Principles of Signals and Systems
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Lecture - 46
Fourier Analysis Examples - Fourier Transform of Exponential, Cosine, Sgn, Unit-Step Signals, Even and Odd Components

Hello welcome to another module in this massive open online course. So, we are looking at a example problems for the Fourier analysis or the Fourier transform of a periodic signals all right.

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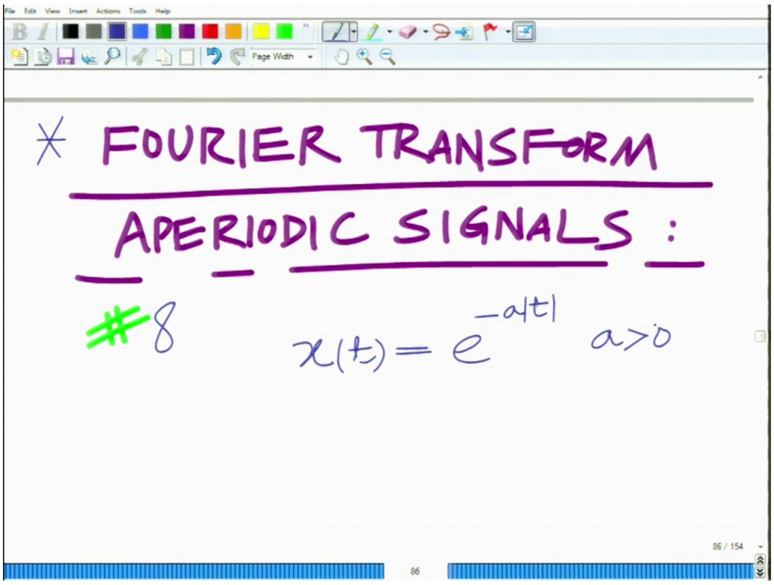

So, we are looking at the the Fourier transform and this is for a periodic signals ok. So, let us continue your discussion let us look at problem number let us look at problem number 8 and in this problem, we have a simple signal x t equals e raise to minus a magnitude t modulus t for a greater than 0. .

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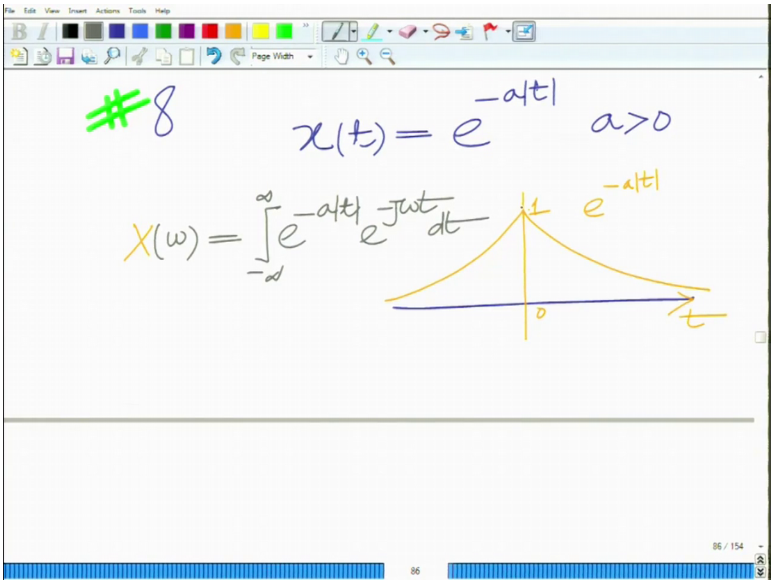

And therefore, if you plot it just basically in exponential; which is decaying on both sides of this looks something like this this i s e raise to minus mod tends to 0 as t tends to either infinity or minus infinity and t equal to 0.

This is unity this is the decaying exponentials on both sides of the real axis this signal is decaying exponential ok. And the Fourier transform of this can be evaluated as follows that is x of omega equals integral minus infinity to infinity, e raise to minus a mod magnitude t e raise to minus omega t or minus J omega t d t.

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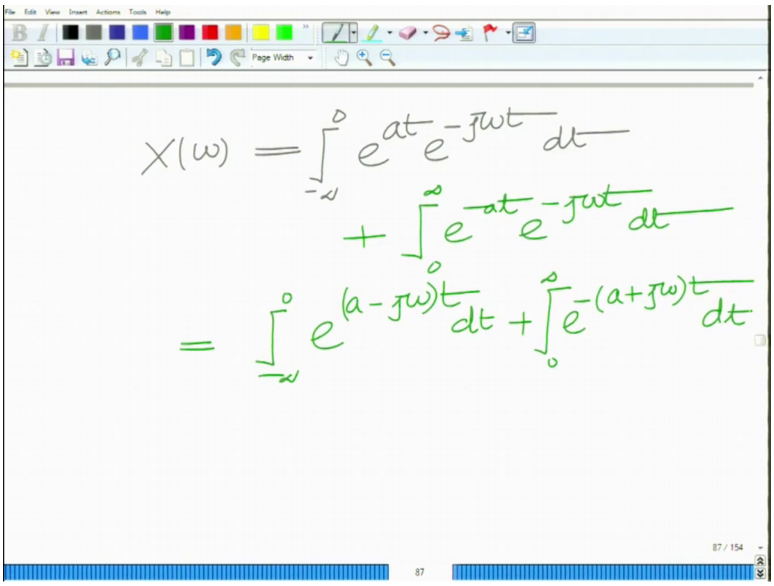

Which is X of omega now split this into 2 integrals 1 from minus infinity to 0 t is less than 0 some magnitude of t is minus t. So, this becomes e raise to a t e raise to a t e raise to minus J omega t d t plus integral 0 to infinity e raise to minus a t e raise to minus J omega t d t.

So, you split this into 2 integrals. Now the first integral is integral minus infinity to 0 e raise to a minus J omega t d t plus integral 0 to infinity e raise to minus a plus J minus a plus J omega t d t.

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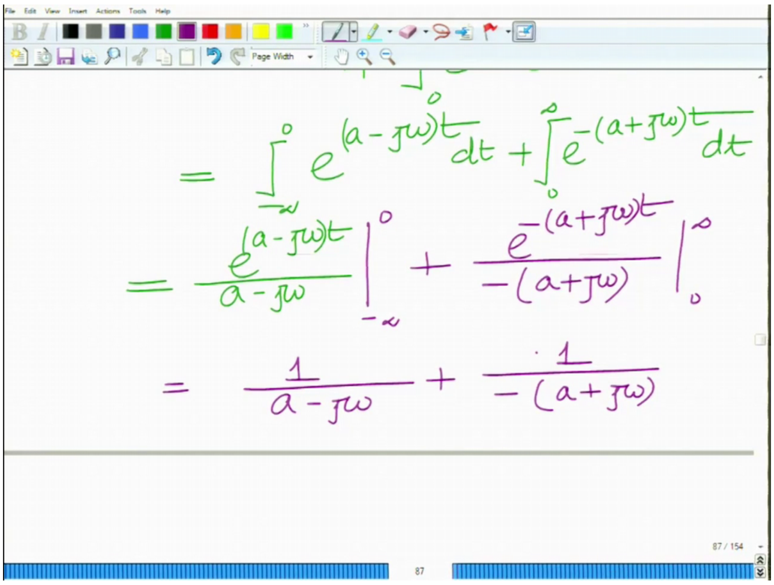

And the first integral is 1 over integral exponential integral of exponentially integral e raise to a minus J omega t is 1 over a minus J omega, e raise to a minus J omega t evaluated between the limits minus infinity to 0 plus or rather or plus divided by minus a plus J omega e raise to minus a plus J omega t evaluated between the limits 0 to infinity.

And now you can see the first integral because remember a is greater than 0. So, e raise to a minus J omega t evaluated in zeros 1 e raise to a minus J omega t evaluated at t equals minus infinity is 0. So, this becomes 1 over a minus J omega plus minus a plus J omega e raise to minus a plus J omega t evaluated at t equal to infinity 0 minus e raise to minus a plus J omega t evaluated at t equal to 0 as 1. So, this becomes 1 or this becomes minus 1 0 minus 1 is minus 1.

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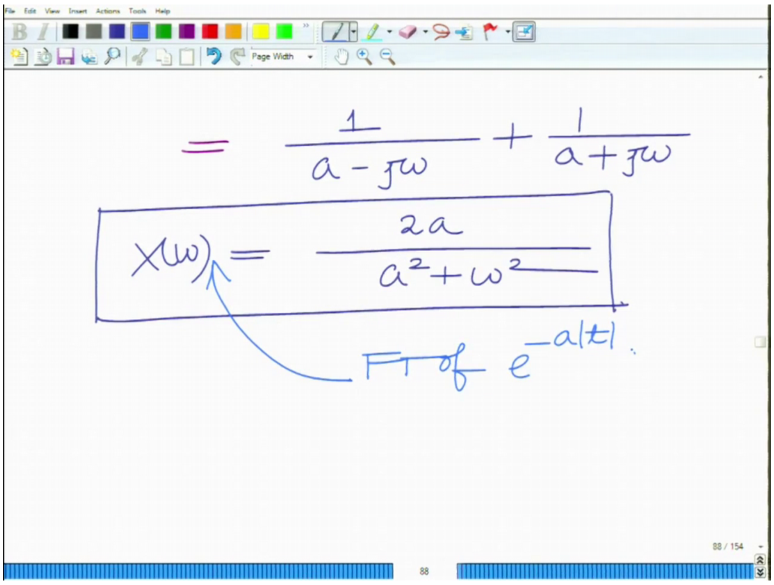

So, this finally, evaluates as 1 over a minus J omega plus 1 over a plus J omega, which is a plus J omega plus a minus J omega that is 2 a over a minus J omega into a plus J omega that is a square plus omega square.

. So, that is basically your the Fourier transform of e raise to minus a magnitude of t ok. So, this is the Fourier transform of that is the Fourier transform e raise to minus a magnitude t that is the exponential which is decreasing on both side of the real axis is 2 a over a square plus omega square ok. Let us go to the next problem.

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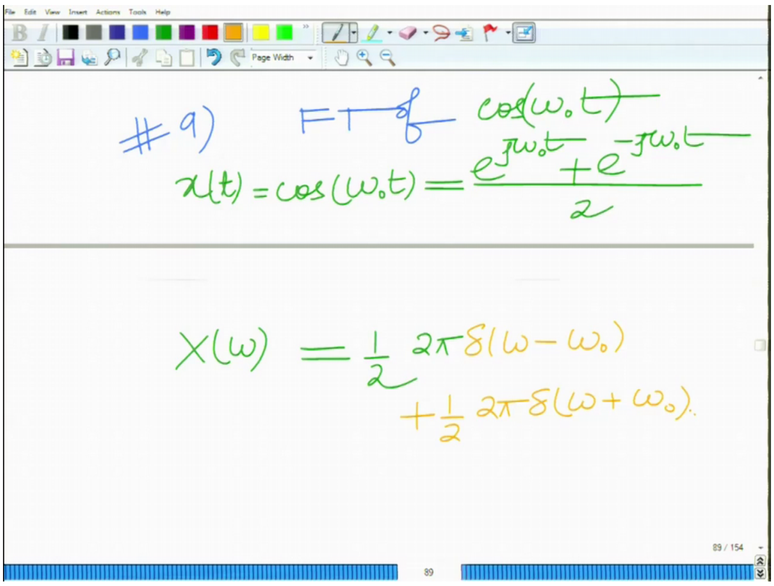

We want to evaluate the Fourier transform of 1 of the most frequently occurring signals which is cosine omega naught t and you can see cosine omega naught t or you can write this as e raise to J omega naught t plus e raise to minus J omega naught t divided by 2 now e raise to J omega naught t has Fourier transforms. So, this is half.

So, Fourier transform of this if you call this as x of t the Fourier transform of this is half Fourier transform of e raise to J omega naught t that is 2 pi, delta omega minus omega naught plus half Fourier transform of e raise to minus J omega naught t that is 2 pi delta omega plus omega naught.

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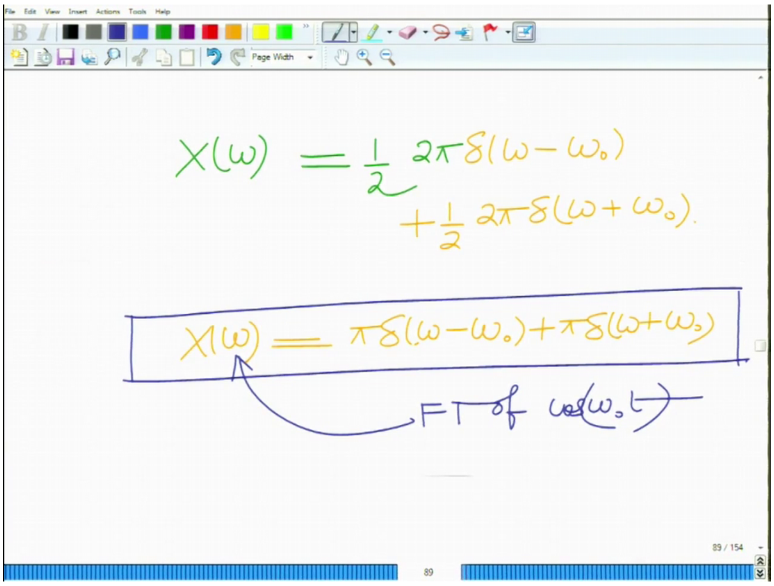

Which is basically the Fourier transform of omega naught t which gives raise to x of omega is pi delta omega minus omega naught plus pi, delta omega plus omega naught. This is the Fourier transform of cosine omega naught t, that is the Fourier transform of cosine omega naught t is pi delta omega minus omega naught plus pi delta omega plus omega naught. And similarly we can evaluate the Fourier transform of another very commonly occurring signal that is the pure sinusoid sin of omega naught t.

All right let us look at the next example.

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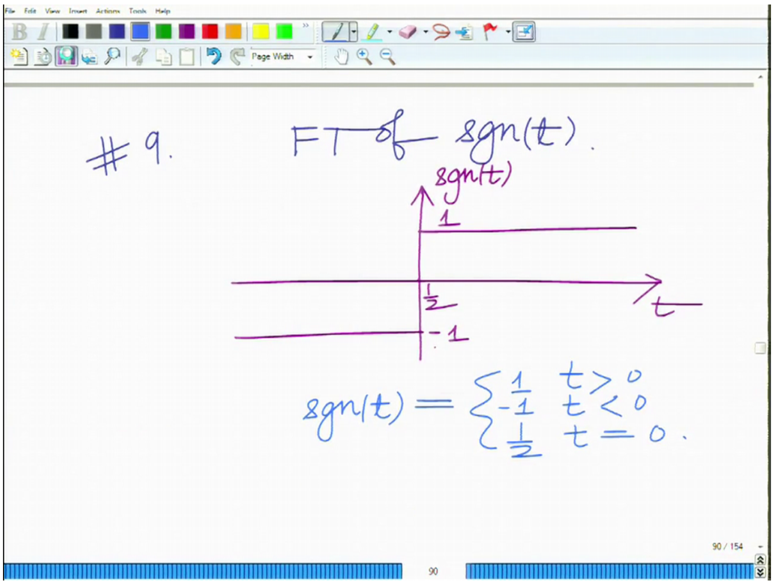

Fourier transform of the sign function s g n of t sign of t and we have seen the sign of t simply basically the sign of a real number that is f f t is greater than 0 then sign of t is plus 1, if t is less than 0 then sign of t is minus 1 and at 0 it is actually half and it makes a transition at 0 from minus 1 to 1 ok.

So, sign of t the signal is 1 for t greater than 0 minus 1 t is less than 0 half at t equal to 0 ok. .

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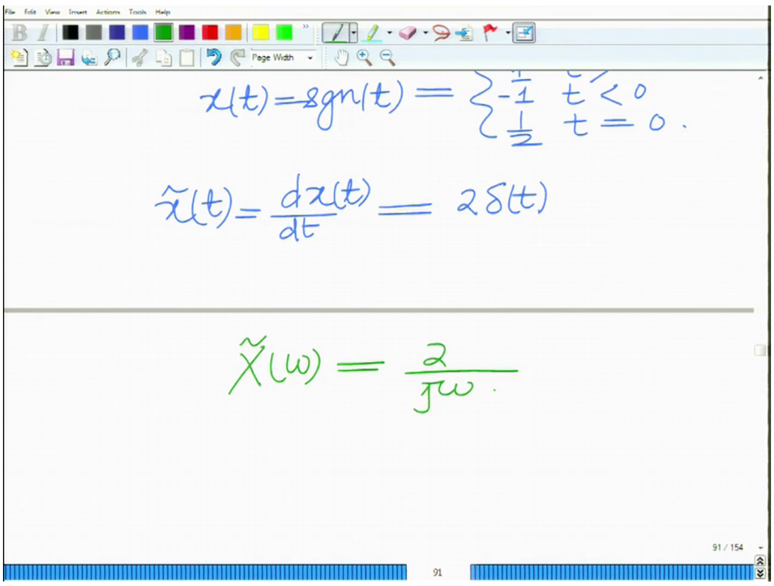

And now you can see if you take the now evaluate this, let us consider the derivative of sign d derivative of this the derivative of this is, basically you can see if t is greater than 0 it is constant it is 1. So, the derivative is 0 if t is less than 0 that is minus 1 which is constant.

So, derivatives once again 0 at t equal to 0 the derivative is an impulse because it makes a transition from minus, which is step change from minus 1 to 1 of magnitude is a step change of basically magnitude 2. So, it is the derivative is an impulse of magnitude 2.

So, the derivative of this is twice delta t. Let us call this x of let us denote this x tilde t. Now, therefore, X tilde of omega which is a Fourier transform of X tilde t this is equal to twice the Fourier transform of [de/delta] delta t, which is twice over J omega. And that is X tilde now remember X tilde t is the derivative of X of t therefore, the Fourier transform X tilde omega is J omega times the Fourier transform X omega this is we knows from the properties of the Fourier transform ok. So, from properties of Fourier transform.

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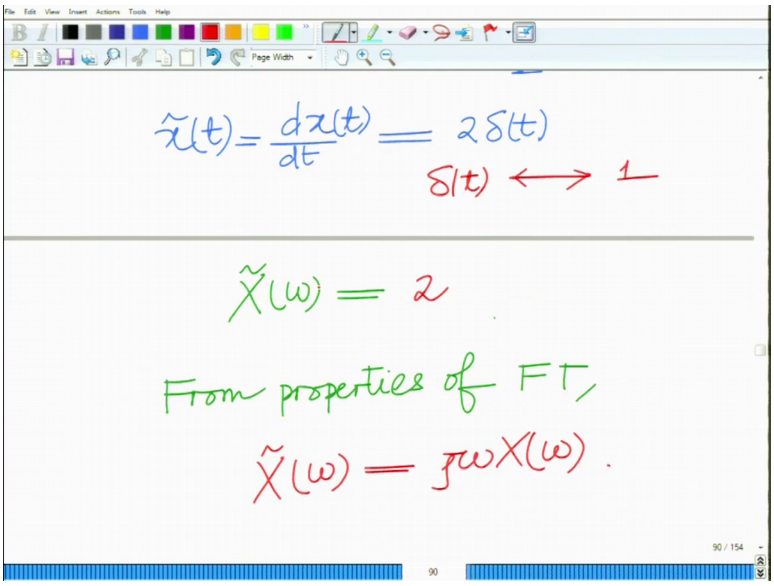

So, from the properties of Fourier transform, we know that X tilde omega equals J omega I am sorry x tilde omega equals J omega times, X I am sorry this is simply 2 X tilde omega the Fourier transform twice the Fourier transform of delta delta t delta t has Fourier transform of unity ok. Since delta t has Fourier transform of unity. So, X tilde omega is simply 2. So, X tilde omega is J omega times X of omega.

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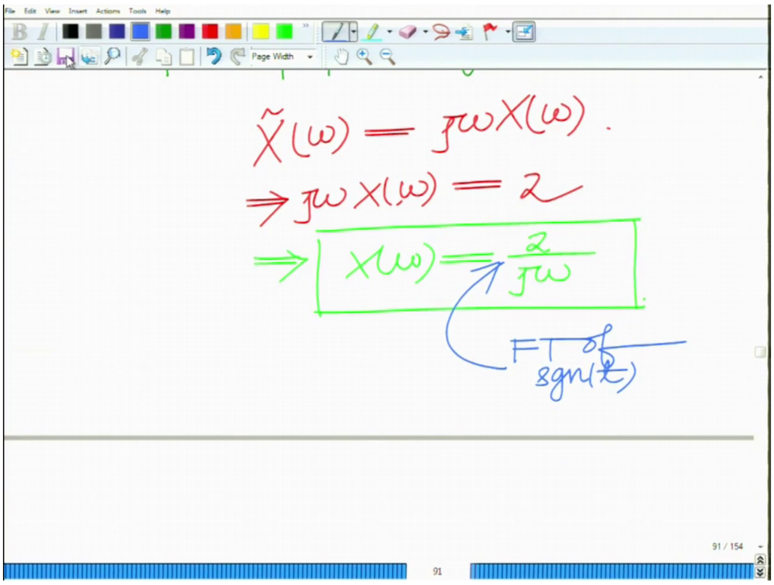

So, this implies J omega times X of omega equals 2 which implies that X of omega that is the Fourier transform sign t is twice 2 over J omega. So, this is the Fourier transform of sign t this is the Fourier transform of sign t ok. So, the Fourier transform of the sign function all right is basically 2 over J of omega. And this is a very interesting result because we will see now that we can use this to derive the Fourier transform the unit step function which we will simply noted before ok.

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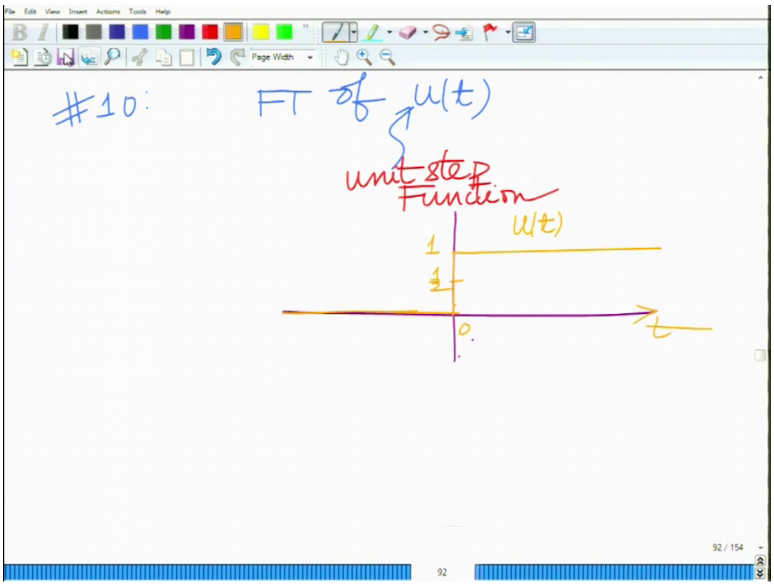

So, now we want to use this result to derive the Fourier transform of u t that is your unit step function and the unit step function note that the unit step function must be very familiar to all. So, the unit step function is basically it is 1 if t is greater than 0 at 0 at 0 it is half and for t less than 0 it is 0 . So, this is the unit step function which is basically u of t 1 t greater than 0 half t equal to 0 0 if t is less than 0 ok.

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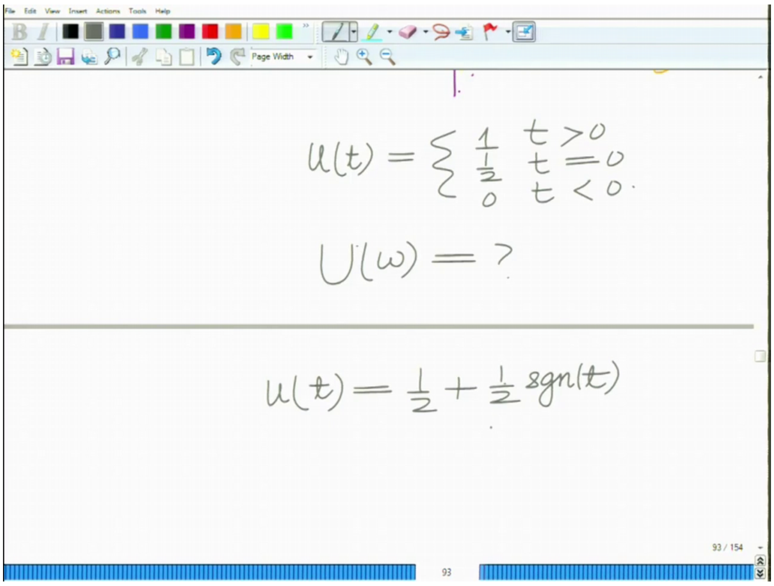

Now we want to derive the Fourier transform of the unit step function that is denoted by u of omega. And now to do this first realize that the unit step function can be expressed in terms of the sign function as u of t and this is once you realize that it is very simple.

So, we have u of t is half plus half sign of t for instance you can check this if t is greater than 0 sign of t is 1. So, this is half plus half into 1 which is 1 at t equal to 0 this is half this becomes half plus half into 0. So, that is half at t t is negative that is t less than 0 this becomes half plus half into minus 1, which is half minus half which is 0. So, this verifies that u of t equals half into half plus sign half plus half into sign of t.

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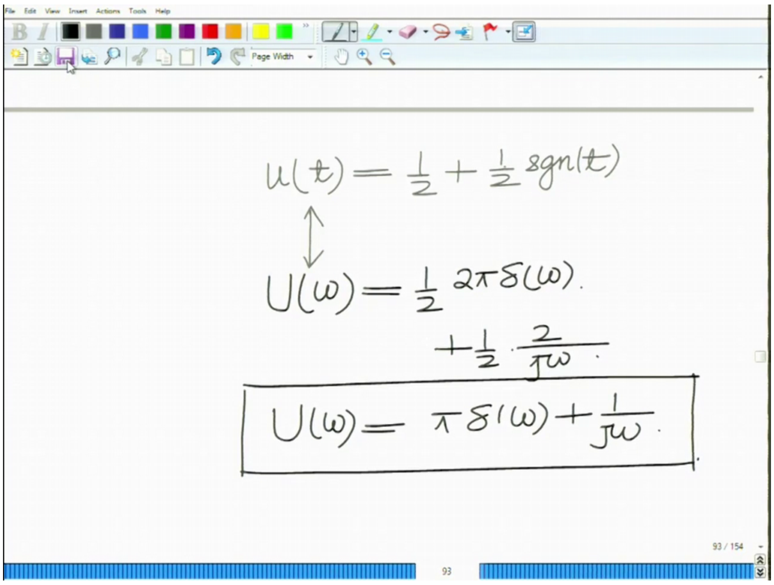

Which implies that is now if you take Fourier transform of both sides you have u of omega the Fourier transform of simply half is half times delta t or half times, sorry this is the half times 2 pi delta omega plus half times the Fourier transform of sign t is 2 by J omega. So, you get this simplify this is pi of delta omega plus 1 of 1 over 1 over J omega ok. So, this is the Fourier transform of the unit step function.

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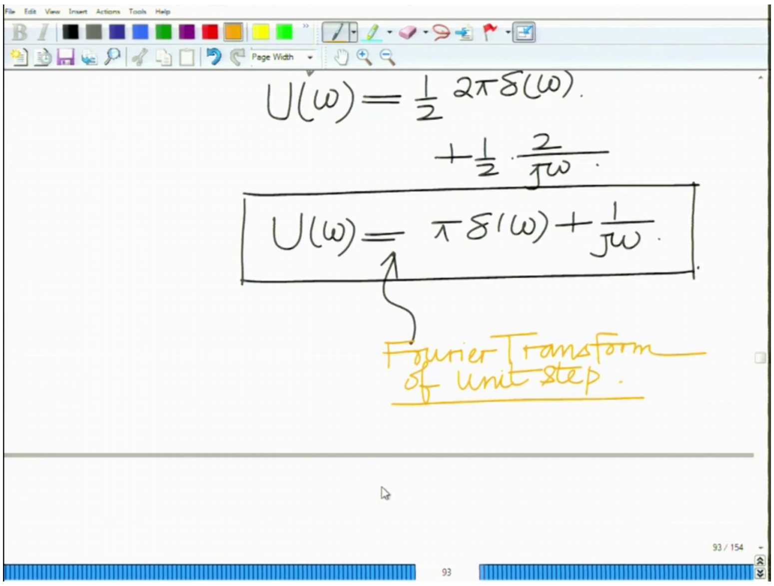

This is the Fourier transform of the unit step function ok. Let us now look at a another interesting problem in which we want to look at.

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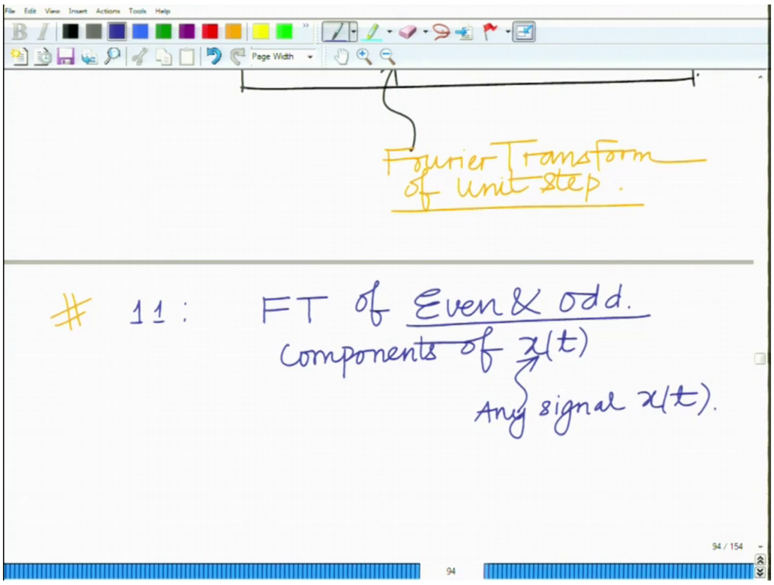

A new concept that we have not seen before, that is a Fourier transform of the even we have a single any single x t can be written in terms of it is even and odd components ok. That is the components which explicit even and odd symmetric respectively. And we want to express the Fourier transform or we want to derive expressions for the Fourier transform of the even and odd components of this signal x of t.

So, given x of t signal x of t we want to find the Fourier transform of the even and odd components that is even and odd components of x of t and this is any signal x of t.

Now the Fourier transform, now first we have to first and the Fourier transform and this is any real signal let me correct this, this is any real signal.

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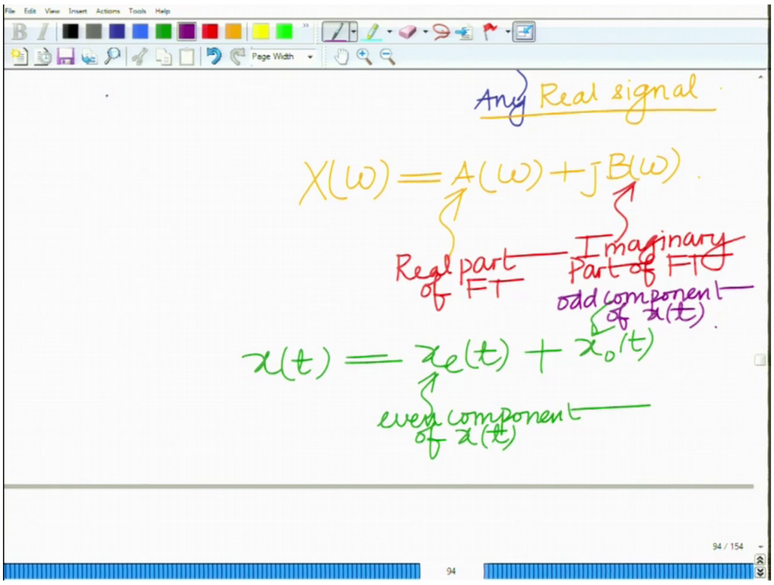

So, x t is a real signal ok. And we have let say X of omega equals a omega plus J times B omega that is a omega denotes the real part of the Fourier transform and B omega denotes the B omega denotes the imaginary part of the Fourier transform.

Now what we can show is that the Fourier transform the even and odd components now first we want to derive the expression for the even and odd components of this signal x t. Let us denote this even and odd components by x e t and x o t for the even and odd components ok.

So, I want to write now x e t x of t that is signal x of t equals x e t sum of the even. So, this is even component and this is odd component of x of t, even component and odd component of x t.

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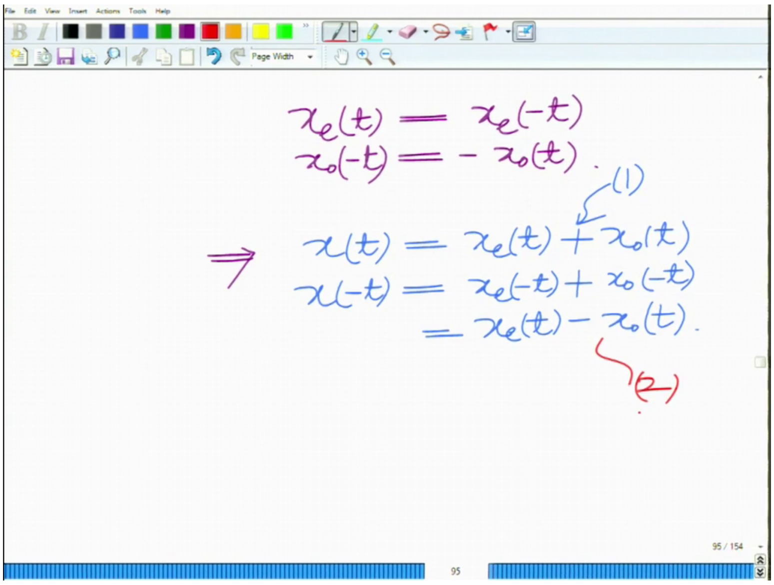

Now naturally these are even and odd we have to have x e t remember this must execute even symmetry, which means x e t equals x e of minus t x o of t that is odd component has to exhibit odd symmetry that is x o t x of minus t is minus of x of t.

So, these have to exhibit even and odd symmetry. So, this implies now first we have x t equals x e t plus x o t. Now x of minus t is x e of minus t plus x o of minus t which is naturally equal to because x o of minus t is x e of t. So, this is x e of t and x of minus t is minus x o of t. So, this is minus x o of t ok.

And therefore, now if you look at these 2 equations let us say we call this as equation 1 and we call this as equation 2 we have solving for x e t.

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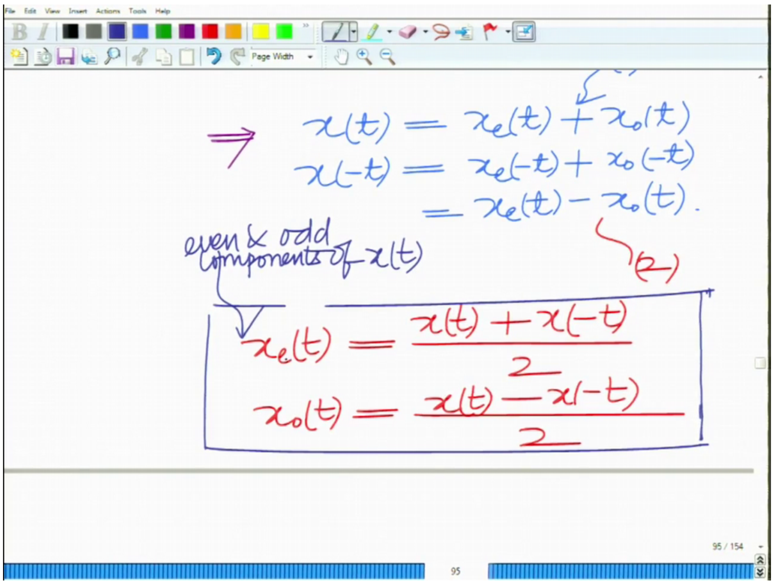

We have x e t equals x t plus x of minus t divided by 2 and the odd component is x t minus x of minus t divided by 2, this is a general result that is for a real signal x t these represent the even and odd components that is any signal x t can be decomposed as the even and odd components.

So, these are even and odd components and you can verify that these x e t that x e t is indeed even, because if you look at x e of minus t that is a x of minus t plus x of minus of minus t that is x of t divided by 2 this is a again x e of t. Similarly we can verify that x o of t is indeed odd and you can also verify that x e t plus x o t is indeed the signal of x t.

This is a general expression for the even and odd components of a real signal x t. Now let us consider the Fourier the Fourier transform of these even and odd components.

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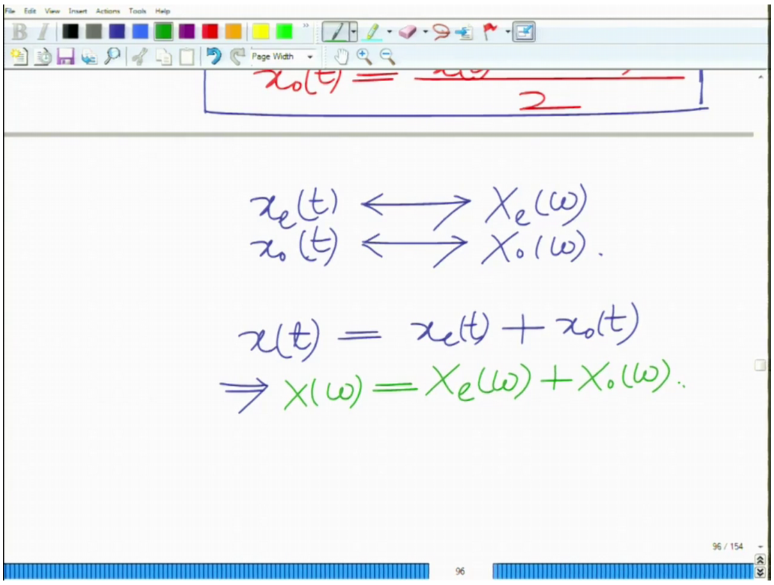

So, let us say we have x e t has the Fourier transform x e of omega, x o t has the Fourier transform x o of omega. Now recall that x t is real now first recall that x t equals x e t plus x o t which implies that the Fourier transform x omega equals x e of omega plus x odd of omega.

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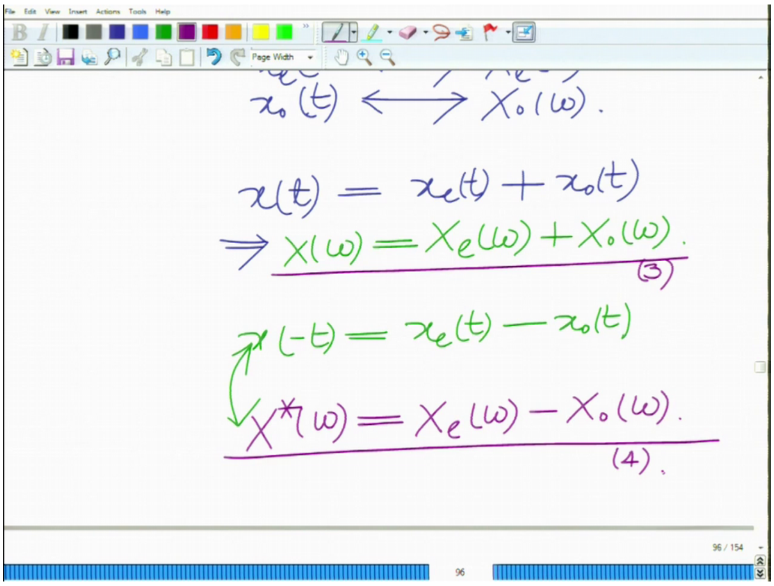

Further, if we look at x of minus t, that is x e of minus t which is x e of t plus, x o of minus t which is minus x o of t. So, this is minus x of t. Now if you take the Fourier transform of this x of minus t of; obviously, this is the real signal the Fourier transform x of minus t remember is X conjugate of omega for real signal the Fourier transform of x of minus t is X conjugate of omega ok.

So, this is X conjugate of omega X conjugate of omega equals now the Fourier transform of X e t equals X e of omega minus X o of omega ok. Therefore, we have these 2 results X of omega is X e omega plus X o of omega let us call this result, since we have already denoted let us call this result 3 and let us call this X conjugate of omega is X e of omega minus X o of omega X odd of omega and from 3 and 4.

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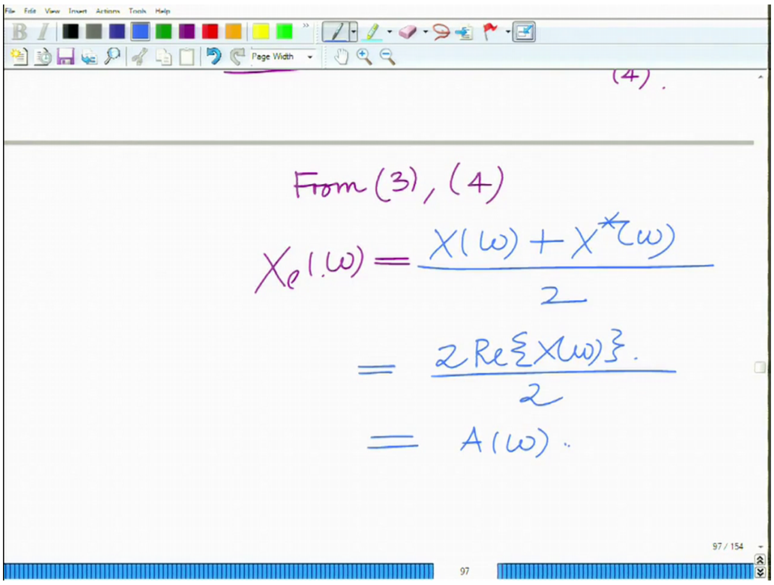

Once again we have X e of omega equals x of omega plus X conjugate of omega divided by 2.

Now look at this X of omega plus X conjugate of omega is nothing, but the twice the real part of X of omega. So, twice real part of X of omega divided by 2, which is the real part of X of omega and X of omega we have already defined it as a of omega plus J times B omega. So, the real part of X omega is a omega. So, therefore, this is twice real part of X omega divided by 2 which is a omega.

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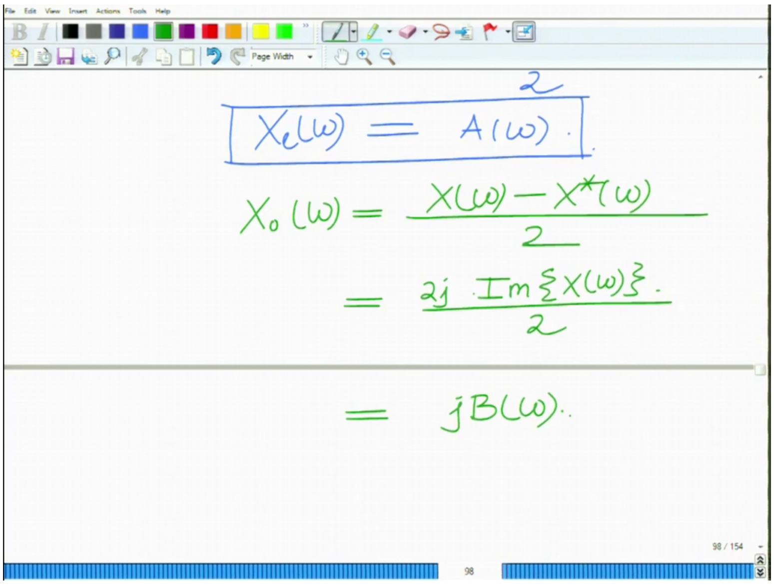

So, if X of omega is A omega plus j B j times B omega X e omega that is the Fourier transform of the real part of the X e t is a omega. And similarly X of omega is X odd of omega is X of omega minus X conjugate omega divided by 2, which is twice j imaginary part imaginary part of X of omega , which is twice J imaginary part of X of omega divided by 2 which is basically your j times B omega.

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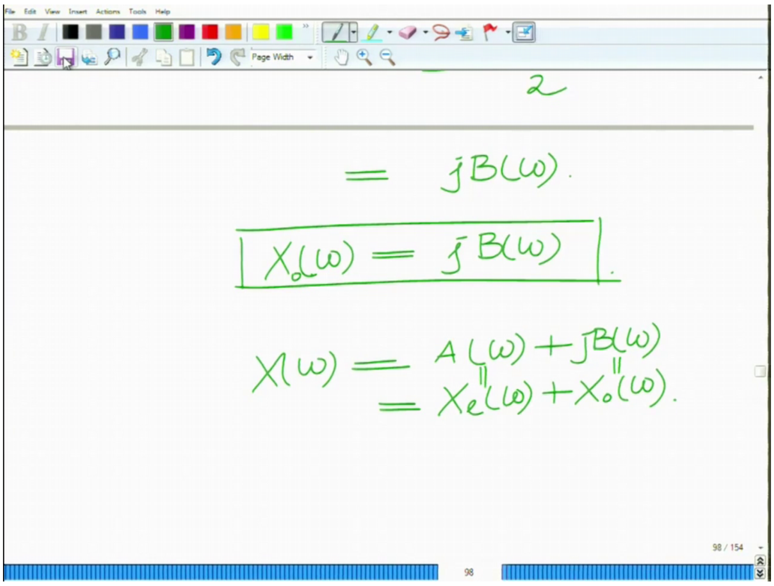

So, X of omega we get is basically j times B of ok. And therefore, we have X omega equals A omega plus J times B omega which is similarly X even of omega plus X odd of omega. So, A omega is equal to X e omega at J times B omega equals X odd of omega that is the Fourier transform of the odd component.

That is the Fourier transform of the odd component of this thing ok. And therefore, so basically that completes these interesting problems. So, we have X even of omega has Fourier transform a of omega that is the, even component of X e t as Fourier transform A omega that is the real part of X omega and the odd component X omega odd component X o t has the Fourier transform of X o omega, which is J times B omega, where B omega is the imaginary part of the Fourier transform of X e t all right.

So, with these examples let us stop this module here will continue with other examples in the subsequent modules.

Thank you.