Principles of Signals and Systems
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Lecture - 53
Sampling: Reconstruction from Sampled Signal

Hello. Welcome to another module in this massive open online course. So, we are looking at sampling correct and in particular we are looking at impulse train sampling, to convert a continuous time analog signal to a discrete time signal all that we are be multiplying the signal xt by an impulse train all right and we have also derived an expression for the spectrum of the sampled signal in terms of the spectrum of the original signal and what we have seen is the following thing.

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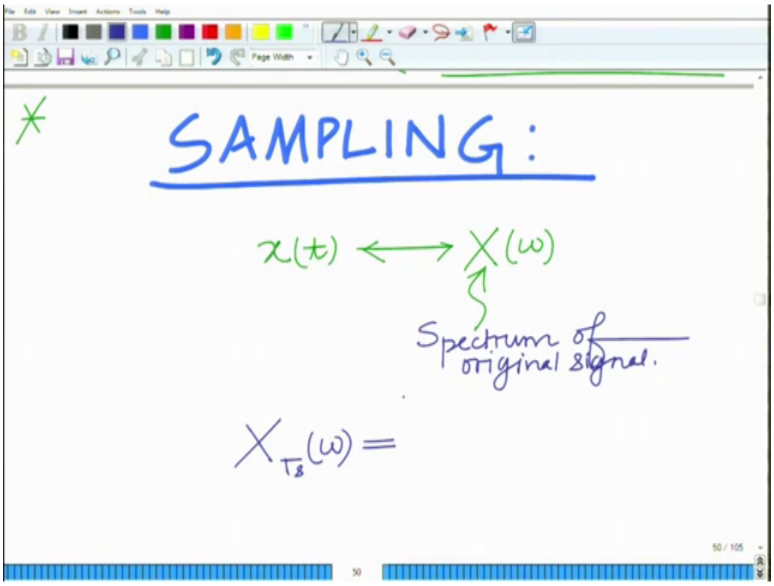

So, what we are doing is we are discussing sampling, in particular we have seen if x of t is the original signal this is your continuous time analog signal, which has spectrum x of omega this is spectrum of the original signal this is your spectrum of the original signal. The spectrum of the sampled signal that is your x Ts of omega, this is the spectrum of the sampled signal that you can see is basically given as 1 over t as summation x omega minus k omega k omega s.

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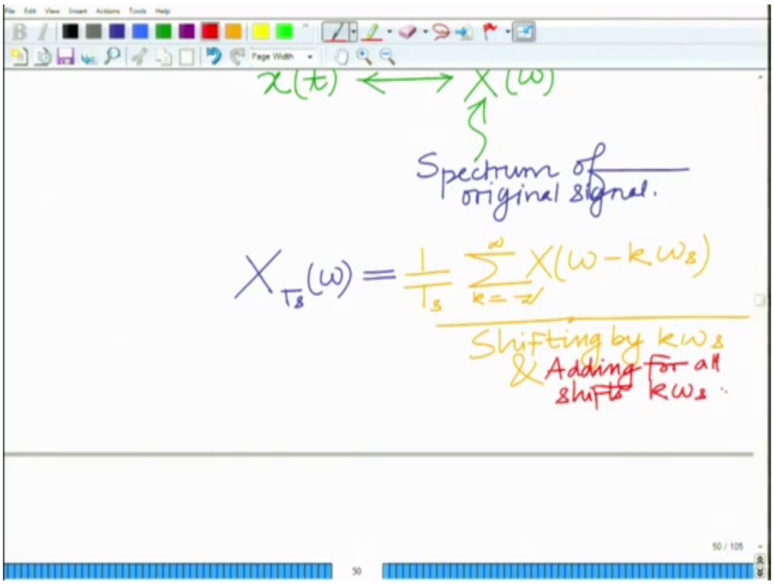

So, that is 1 over Ts summation k equals minus infinity to infinity x of omega minus k omega s, that is basically shifted k omega s and adding for all k that is adding for all shifts And therefore what we have seen is that if your original spectrum, let us say this is your original spectrum if your original spectrum X of omega is given as follows. So, this is your x of omega this is minus omega m this is omega m where omega m remember this is the maximum frequency.

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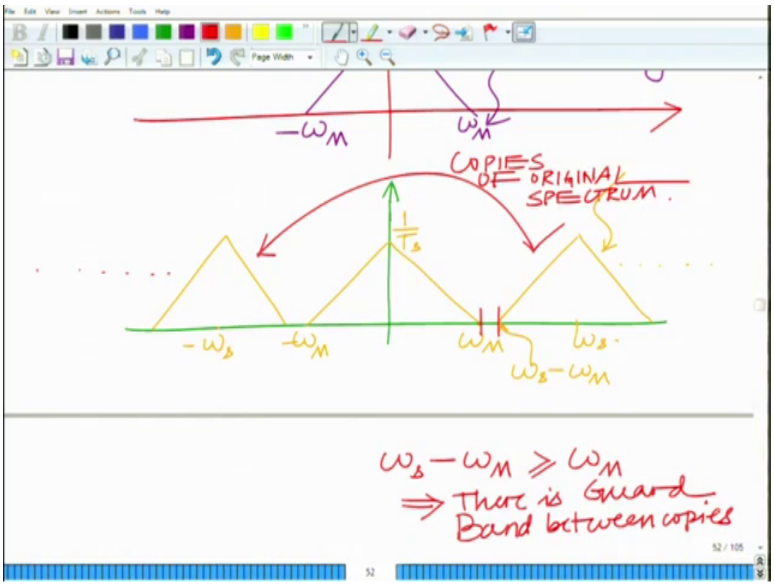

Now, if omega s is greater than or equal to omega m, now what happens is now you see the sampled signal will a spectrum that looks as follows, that is it will have of course, the original copy that is. So, this is scaled by 1 over Ts this is minus omega m to omega m and then you will have yet another copy that is shifted by omega. So, this point here is omega s minus omega.

So, this is basically and you will have several other copies for instance here you will have another copy that is shifted by minus omega s ok. So, these are basically all the copies and then you will have again several other copies. So, these are your copies of the original, so you can think of these as imposters basically someone is trying to pass off that is basically just not the original spectra, but copies of the original spectra someone is trying to imitate the original spectrum.

So, basically what happens is now when you have these copies and if there is no gap between the copies that is omega s minus. So, if there is a gap correct omega s minus omega m is greater than or equal to omega m, the maximum frequency implies there is a guard band correct there is a guard band there is a guard band between copies implies no interference or no distortion, that is and this distortion is term what is termed as aliasing.

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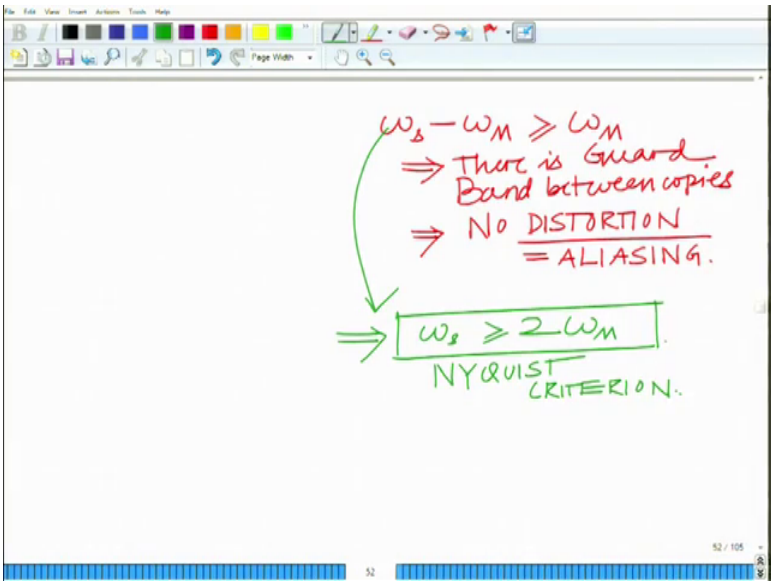

So, there is no aliasing which means aliasing basically an alias, it basically means an alternative name correct. So, aliasing means basically some other frequency is trying to pose right with an alternative name trying to pose as the original frequency. So, that does not happen, so this the spectral copy does not over the overlap with the original spectral band. So, there is no aliasing and there is that is some that is some other copy is not superimposing itself on the original copy of the spectrum.

So, they in that sense there is no aliasing, if omega s is greater than or equal to omega s minus omega m is greater than or equal to omega m, the maximum frequency which implies that omega s is greater than equal to twice omega m, which is termed as a Nyquist frequency ok. So, this implies no aliasing if omega s greater than equal to twice omega m and the sampling rate is greater than equal to s omega n, this is called basically the Nyquist criterion.

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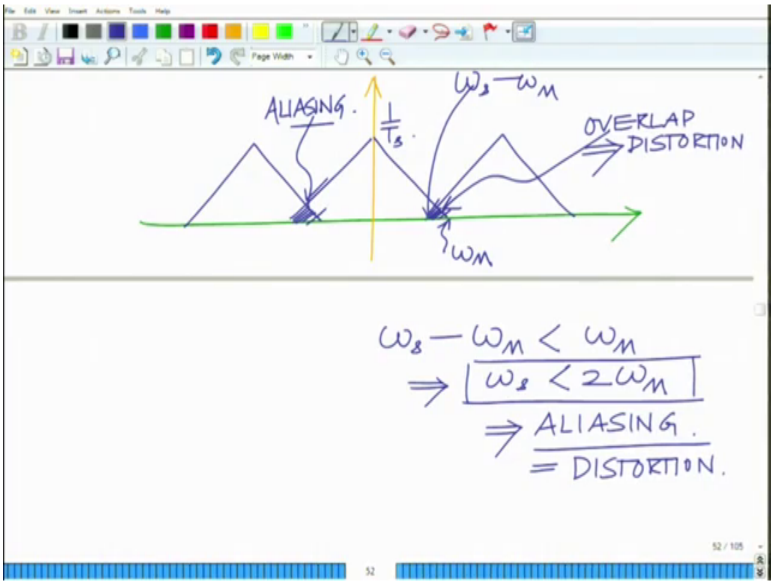

On the other hand, if there is no guard on the other hand if there is no guard band which happens, so this is your 1 or Ts this is your omega m and this is your omega s minus omega. In this case you can see there is overlap which implies distortion when you sum them. So on the other hand, when there is overlap that is omega s minus omega m less than omega m, which implies omega is less than twice omega m. This implies there is aliasing which is equal to basically or which is a form of distortion.

So, what happens when omega is the sampling frequency omega s is less than twice the maximum frequency. Then we have a distortion that is used that is arising because the aliasing of these various spectra back to the original spectral band and the copy of the same spectral band that is shifted by omega s and for that matter also shifted by minus omegas. So, you will have aliasing in fact in every spectral band. So, again here you will have aliasing ok.

So, because you have aliasing you will not be able to recover the origin spectrum, which brings us to the next topic that we are going to talk about how to reconstruct the original message signal from the sampled signal. So, what we want to also look at now is a reconstruction of the original signal from the sampled signal. So, what we want to talk about is basically how to reconstruct the original the reconstruction aspect.

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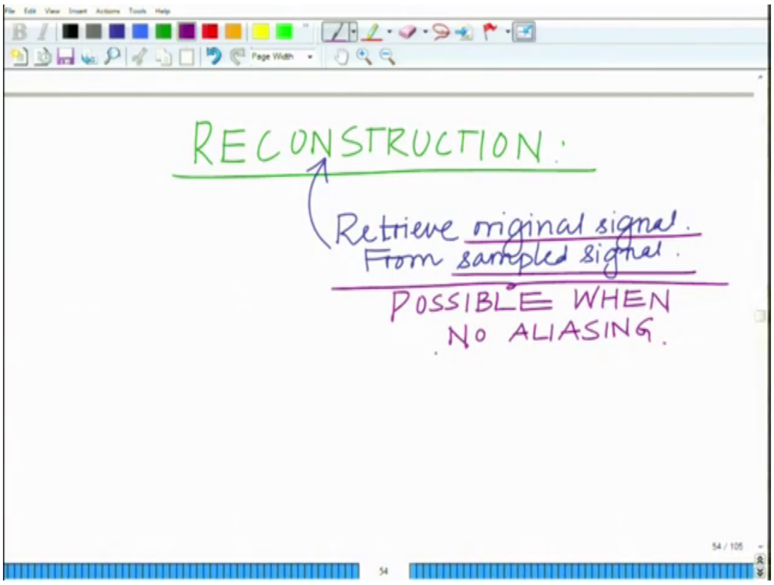

So, how to reconstruct the by reconstruction we means basically retrieve, retrieve the original signal retrieve the original signal from the sample. So, so how do you retrieve the original signal from the sampled signal and obviously this is only possible when there is no distortion correct ok.

So, this reconstruction of the original signal from the sampled signal the exactly, we talking about perfect reconstruction such the original signal is equal to the reconstruction or the re-consist reconstructed signal is equal to the original signal, this is only possible when there is no aliasing all right. So, possible when there is no aliasing and when there is no aliasing how we can do it is fairly simple.

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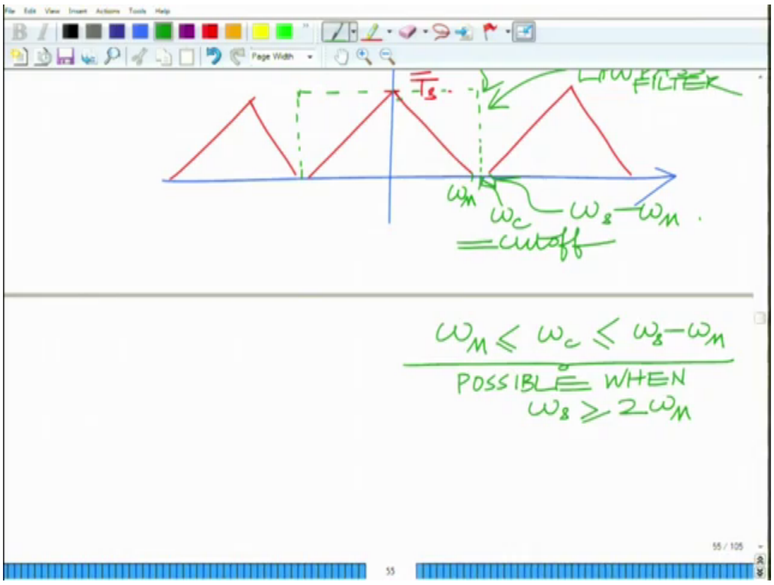

So, what we have over here is basically you have your copies of the spectrum, which are not aliased remember you can see there is a guard band ok. So, this is 1 over Ts which is simply a scaling factor.

Now, I can in principle low pass filter this using an ideal low pass filter, I can low pass filter of this using an ideal low pass filter; such that I extract the spectral components corresponding to the original spectral. So, this is your ideal low pass filter gain is unity, so this is your omega m and the cutoff basically is omega c such that and now look at this is your omega s minus omega m.

So, this is basically your omega c which is your cutoff of the ideal low pass filter ok, this is your ideal low pass filter again and I can choose omega c such that I can choose the cutoff frequency of the ideal low pass filter such that omega m less than or equal to for that matter, omega c less than or equal to omega s minus omega n ok. Only possible when there is a guard remember for this to be possible when omega s when less than omega s omega m is less than equal to omega s minus omega m which means omega s is greater than equal to that, is when there is a gap between those 2 bands I can design a low pass filter and ideal low pass filter such that it extracts only the original spectral band, original spectral corresponding to x x corresponding to x t by appropriate suitable scaling.

Of course while it suppresses it cuts off the other copies the other aliased, the other not alias the other copies that is other shifted spectral copies of the original signal xt ok. So, basically we want to multiply it by the ideal low pass filter pass, this signal the sampled signal through the ideal low pass filter omega c minus omega c gain equals 1 omega c equals your cutoff frequency.

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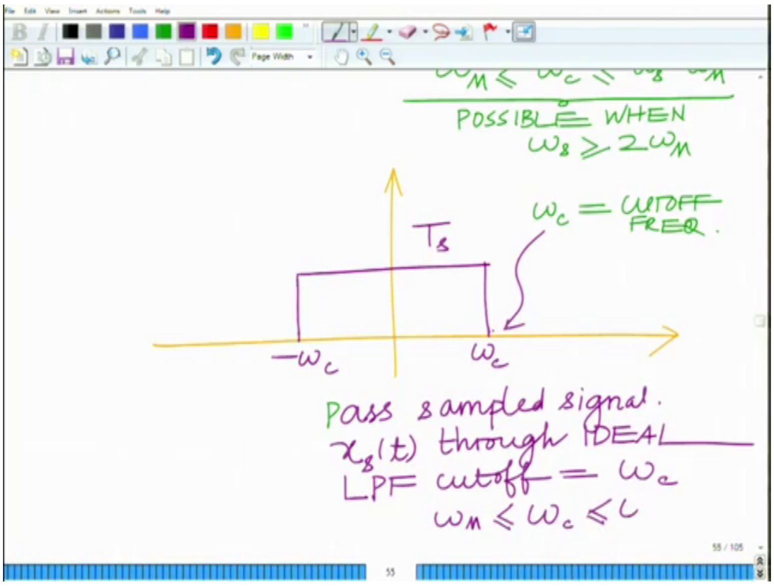

So, pass this and to invert to scale it back by the original amount we can always choose this to be Ts. So, that we have 1 by Ts multiplied by Ts that will give a scaling of 1 ok, so you get back the original spectrum ok. So, pass your sampled signal ideal LPF cutoff equals omega c omega m less than equal to omega c less than equal to omega s minus m omega for reconstruction. And when there is no aliasing you can pass it as ideal low pass filter and note that will give you again original spectral copy, because when you pass it through the ideal low pass filter the resulting spectrum of the output.

The spectrum of the resulting output is simply the multiplication of the spectra of the input that is a sampled signal with that of the low pass filter, which only extracts that original which extracts only the original message spectrum in the band minus omega m into omega m ok, because the cutoff frequency is greater than omega ok. Now, without loss of generality we can choose omega c equals omega that is I can set the cutoff frequency omega s omega c equals omega.

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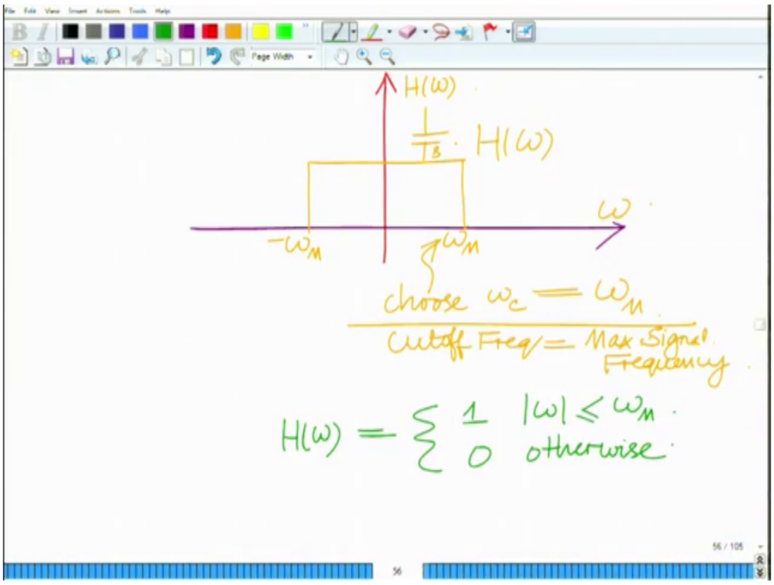

So, I can choose in fact, 1 possible choice is to choose omega s equal to omega m. So, that you can be absolutely sure that you get do not get any of the spurious spectral components of the sampled signal.

So, I am choosing omega c that is cutoff frequency equals maximum message frequency and gain is 1 over Ts cutoff maximum cutoff frequency is the maximum frequency of the signal ok. Now let us look at now this is your low pass filter. So let us call this spectrum as h of omega correct. Now H of omega equals 1 magnitude of omega less than or equal to omega c or basically your omega m 0, otherwise.

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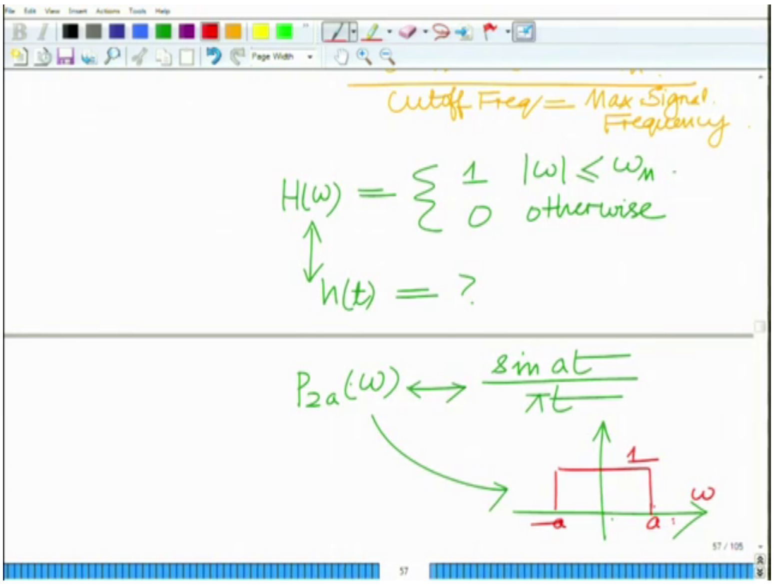

Now, what can we say about the corresponding impulse response H of t, now we know that if you look at the pulse in frequency of width 2 omega centered at 0, that has Fourier transform sin at and this we have derived before this has Fourier transform sin at divided by pi t, what is this is basically your if you remember this is basically you are a pulse of width 2 a centered at 0 that is from and height 1 minus a to a ok. So, this is your period so that has the impulse response sin at by pi t.

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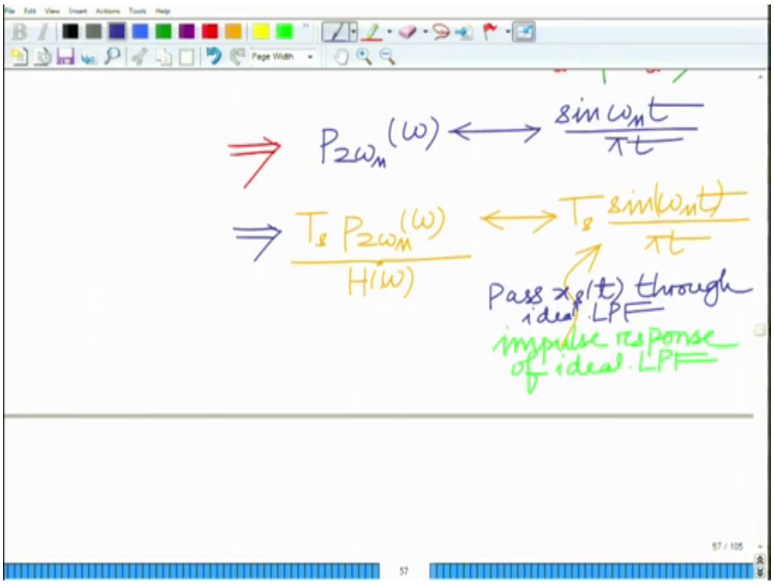

Now this implies that if you consider the pulse of width 2 omega m centered at 0, omega that has impulse response sin omega Mt divided by omega divided by pi t, I am sorry divided by pi t which implies. Now if you multiply this by Ts, Ts is remember nothing but a scaling factor. So, Ts into p of 2 omega m omega that is basically this is your h of omega this has the impulse response sin Ts.

So, you multiply on the left by Ts you multiply on the right also by Ts ok, so in the Fourier transform is linear ok. So, this sin omega m t divided by pi ok. So, that is basically your impulse response of the ideal LPF required for reconstruction, this is the impulse response of an idea ideal LPF.

So, pass the signal so you pass xs t through ideal LPF ok. So, idea you pass it through the ideal low pass filter basically we cut off frequency given by omega n and gain is Ts all right to invert the factor 1 over t x all right. So, Ts x monotheist gives an overall net gain of unity all right and therefore now when you pass it.

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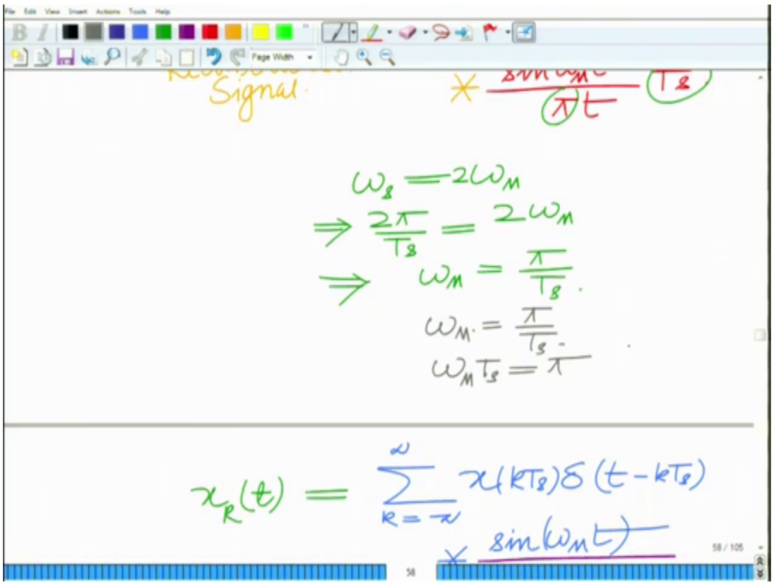

So, let us denote the reconstructed signal by x R of t. Remember this is your sampled signal x S t times h of t. So, this is basically your reconstructed signal, this is which is equal to summation k equals minus infinity to infinity x of k Ts delta t minus k Ts convolved with sin of omega Mt divided by pi t into Ts.

Now, of course you also can also note that we are choosing, we can also choose the sampling frequency equals twice the max that is the Nyquist rate all right is the sampling frequency equal to twice omega m, which implies 2 pi by Ts that is if you choose 2 pi by t s is equal to twice omega m, which means omega m equals pi over Ts ok. So, which means pi over Ts is omega m.

So, you have pi this factor pi over Ts which basically reduces to omega m, that is when you choose the sampling frequency of omega s such that it is exactly the minimum sampling frequency that is required to avoid distortion, that is omega is equal to twice omega n. In this case we have omega m equals pi over Ts where Ts is a sampling duration of the sampling interval the interval between 2 consecutive samples ok.

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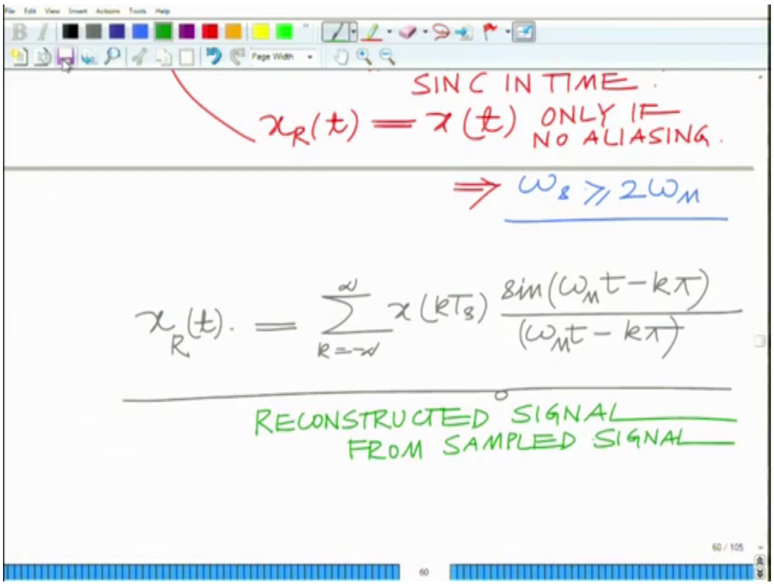

And in this case what we have is x the reconstructed signal is summation k equals minus infinity to infinity x of k Ts delta t minus k Ts convolved with sin of omega Mt divided by omega Mt, which is basically; now again simplifying this summation k equals minus infinity to infinity x of k Ts.

Now, remember you can take the sin the sin function inside. So, the sin of omega Mt divided by omega Mt convolved with delta t minus k Ts that gives me sin of omega M t minus k Ts that is basically shifting it to k Ts. So, this is your reconstructed signal and you can see this basically interpolated this basically taking the samples and interpolating them we are performing linear combinations of these samples x x k Ts by interpolating them by a sinc filter all right.

So, you are basically interpolating the samples using a sinc function in the time domain ok. So, interpolation using a sinc in time domain or basically a sinc filters and further you can simplify it by noting the following thing you can also write it. Now of course, you can realize that x r of t this reconstructed signal is equal to the original signal only if there is no aliasing correct, only if no aliasing this implies that omega s is greater than or equal to twice omega And you can also further simplify this as summation k equals minus infinity to infinity x of k Ts.

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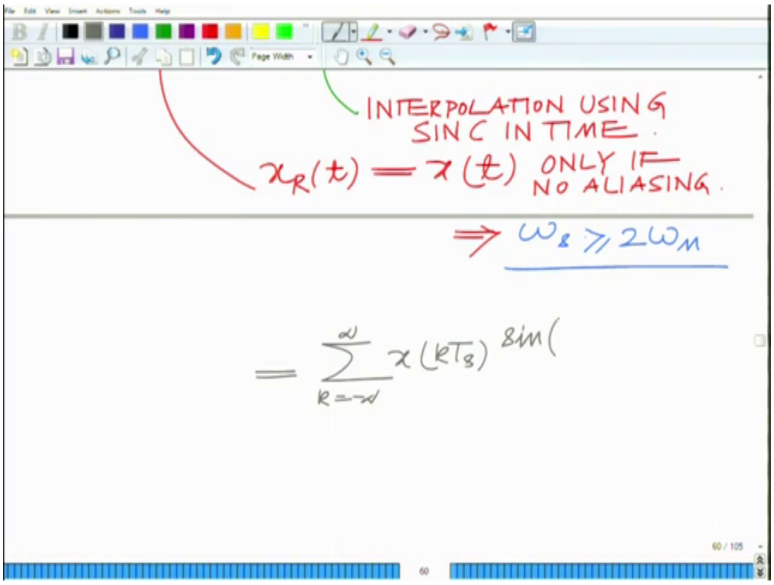

Remember, we have omega m omega m equals pi over Ts which means omega m into Ts it is equal to pi. So, this will be sin of omega m into t sin of omega m into t minus k omega m into Ts minus k pi. So, divided by omega m into p k pi omega m is equal to pi correct. So, that is basically your expression for the reconstructed signal ok.

So, this is basically your again the reconstructed signal in time, this is the reconstructed signal from the sample signal and perfect reconstruction is possible of course when there is no aliasing all right. So, you sample it at a sampling rate that is greater than or equal to Nyquist rate, then pass it through an ideal low pass filter which satisfies the given cutoff frequency requirements. And basically based from and from that you can reconstruct the original signal by extracting the spectrum copy and suppressing extracting the spectrum corresponding to the original signal and subsidizing all the spectral copies correct or the aliases that arise because of the sampling process and this is only possible when there is no aliasing when there is no distortion that arises due to aliasing or the sampling frequency is basically greater than the Nyquist rate, that is twice the maximum frequency of the signal.

So, that completes our discussion of the sampling as well as the reconstruction process which as we have said is one of the fundamental aspects of signal processing and communications since most of the processing is digital.

So, we would like to take while the naturally occurring signals are analog. So, 1 would like to process or convert the first step in the storage and transmission or 6th of signals is usually acquiring the analog signal and converting it into a digital signal. And the first step is sampling converting continuous time signal into a discrete time signal.

So, I will stop here and continue with other aspects and subsequent modules.

Thank you very much.