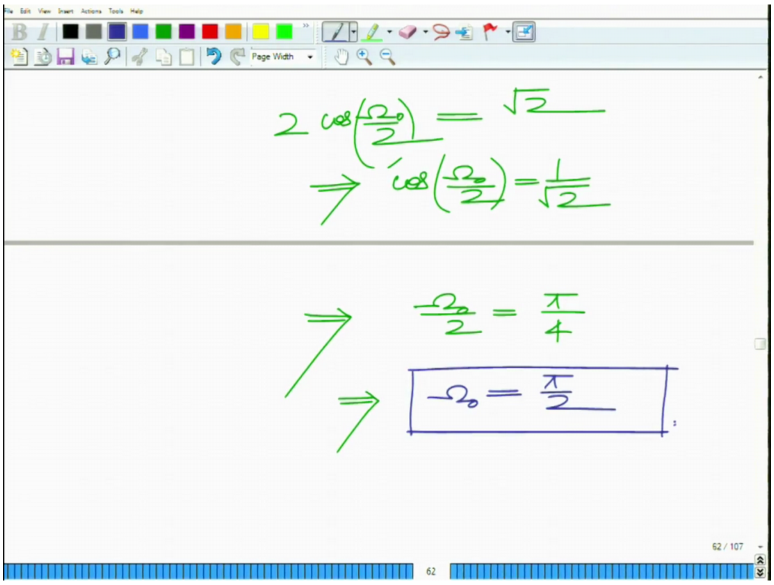

And now this is very simple, so we have two assuming omega greater than 0 for omega greater than 0, for omega less than 0 it is symmetric, 2 cosine omega by 2 equal square root of 2 implies or in fact, this is omega naught, implies cosine omega naught by 2 equals 1 over square root of 2 implies omega naught 2 is now cosine inverse 1 over square root of 2, which is pi by 4 implies the 3 dB frequency omega naught equals pi by 2 very simple ok. So, the 3 dB frequency omega naught equals to pi by 2.

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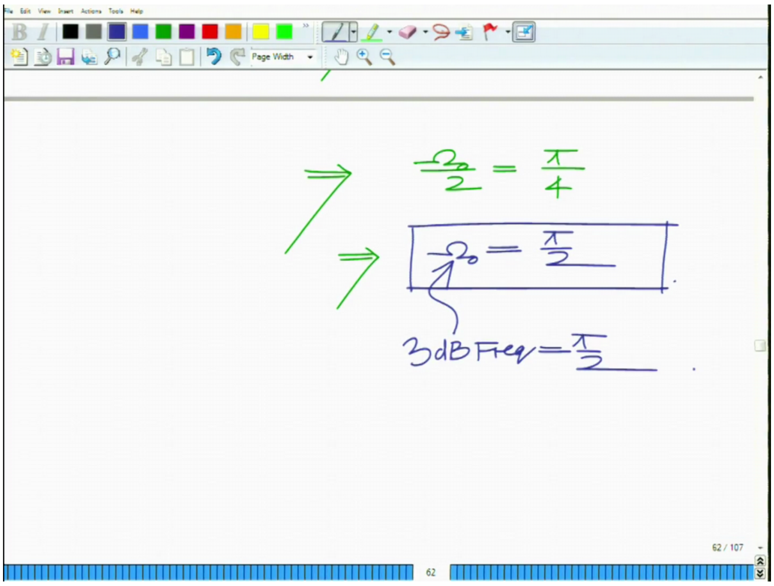

So, the 3 dB frequency is pi by 2. And if you look at this you can indicate the 3 dB frequency over here, if you look at pi by 2 what this is saying, is that if you look at pi by 2.

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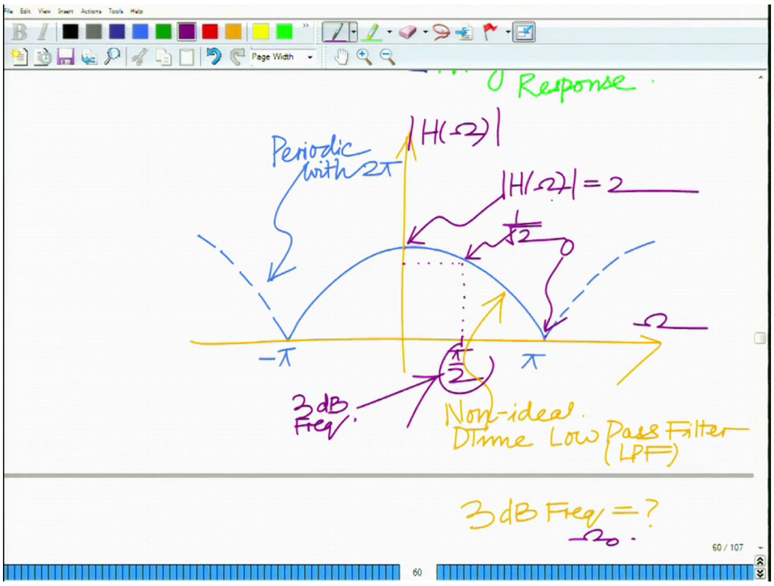

This will be 1 over square root of 2 and therefore, this will be your 3 dB frequency. And 3 dB you can also say that the 3 dB bandwidth ok, the 3 dB bandwidth of the filter.

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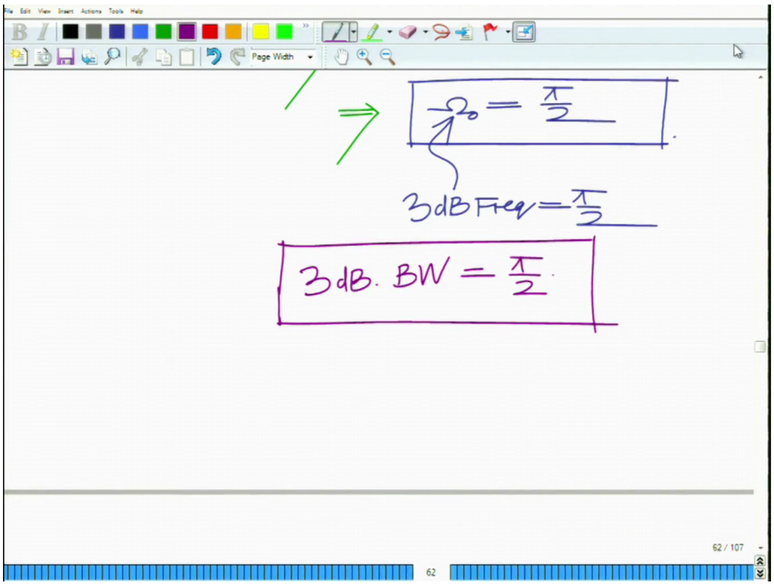

You can also say for this non ideal low pass filter, the 3 dB bandwidth is basically pi over 2 all right.

So, that completes the analysis of this problem. Let us now move on to the next problem which is also very interesting. Let us look at how to derive a high pass filter from a low pass filter.

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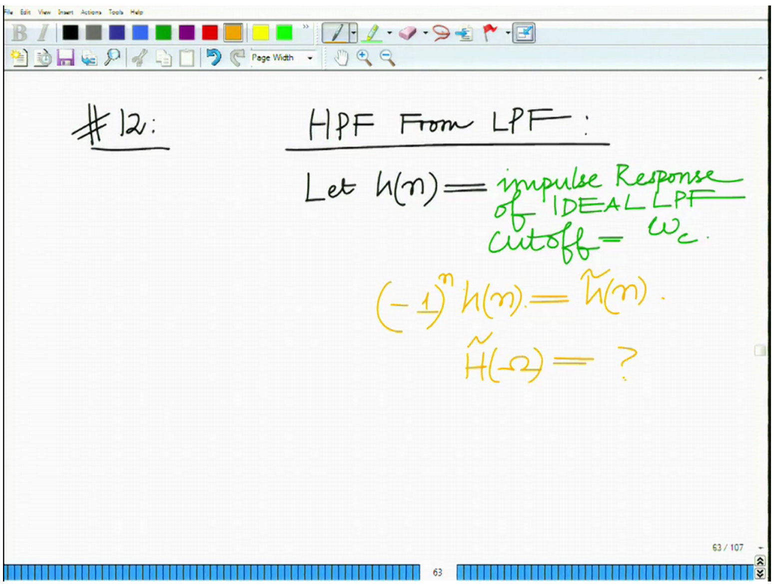

So this is my problem number 12 and we will see an interesting technique to derive a high pass filter given a low pass filter, a discrete time high pass filter. So, let hn be the impulse response of ideal LPF low pass filter and the cutoff equals omega c, remember every ideal low pass filter we will have a cutoff frequency.

Now, what we want to do is, we want to consider minus 1 to the power of n hn, let us denote this by h tilde n, and we want to demonstrate that is. In fact, a high pass filters or question is, what is h tilde of omega. And in fact, if since hn is the ideal low pass filter.

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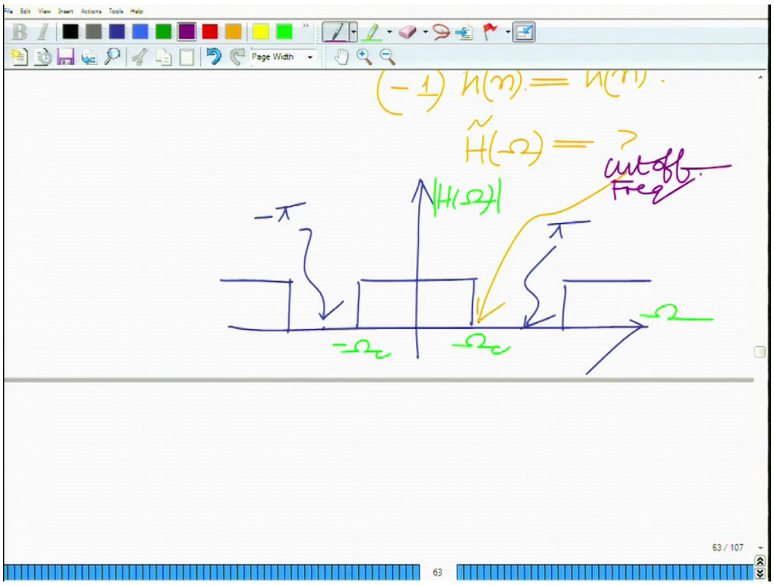

If you look at um, for instance if you look at H of omega that will looks something like this, and remember its all periodic with respect to pi or 2 pi. So, this will be something like this. So, its not necessarily according to scale. So, this is pi, this is minus pi and this is omega c, the cutoff and minus omega c or let us denote this by omega c minus omega c and this is omega c and this is your magnitude H of omega and this is omega ok.

And this is basically your cutoff frequency ok, this is your cutoff frequency, let us call this omega c ok. Now what is the cutoff frequency in this case minus. So, h tilde n equals minus 1 n hn.

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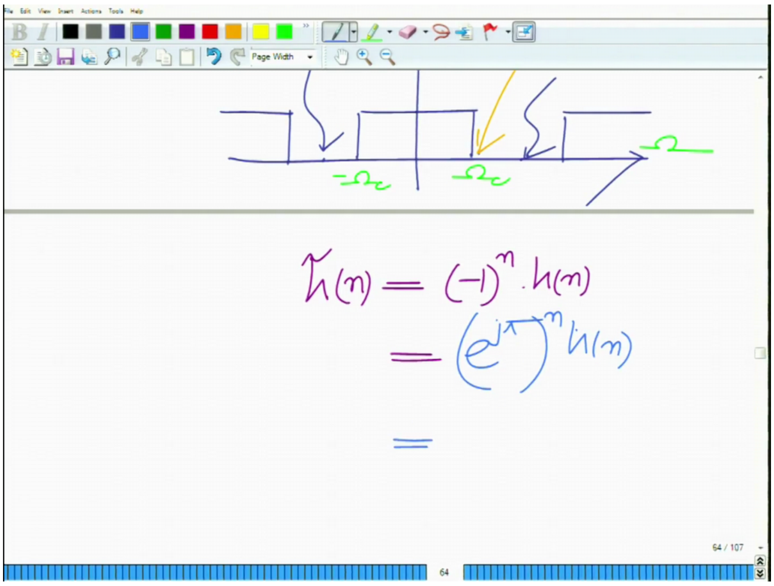

If you look at h tilde n equals minus 1 n hn. Now you can write this minus 1 is e raised to j pi or let us say minus 1 raised to n is e raised to j pi n and then you will observe something interesting. So, this is e raised to j pi n times hn.

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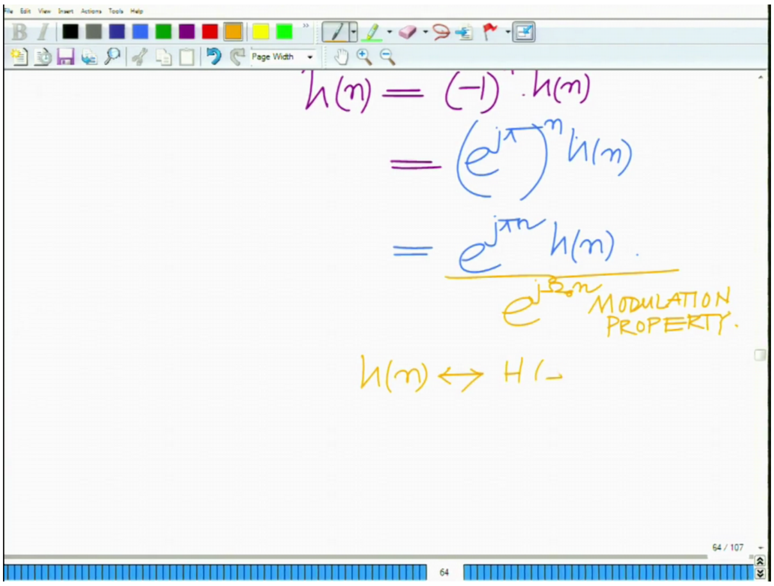

Which is e raised to j pi n times hn and you can see this is modulation in time this is of the form e raised to j omega naught n or this is of the form e raised to. So, we use the modulation property ok.

So, the modulation property states that remember you can recall if h of n has DTFT H of omega, then e raised to j omega naught n has DTFT e raised to j omega naught n has DTFT H of omega minus omega naught, this is your modulation property ok.

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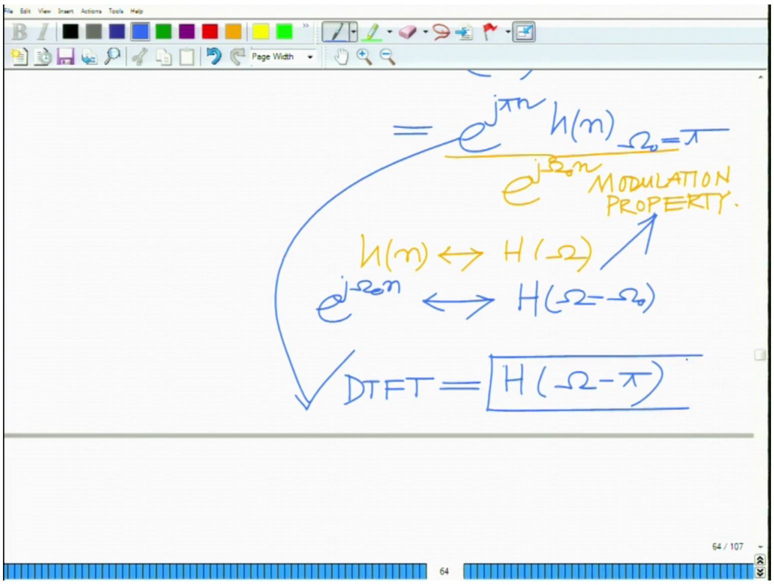

So, which means e raised to j pi n has DTFT equals H of omega minus omega naught. So, here omega naught equals pi, so DTFT will be H of omega minus pi and that is your H tilde of omega. So, H tilde of omega corresponding to minus 1 raised to n hn.

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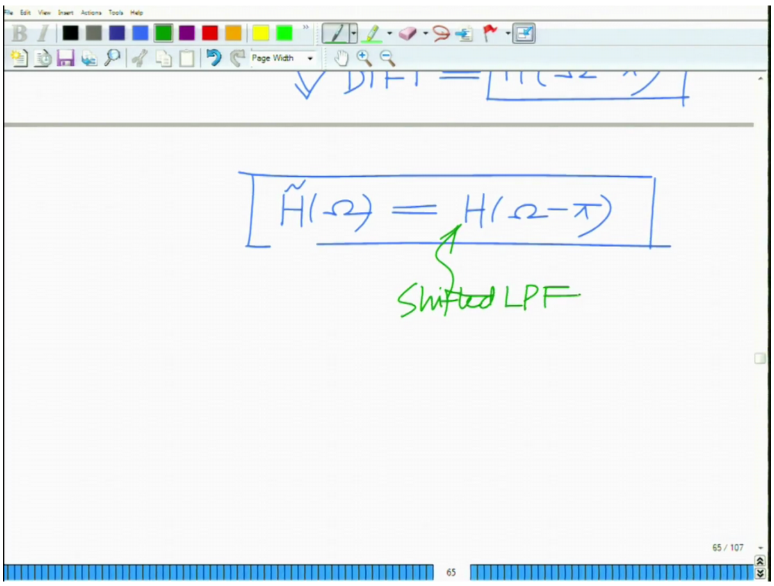

So, H tilde omega is H of omega minus pi, which is basically what you doing is you are taking the low pass filter response and you are shifting it by pi. And remember its periodic ok. So, the shifted version that is H tilde omega will also be periodic in fact, with period 2 pi again ok. So, simply shifting does not either induce or destroy periodicity all right. So, the original signal is periodic, the shifted signal will also be periodic. In fact, the period will also be the same ok.

So, this is the shifted LPF and once you shift the LPF remember with cutoff frequency, cutoff frequency omega c and it is very easy to derive that; that is going to looks something like this.

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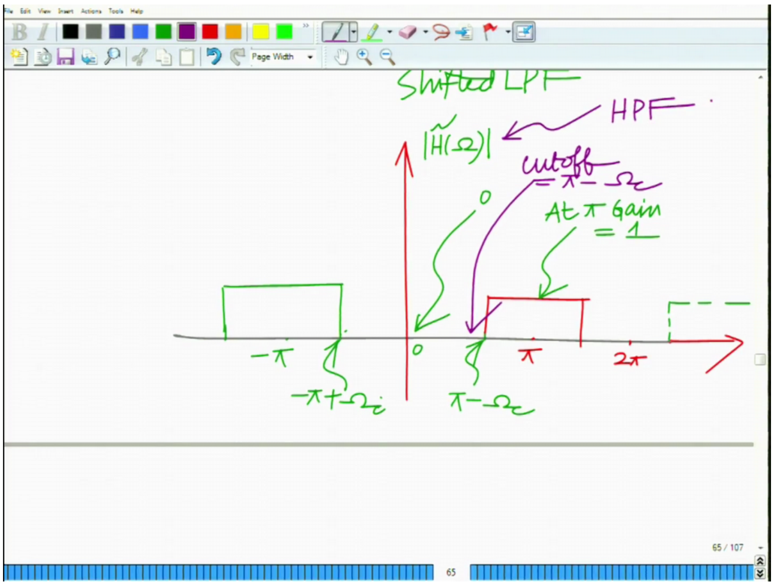

So, we take the original LPF you shift it to pi. So, it will be now centered around pi and. So, this is pi and this is 2 pi at 2 pi it will be 0. So, at 2 pi it will be 0, because originally what was it because originally, what is that pi will come to 2 pi all right you are shifting it or originally what is that 2 pi will come 2 pi what is the 3 pi will come to 2 pi which is also anyway 0.

So, and rest it is periodic with omega ok. So, this is pi. So, this is minus pi and you will also have something; that is basically looks like this. And now this point if you realize this is basically shifted by pi. So, originally, so this is omega c when you delay it by pi this will become omega c minus pi and this will become pi minus omega c. So, this point will become. In fact, I am sorry this point will become, so this is basically pi minus omega c and this is omega c minus pi or minus pi plus omega c. So, now, you can clearly see, gain is 0, if you look at magnitude h tilde omega at 0, this is equal to 0 and gain at pi is equal to the at pi gain is equal to 1.

. So, what you can see at 0, earlier at 0 it had the maximum gain at pi the gain was 0, cutoff was omega c all right. And now in fact, let me just write it as capital omega c. And now what is happening because of the shift by pi all right the maximum gain is now at pi all right. So, maximum gain is unity and in fact, at 0 all right the gain is 0 all right.

So, this is now a high pass filter ok. So, using this modulation minus 1 raised to n hn, you are converted we have converted a low pass filter into a high pass filter into an equivalent high pass filter with the cutoff frequency pi minus omega c ok. So, that is now the cutoff frequency ok. So, the cutoff frequency of this will be pi minus omega c and this is a high pass filter. So, H tilde and you can see will have the DTFT. So, we have minus 1 to the power n H tilde hn equals H tilde

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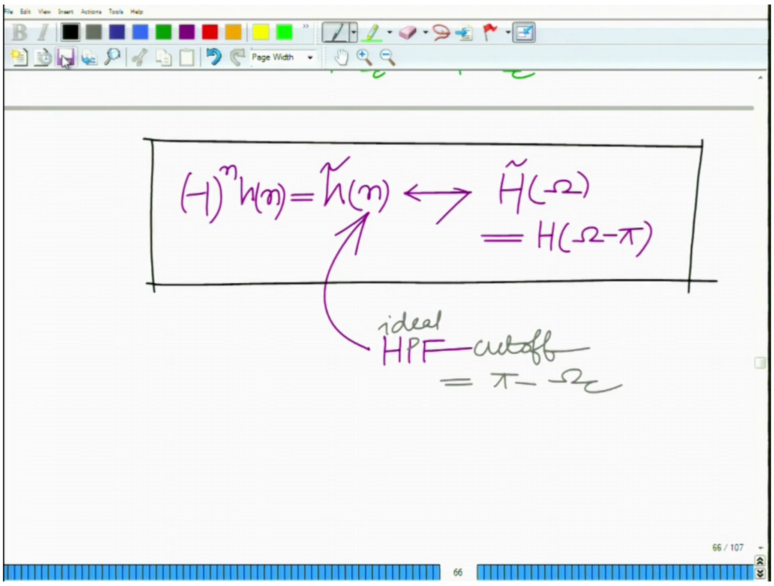

And which is the DTFT H tilde of omega which is H of omega minus pi and this is basically therefore, this is a now this is now hn is a low pass filter and H tilde n, this is a equivalent high pass filter. In fact, an ideal high pass filter, ideal HPF, this is a ideal HPF, cutoff equals pi minus omega c ok. So, that is what we are able to, that is what we are able to show all right.

So, that is a very interesting problem and it shows an interesting trick of how to derive an equivalent high pass filter correct, once you have a ideal low pass filter, or for that matter it works also for non ideal; that is corresponding to a non ideal low pass filter you one will derive a non ideal high pass filter all right.

So, let us stop here and we will continue with other problems in the subsequent modules.

Thank you very much.