Principles of Signals and Systems
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Lecture – 06
Linear Systems – Additivity/ Homogeneity Properties, Time Invariant Systems, Linear Time Invariant (LTI) Systems, BIBO Stability

Hello, welcome to this module in this massive open online course alright. So, we are looking at classification of systems and their property. So, let us continue our discussion on the various kinds of systems. So, another important class of systems is what are known as is what is known as linear systems.

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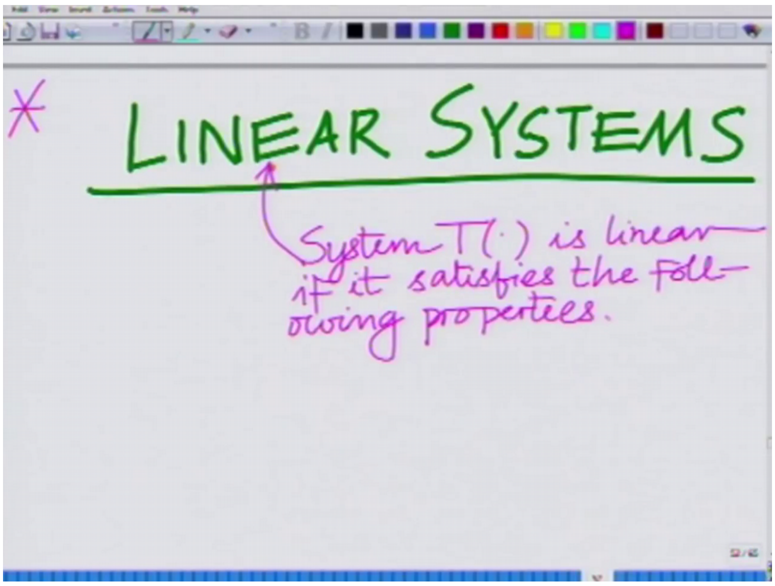

So, another very important class of systems which we will frequently encounter and which we will almost always encounter is that of linear systems. Now, a system is linear now the system T is linear if it satisfies the following properties.

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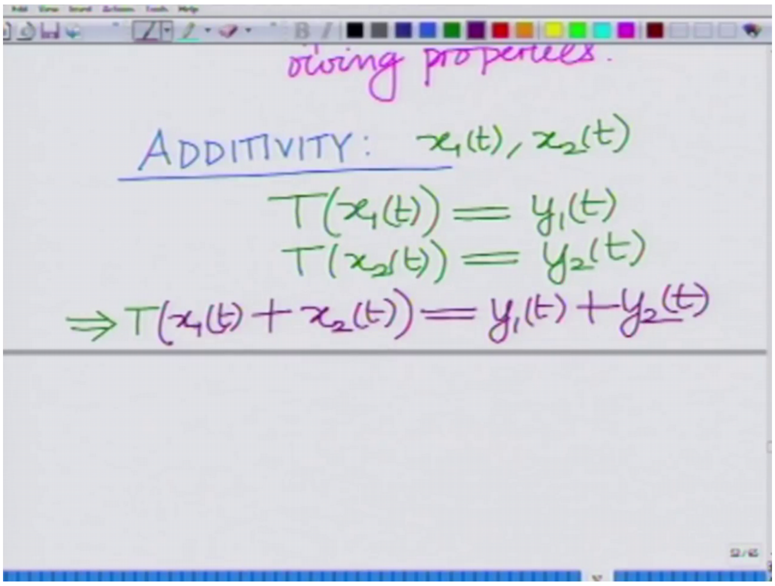

Now, the first property is what is known as additivity. Additivity is very simple, consider 2 input signals x 1 t, x 2 t such that input x 1 t that is T of x 1 t equals y 1 of t that is input x 1 t produces y 1 of t, input x 2 t produces let us say output y 2 t.

Now, the system the additivity property or the system satisfies additivity if this implies T of x 1 t plus x 2 t, if this implies T of x 1 t plus x 2 t equals y 1 t plus y 2 t, that is, if x 1 t produces the output y 1 t, x 2 t as input produces output y 2 t then if x 1 t plus x 2 t produces the output y 1 t plus y 2 t and this is true for all possible inputs x 1 t and x 2 t the system is said to be additive.

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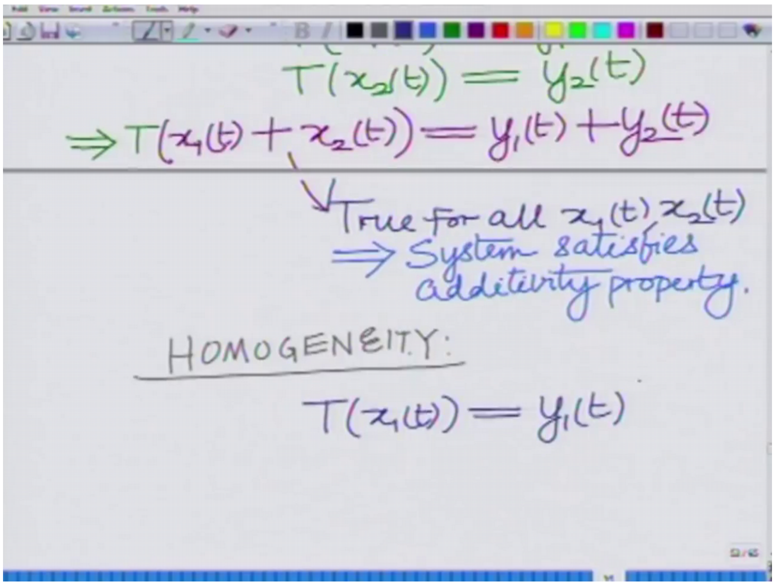

And, this has to be true for all not just a chosen x 1 t, but true for all x 1 t comma x 2 t. This has to hold true for all x 1 t, x 2 t and if this satisfies. If this is satisfied this implies the system is additive, our system is additive, system satisfies additivity. Now, similarly the next property is what is known as homogeneity.

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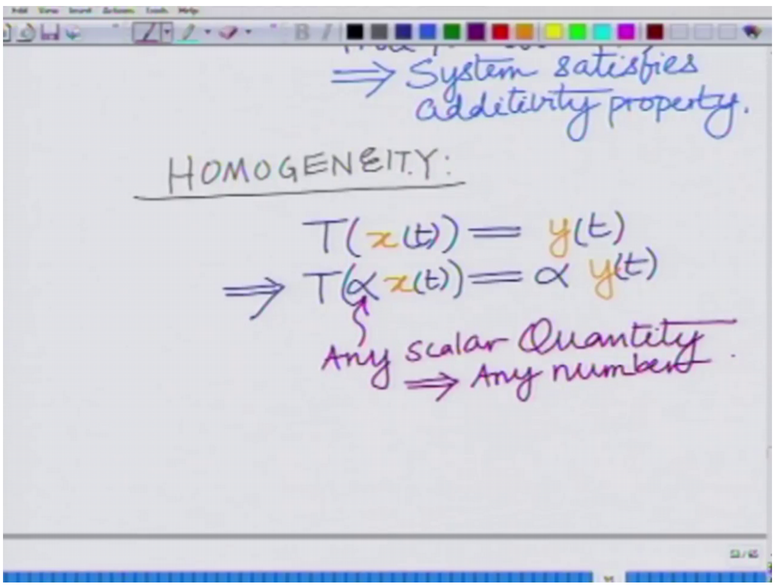

Homogeneity simply implies that if x 1 t produces the output y 1 t. This implies t of alpha times x 1 t produces the output alpha times y 1 t or let us, so, alpha times x 1 t or let us you can simply write it as x t and y t that is if any input x t produces output y t alpha times x t produces the output alpha times y t, where alpha is any scalar quantity, alpha is any number; implies any basic number either can be real number for real some signals or can be a complex number for complex signals. So, scaling by a scalar quantity alpha, if the input is scaled by a scalar quantity alpha then the output is also scaled by a scalar quantity alpha.

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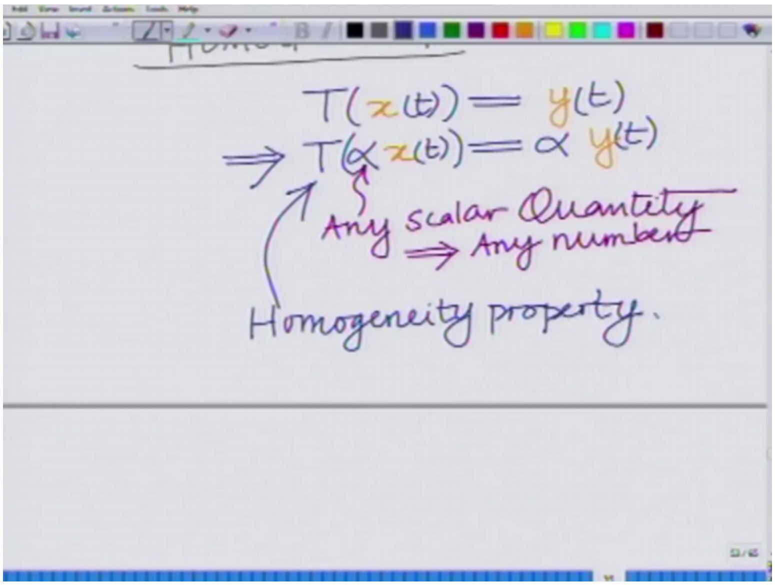

If the system satisfies this property, it basically satisfies the homogeneity property. So, this is the homogeneity property and systems that satisfies.

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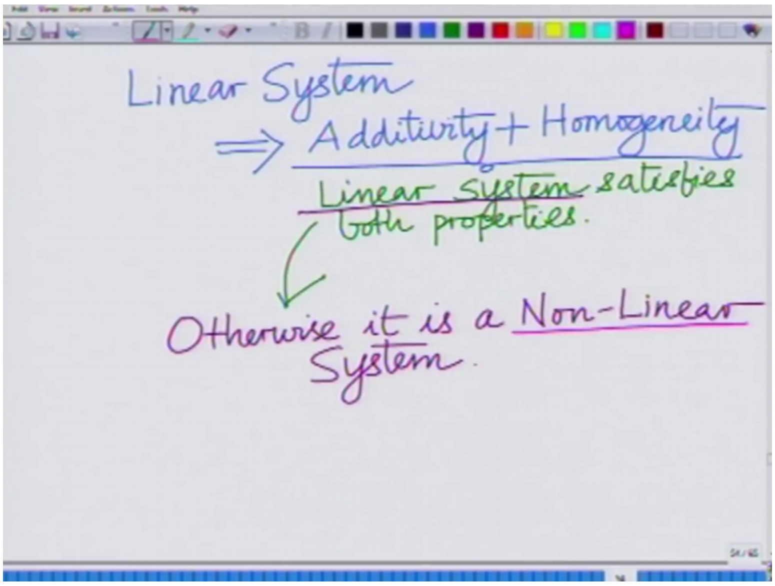

Now, a linear system is simply one implies it satisfies both, additivity plus homogeneity, that is linear system satisfies both the properties that is additive and homogeneous. So, such a system is known as a linear system, if it does not satisfy otherwise it is known as a non-linear system. Linear system satisfies both the properties otherwise it is known as a it is a non-linear. So, this is a linear system, non-linear system does not satisfy either property or both properties.

So, a linear system is 1 that satisfies both the additive properties that is if you add 2 input signals the resulting output signal should correspond to the sum of the output signals of these 2 input signals and if you scale an input signal the resulting output signal should be correspondingly a scaled version of the output signal and a system that satisfies both these is known as a linear system.

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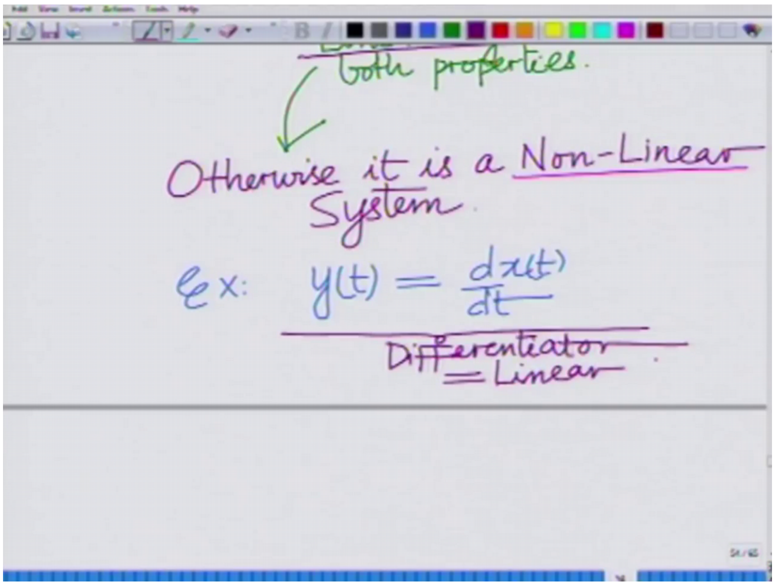

And, for example, let us take a very simple example y t equals dx t by dt; this is known as a differentiator, you can see that this is a linear system.

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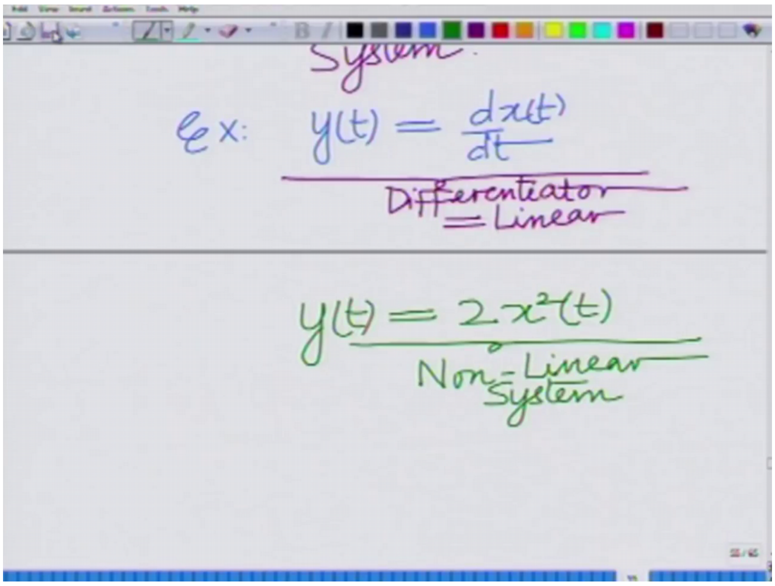

Now, if you take for example, y t equals 2x square of t, this is linearly this is a non-linear system. You can check that it will not satisfy the properties that are related to linearity. So, that basically is a definition for a linear system which as we said is the class of linear systems is a very important class of systems because most of the time these are the kind of systems that we are going to be encountering.

Another important class of systems in the same way is what is known as time invariant systems. So, time, the class of time invariant systems.

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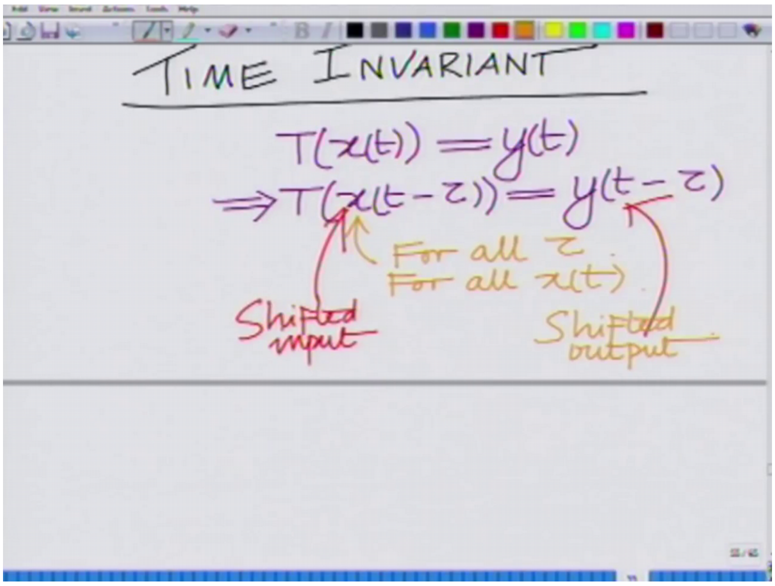

So, time invariant implies if T of x t equals y t, that is, if input the system x t gives output signal y t; if this implies t of x t minus t naught equals y of t minus t naught and this holds for all shifts t naught and for all input signals x t. That is, for any or let us put it this way x t minus tau equals y t minus tau, that is, for any shift tau that is if input x t yields output y t then if I shift it by tau x t minus tau is basically a delayed version of x t. If I shift x t x t minus tau then the output y t is corresponded yields an output signal y t which is correspondingly shifted.

So, this is x t minus tau this is shifted input and this is basically y t minus tau is basically which is shifted output. Of course, if tau is positive it is a delay, if tau is negative then it is an advanced signal.

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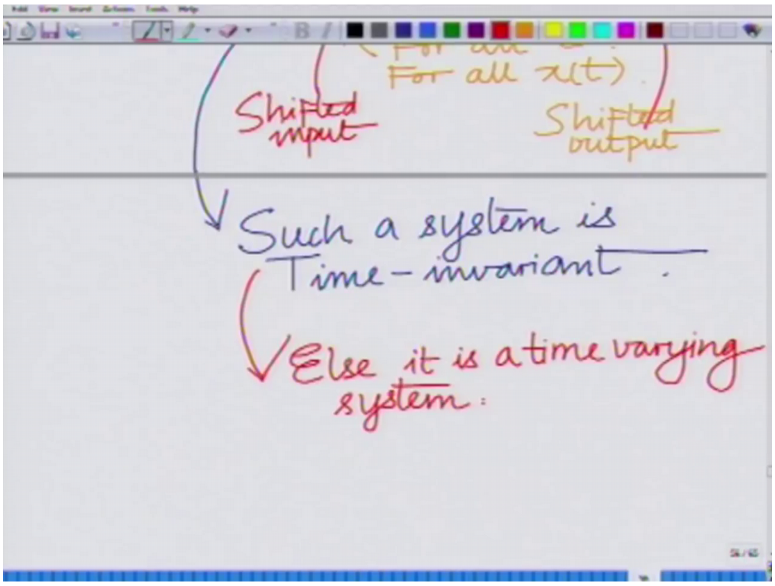

So, it is a shifted, if it satisfy this property such a system is said to be time is a time invariant system otherwise it is a time variant or a time varying system, else it is a time varying system for example.

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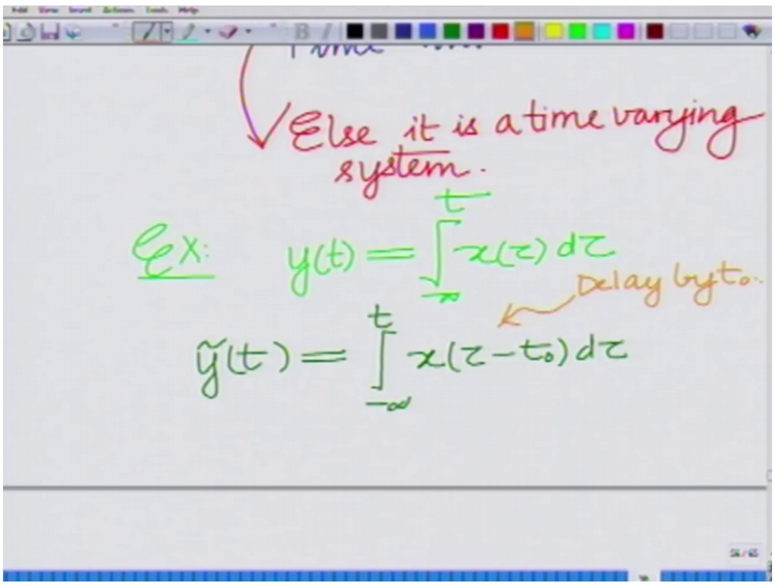

Again let us take another example, if you have y t equals minus infinity to t x of tau d tau you can show this as time invariant for instance if you substitute x of t minus tau let us call the output y tilde of t, which is simply minus infinity to t minus tau. Let's make this as tau naught just to distinguish it from the integration variable just make it t naught. So, I am shifting this by naught this is going to be x of tau d tau or basically I am just shifting I am sorry I am just shifting x of tau, I am delaying it by t naught x of tau minus t naught. So, if I am delaying by t naught, let us see what the impact on the output signal is.

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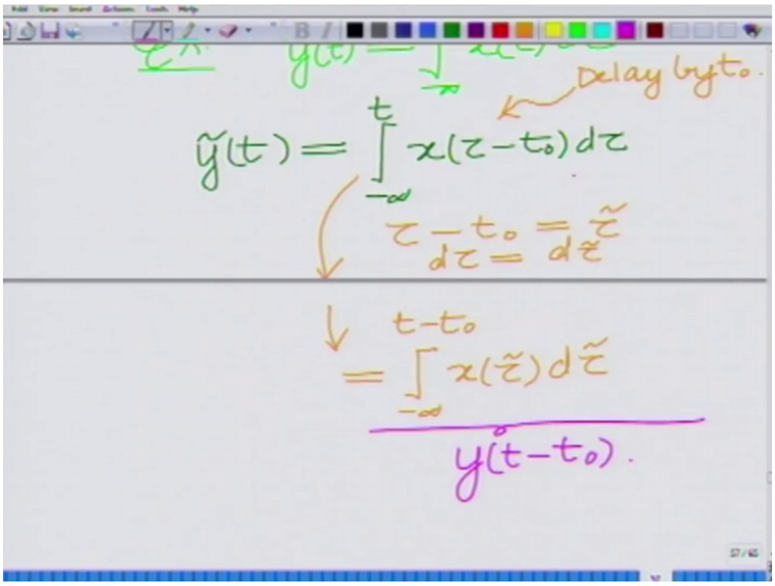

Let us set tau minus t naught equals tau tilde implies d tau equals d tau tilde. So, this integral y tilde t will become integral minus infinity to infinity, tau minus t naught. So, this will be t minus t naught integral minus infinity to t minus t naught x of tau tilde d tau tilde and you can see that this is nothing, but y of t minus t naught. So, if I shift the input by t naught the output is correspondingly shifted by t naught, hence this is a time invariant system.

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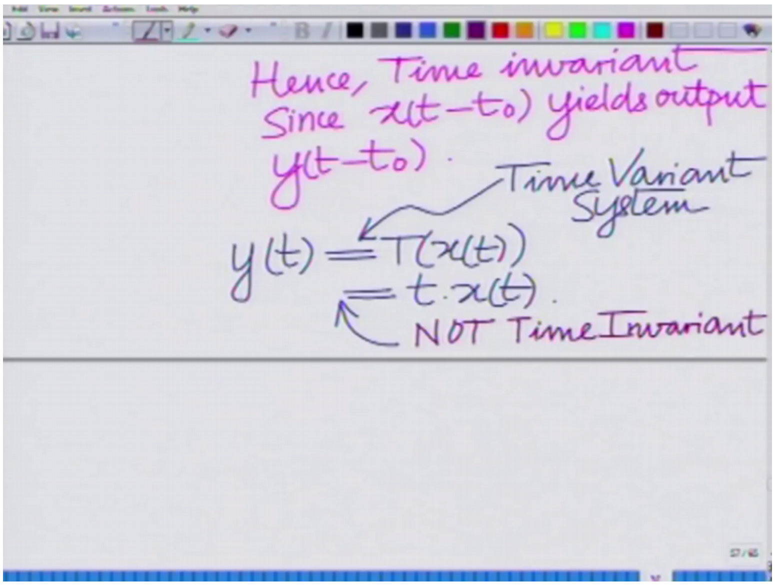

Since, x t minus t naught yields the output signal y t minus t naught, this is a time invariant system. So, this system is a time invariant system. On the other hand, if you can look at for instance, y of t equals T of x t equals t times x t you can show that this is a time variant or a time varying system. You can show that this is not a time invariant or another way to also see this is not time invariant.

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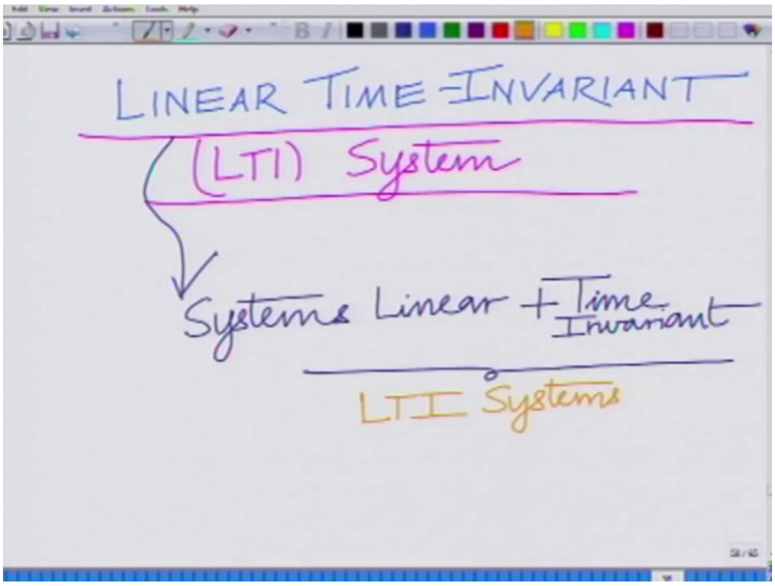

And, that brings us to the most important class of systems which is that of an LTI system or let me write this clearly this is a linear time invariant, this belongs, this is what is known as the linear time invariant or LTI systems which is probably the most important class of systems or the class of systems that we most frequently encounter in practice and which we are going to be most frequently interested in, that is, a class of linear time invariant systems, which as the name implies are systems that are both linear as well as time invariant. So, this as time as the name implies that is systems that are the linear plus time invariant. These are known as LTI systems, linear plus time invariant systems are known as those satisfy linearity as well as time invariant.

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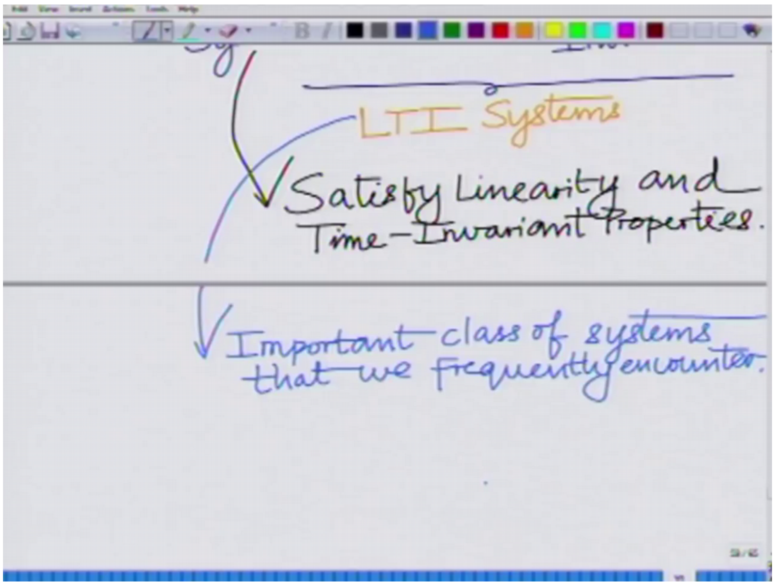

And, these are the type of systems which we will most frequently encounter and most readily analyzed most. So, these are the very important class of systems that we most important class of systems that we are going to frequently encounter. So, this is linear the LTI systems class of LTI systems which stands for linear time invariant system; systems that are both linear as well as time invariant.

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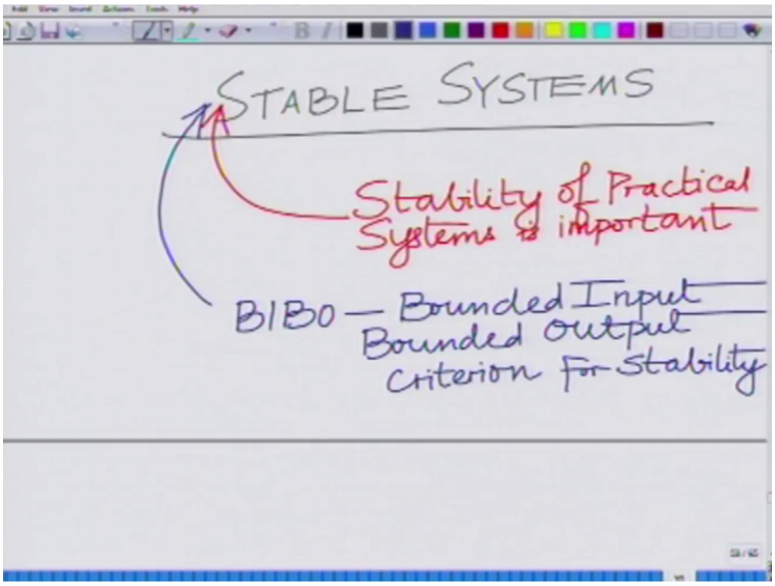

And, some other class of systems that are also important, because these are practical implications for instance the first one such class is the class of stable systems. Naturally, we are interested in systems that are stable and not systems that are unstable or a system that exhibit unstable behavior from a practical perspective interested in stability of systems that is an important property. So, stability let us note that down stability of practical systems is important correct because in practice we would like to have a system that is stable not something that is unstable and a stable system is simply defined as follows; we have 1 definition of stability is the following thing this is known as BIBO or Bounded Input Bounded Output criterion. The Bounded Input Bounded Output Criterion for stability or such systems are known as BIBO stable systems, that is, bounded input bounded satisfy the bounded input bounded output criterion for stability.

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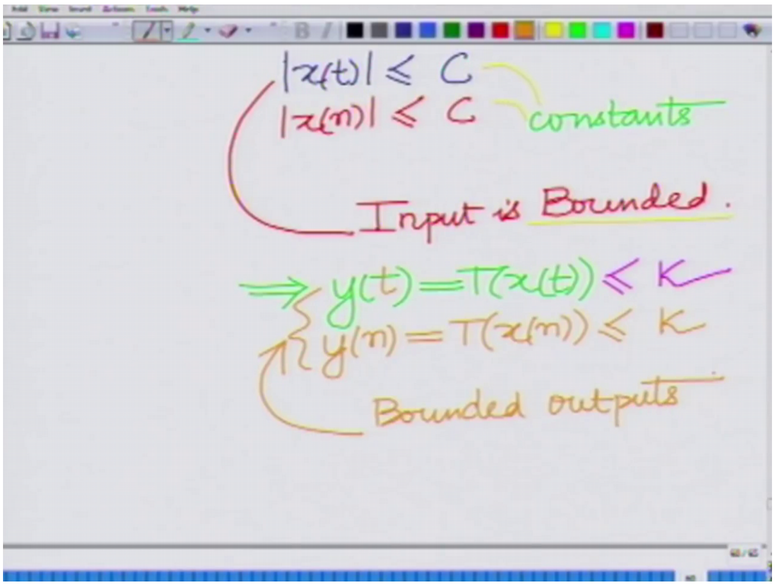

And, the definition of that is, if a system is such that magnitude x t less than or equal to some constant C or for a discrete time system magnitude x n, is less than or equal to C. This is for a continuous time for system, this is what a discrete type system that is input is bounded input is bounded which means that input is less than some constant C these are constants.

If the input is less than some constant C such bounded inputs produce bounded outputs implies y T equals T of x t is less than or equal to some constant K or y n equals T of x n is less than or equal to constant, that is, the outputs are bounded that is they are less. Bounded means they are basically less than or equal to some constant.

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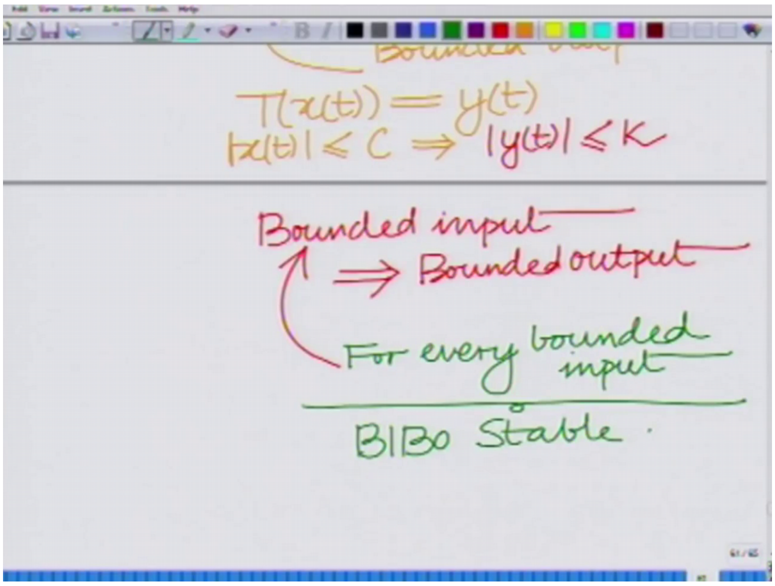

So, if for every bounded input, the output is bounded. So, if for every bounded input the output y t is bounded such a system is known as a BIBO stable system that is every bounded input produces an output y t that is bounded that is T of x t equals y t with magnitude of x t less than or equal to just have the magnitude function over here magnitude of y n equals magnitude of T n is less than or equal to. So, if magnitude of x t less than or equal to C that is bounded input implies that the output. Magnitude of y t is less than or equal to K.

So, bounded input produces bounded output implies or produces for every input, for every such bounded input and mind you this has to hold for not a single bounded input, but every bounded input this implies system is BIBO stable that is every bounded input produces a bounded output, system is a BIBO stable system a bounded input bounded output stable system.

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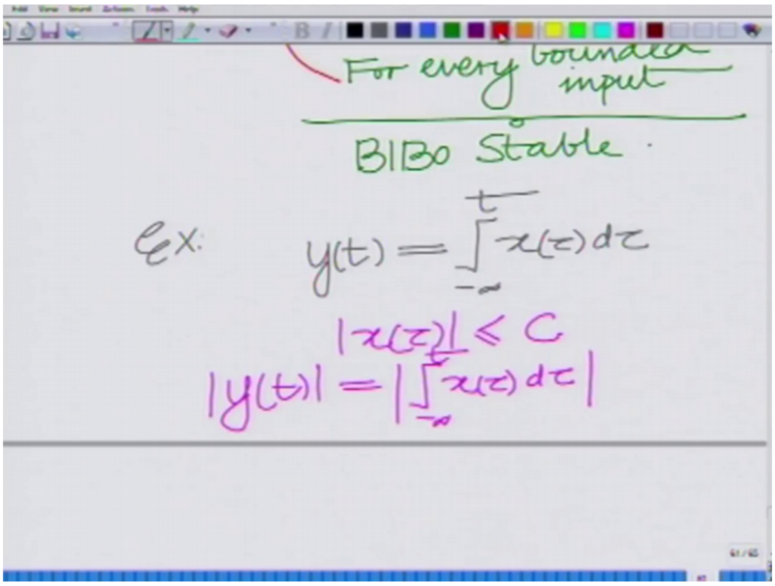

For instance again let us take another example here. Again, a simple example we will do. Let us again go back to our integrator y of t equals minus infinity to t x of tau d tau now, if I consider if I give a bounded input that is magnitude of x of tau less than or equal to C let us look at magnitude of y of t which is equal to magnitude of the integral x of tau d tau, but the magnitude of an integral is less than or equal to the integral of the magnitude of the function that is being integrate or the signal that is being integrated.

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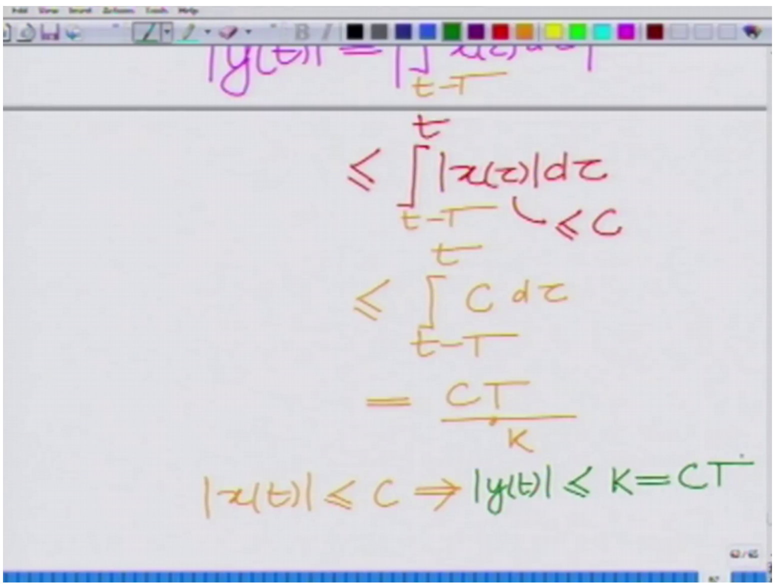

Which means, this is less than or equal to integral of t magnitude x tau d tau, but we know magnitude x tau this is less than or equal to C because we have a bounded input implies this integral is less than or equal to t or let us make this since we are not, so, let us make this as an integral from t minus T, rather than minus infinity, because as you can see from minus infinity to t there is going to be a problem. So, t minus T; that is an integral over a window. So, this is less than or equal to integral from t minus T to t C d tau which is equal to C tau and now, you can set this as your quantity K. So, magnitude of x t less than equal to C, implies magnitude of y t less than or equal to K equal to CT and therefore, such a system.

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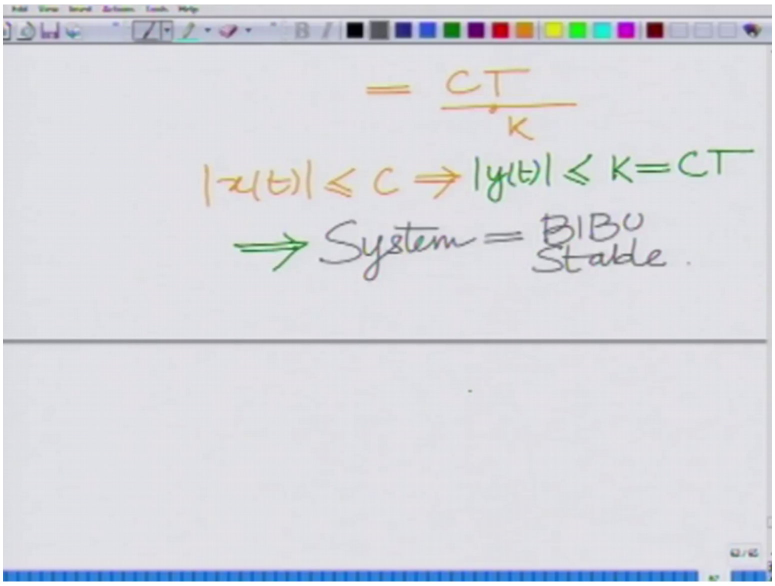

Therefore, implies the system is BIBO stable, implies the system is bounded input bounded output stable. So, it satisfies the BIBO stability criterion.

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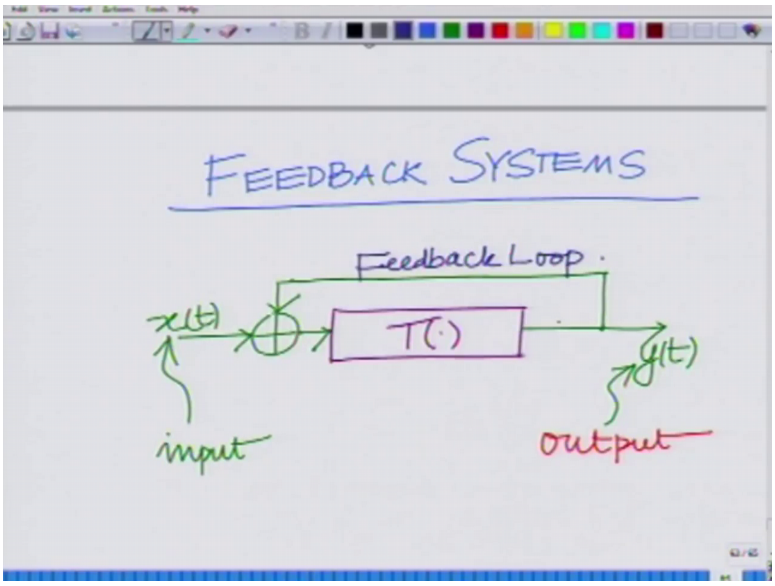

And, the last class of systems finally, which we are also important are what are known as feedback systems. And, in feedback systems, what we have is an output. The output of the system and this is also a very important class of systems especially important from the perspective of stability. So, I have a system T and I have an input x t. So, what I am going to do I have the output y of t. So, what I am going to do is I take the output y of t either directly or after suitably modifying it I input I feed it back to the input. So, this is the input, this is your y t is the output and this is basically your feedback loop. So, I have an input, I have an output and this is a feedback.

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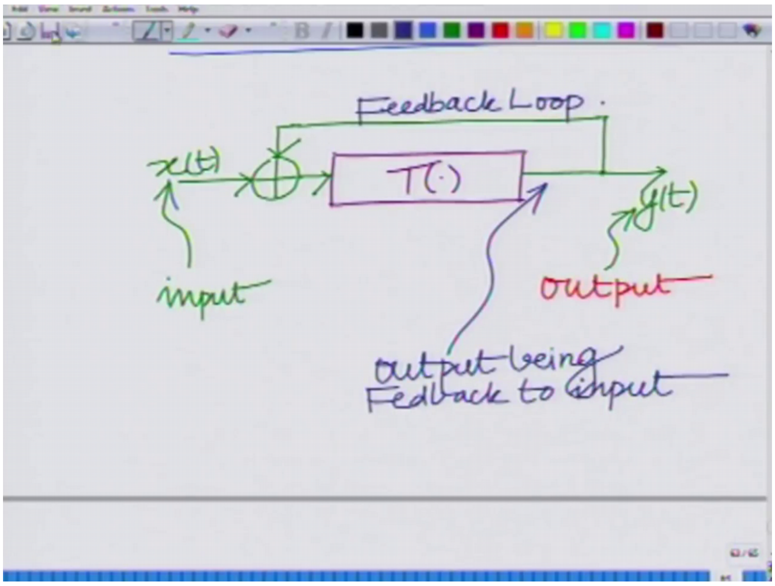

So, output is being fed back to the input. This completes our discussion on classification system. So, we have looked at various kinds or various classes of systems along with suitable examples. These properties at this classification of systems are especially important to understand the behavior and properties of various systems and also understand the various applications, the various important criteria from a practical perspective. So, we will stop here and look at other aspects in the subsequent modules.

Thank you very much.