Principles of Signals and Systems
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Lecture - 45
Fourier Analysis Examples - Fourier Transform of Square Pulse, Fourier Transform of Sinc Pulse

Hello, welcome to another module in this massive of online course. So, we looking at the Fourier analysis, and we looking at the example problems in the Fourier analysis and so far. We focused on example problems for discrete type signals let us now shift or focus to example problems for continuous time signals ok. So, starting this module we were going to look at again example problems for the Fourier analysis and this time specifically focusing on your continuous time signals.

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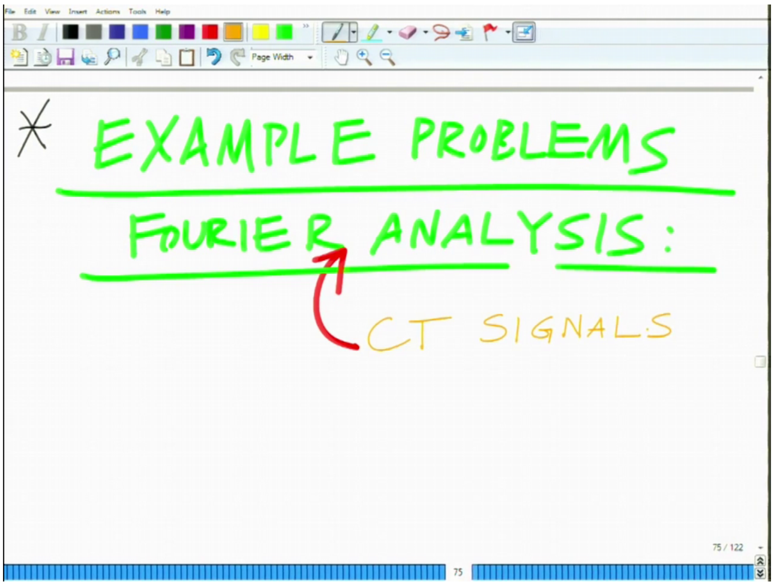

The C T or continuous time signals let us start with example continuing our example numbers.

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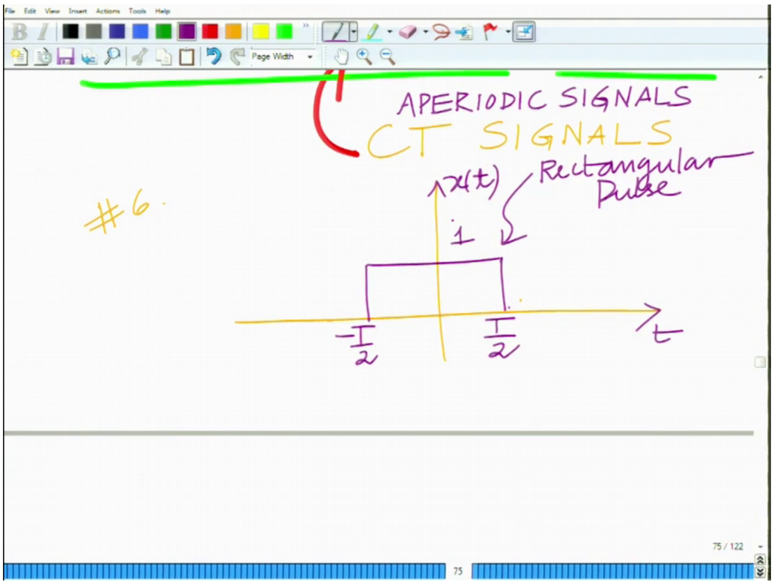

Let us start with example number 6, let us consider a very simple be yet fundamental signal and this we will encounter in various applications, that is let us say I have a signal or rudimentary a very basics signal that looks something like this. So, pulses continuous time signals and aperiodic remember we have looked at, so far. We have looked at continuous aperiodic signal periodic signals, so of the Fourier series.

Let us look at now continuous aperiodic signals ok. So, this is a pulse a rectangular pulse if you well. So, this is a rectangular pulse of width T. So, from minus T by 2 to T by 2 of height unity this is in time domain, this is the signal x t we denote this as the pulse.

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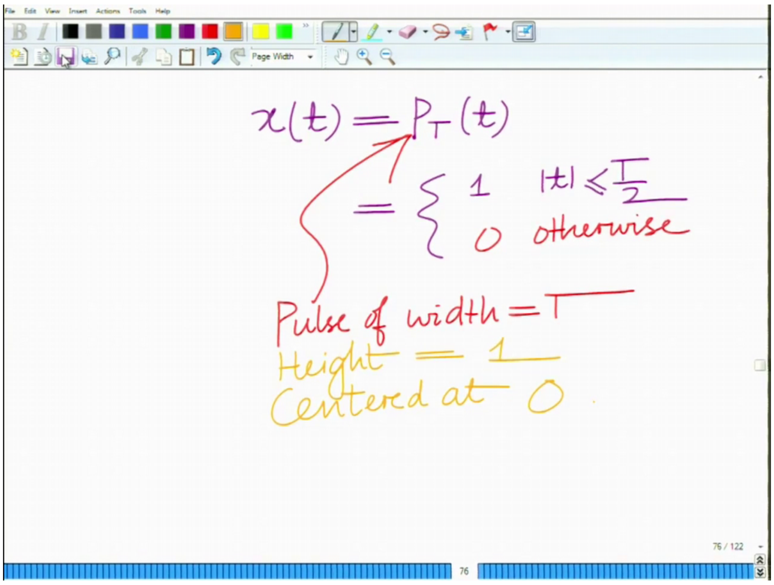

X t equals P T pulse of width t of t P T of t which to define it formally that is denoted by pulse of width t, this is basically 1 for magnitude of t less than or equal to capital T over 2 and this is 0 otherwise, and this is pulse of width t pulse of height unity width t centered at 0. So, 1 of the way to think about is this pulse of width equal to T height equals unity. And this is centered at 0 now we want to find the Fourier transform of this.

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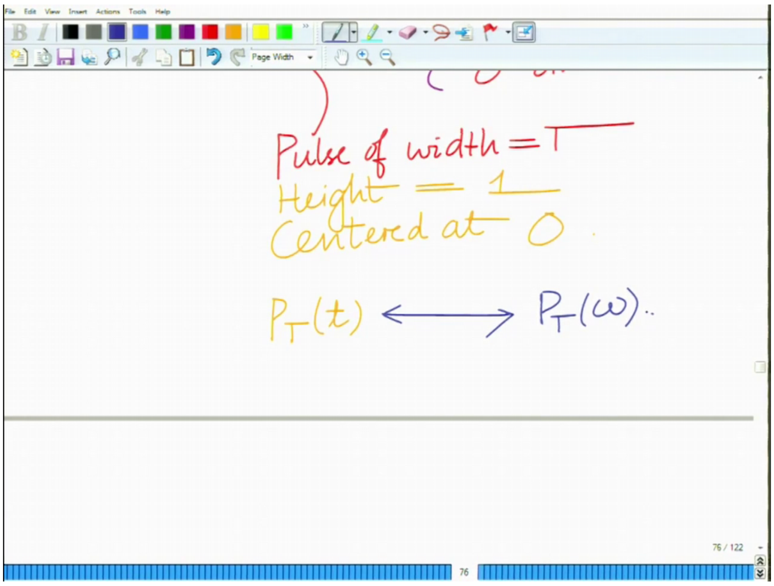

P T of t let us denote this Fourier transform by capital P T of omega that is the Fourier transform of this pulse.

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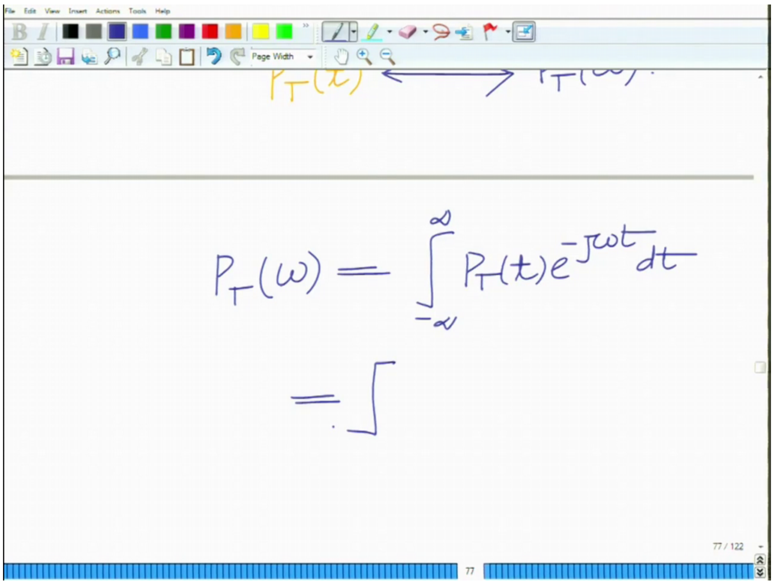

And remember the expression for the Fourier transform, the continuous time Fourier transform that is your integral minus infinity to infinity, P T of t e raise to minus j omega t d t, which is basically non-zero only from minus T to T by 2.

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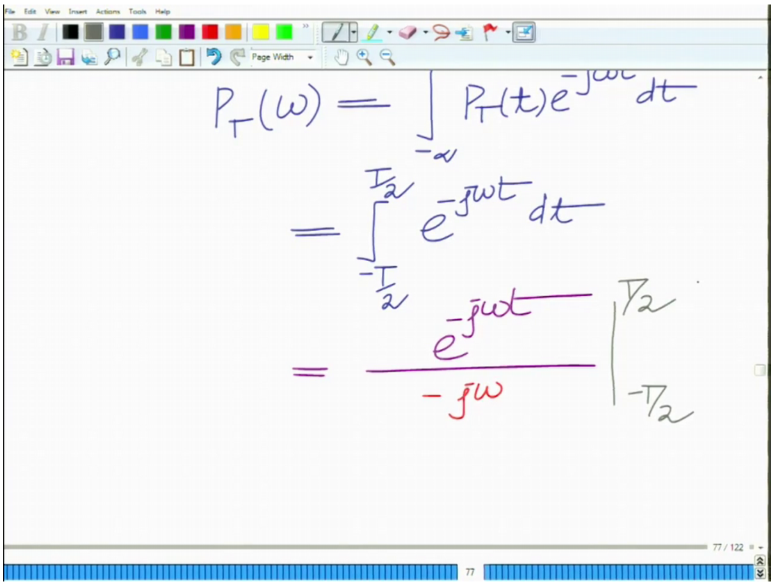

So, I can write this as this becomes your integral minus T by 2 to T by 2 which is 1 in this range. So, it is simply e raise to minus j omega t d t and taking the integral this will be e raise to minus j omega t divided by minus e raise to minus j omega t divided by minus j omega between the limits; obviously, between the limits minus T by 2 to T by 2 .

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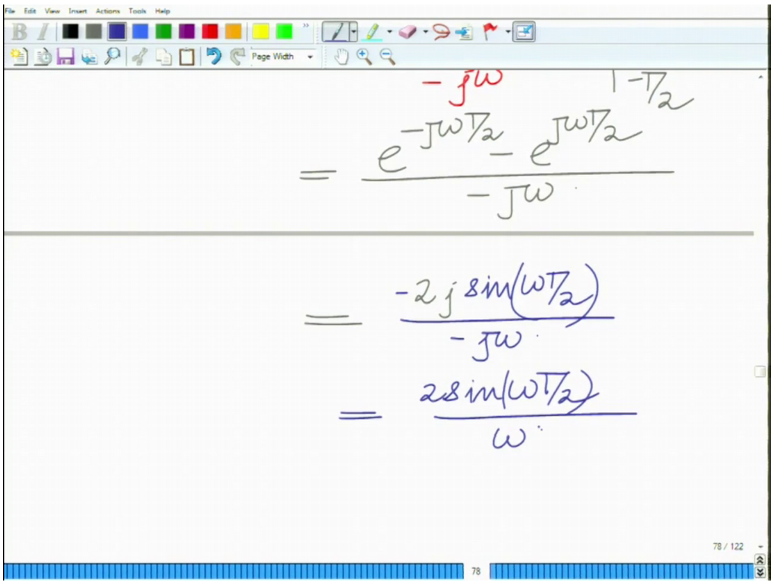

So, this integral is e raise to minus j omega t divided by minus j omega evaluated between the limits minus T by 2 to T by 2. Now substituting the limits we have e raise to minus j omega T by 2 minus, e raise to j omega T by 2 divided by minus j omega, and we know that e raise to minus j omega T by 2 minus e raise to j omega T by 2 T over 2, is basically sin omega or 2 j sin omega T over 2.

So, I can simplify this as 2 j using the properties of the complex numbers that is e raise to minus j theta minus e raise to j theta ok, so 2 j sin omega T by 2 by. In fact, this is minus 2 j sin omega T by 2 by minus j omega which is basically you can simplify this by cancel this will be sin omega T by 2 divided by or twice sin omega T by 2 divided by omega, and this you can mult this is your P T omega.

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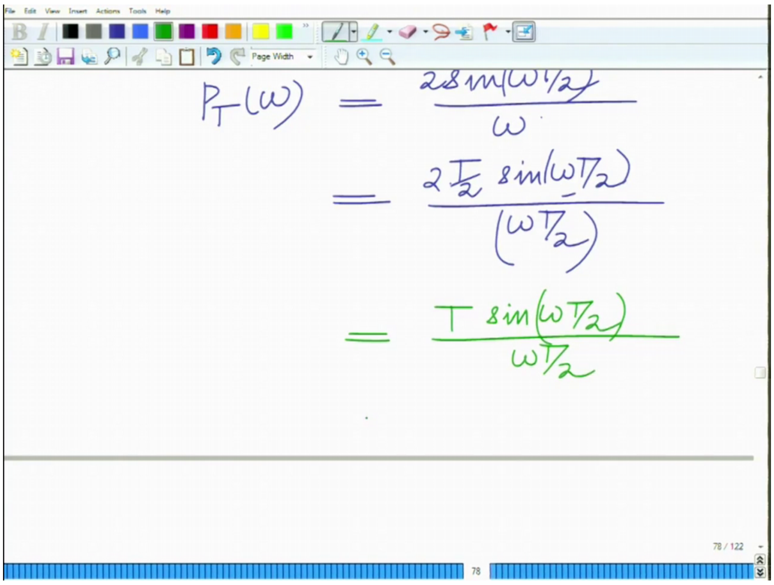

That is the Fourier transform of this pulse and you can multiply and divide by T over 2. So, this is twice T over 2 sin omega T by 2 divided by omega T over 2, which is equal to t sin omega T over 2 divided by omega T over 2. And further I can also write this as remember we still simplifying the same expression.

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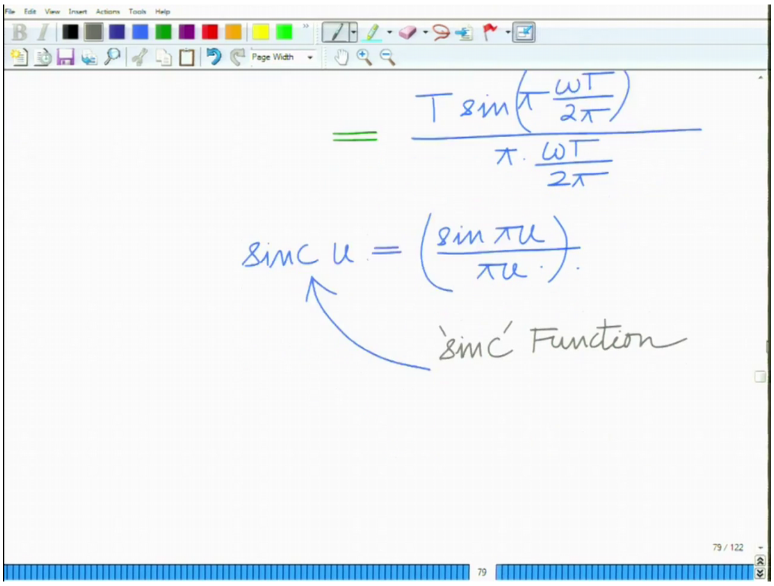

I can write this as T sin pi omega T over 2 pi times pi omega T over 2 pi and we will now define a notation that is a popular notation this is known as the sinc functions sinc of x or sinc of u equals sin pi u divided by pi u. So, this is known as the sinc function and its 1 of the most very convenient and very frequently something that is very convenient use something that arises very frequently in communications and signal processing in general and in general signals and systems. So, this is termed as the sinc this is the sinc function and therefore, this here.

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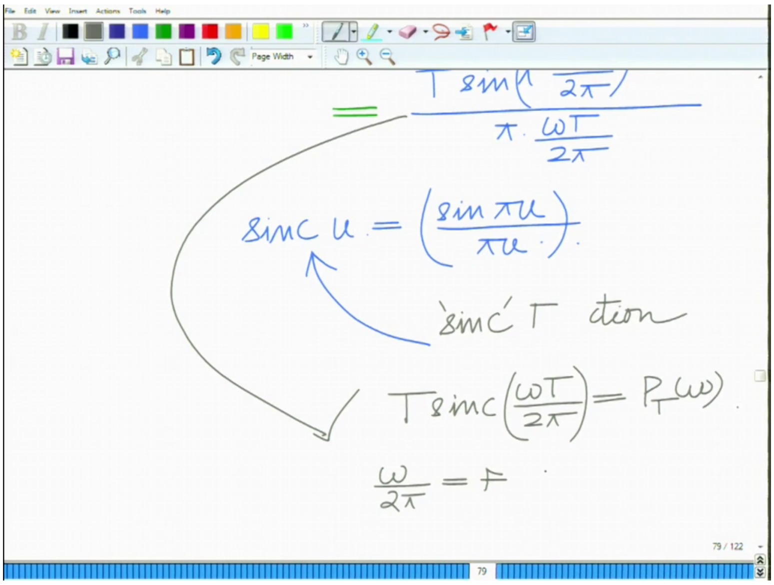

Will simply be T sinc omega t T over 2 pi or T sinc of omega T over 2 pi, and this is your P T of omega. And further if we denote omega over 2 pi as the omega as the over angular frequency omega over 2 pi is a frequency F, then this simply becomes T sinc FT. So, substituting omega over 2 pi equals F you can write.

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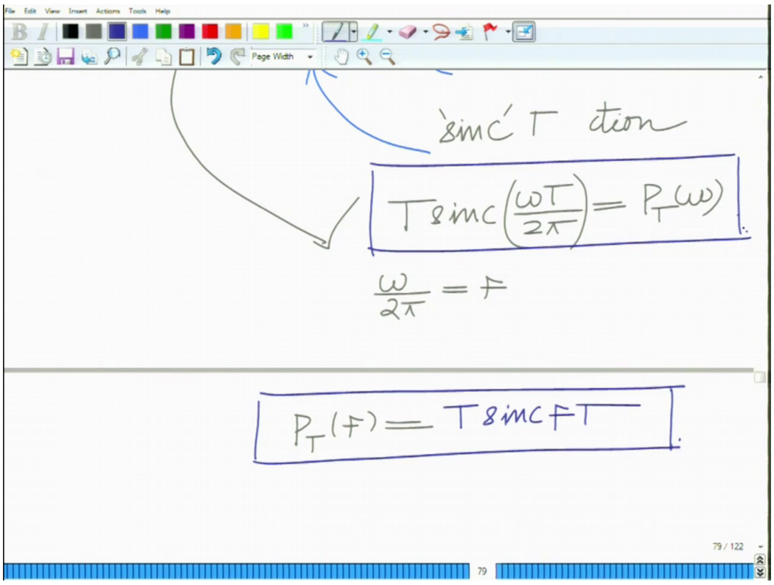

P T that is the Fourier transfer in terms of the frequency P T of F, this will be equal to T sinc of F T, or T sinc of omega T over 2 pi and if you plot this T sinc omega T over 2 pi

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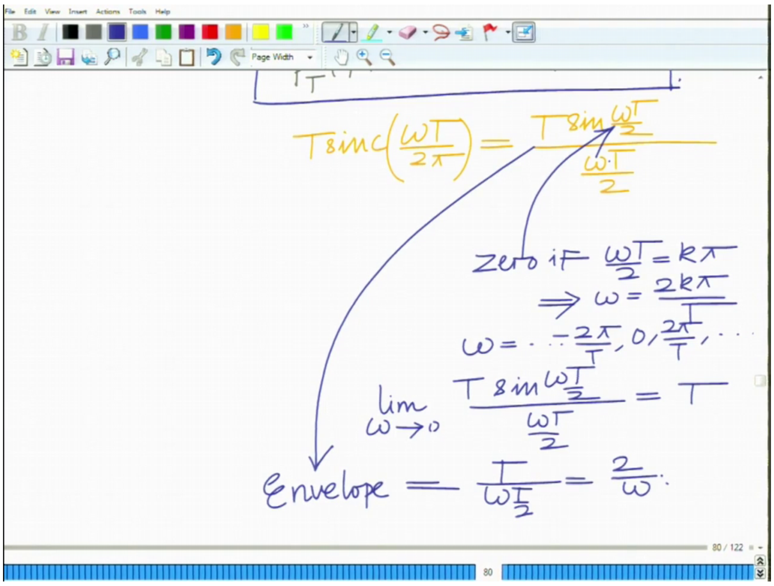

Equals T sin omega T over 2 divided by omega T over 2, and you can see that sin omega T over 2 sin omega or sin omega T over 2 now this is now this is 0. If the numerator is 0 if omega T over 2 equals k pi implies omega equals or implies omega equals 2 k pi over T.

So, this is 0 for omega equals for instance minus 2 pi by T is 0 2 pi by T and so on. For pi by T and so on ok, and as limit, remember limit omega tends to 0 you can check this sin omega T by 2 by omega T by 2 this is equal to T. So, T equal to 0 this is T ok this T sinc omega T over 2 pi at T at omega equal to 0 this is T this is magnitude T and it is 0 at every integer multiple of that is 2 k pi over T, where T is the width of the path at every integer multiple of 2 pi over T ok.

And you can see that the amplitude of this is decreasing this is amplitude is decreasing as T over omega. So, if you look at the envelop of this is something is interesting envelope which is places where the sin has magnitude 1. So, that will be T over omega T over 2 which is basically equal to twice over omega.

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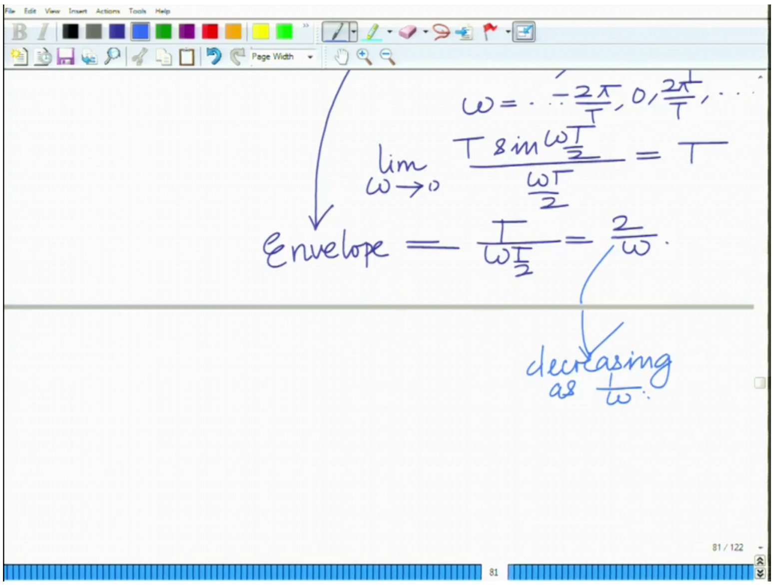

So, this is linearly sorry decreasing in omega linearly decreasing in omega or not linearly this is decreasing inversely in omega, and with that we can plot this.

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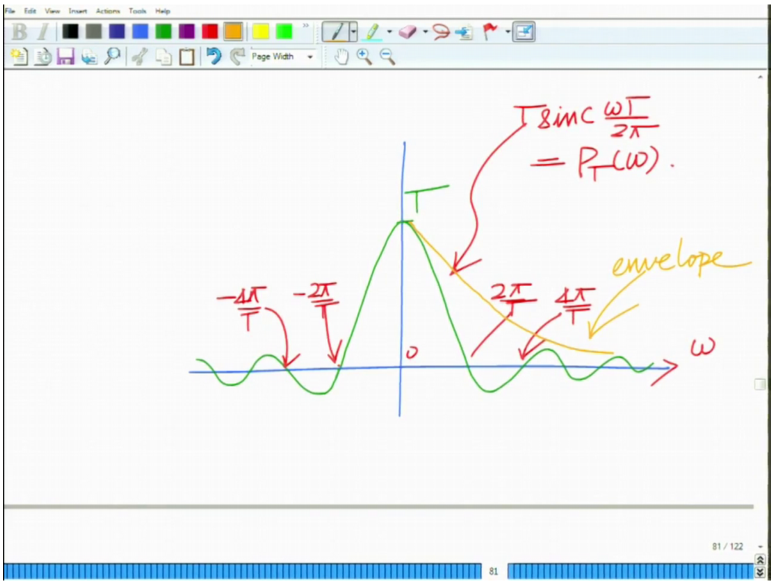

So, if I try to plot this, so this will be something which is of height T with the decreasing envelope and these points. So, this is T and these points where it is 0 2 pi by T at 0 it is T this point will be 4 pi by T and.

So, on this on omega axis and this point will again be minus 2 pi by T minus 4 pi over T and so on. And this is your sinc function, sinc of omega T over 2 pi times T and this is equal to your P T of omega ok, this is what we know as sinc function and you can, you can see the envelope the something that is decreasing. Envelope is decreasing as 2 over omega something that is decreasing that is the omega tends to infinity, the signal or the response of the signal right the Fourier transform tends to 0.

And this is important as said because it arises in a line as a large number of applications related to communication and signal processing, because the pulse where pulse itself something that is very fundamental to several applications ok. Now let us look at something interesting, which is we look at what is the now using the Fourier transform of sinc pulse now let us look at the inverse Fourier transform of the sinc pulse so.

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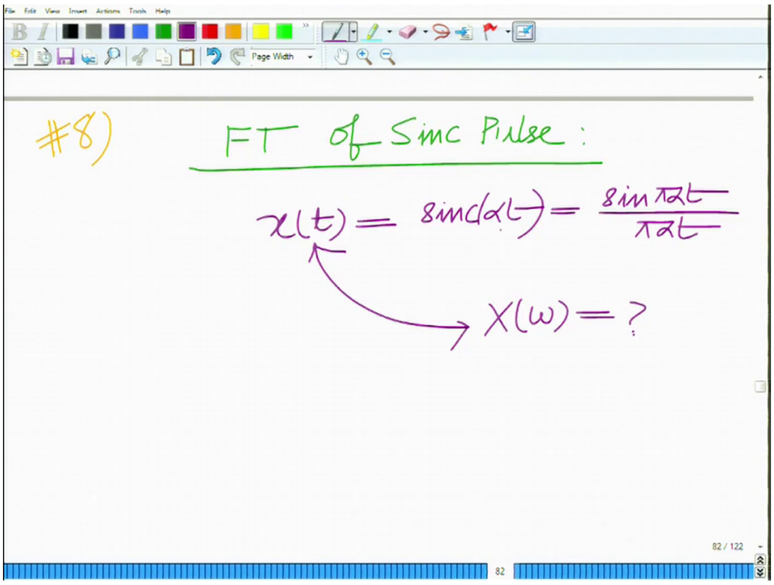

Or let us look at we are looked the Fourier transform the square pulse let us look at the Fourier transform of sinc pulse. And leadless to say we can use the duality principle, that is what we want to do is we want to consider a signal x t, which is sinc of alpha t, which is basically sin pi alpha t over pi alpha t, and we want to ask the question what is the Fourier transform of this sinc pulse.

Now the Fourier transform of this sinc pulse we can use the duality principle as well we know that the Fourier transfer of square pulse is the sinc in the frequency domain. So, using duality you can already see if you look at the sinc pulse at the time domain. Fourier transform must be related to that of square pulse in the frequency domain all right, so we use.

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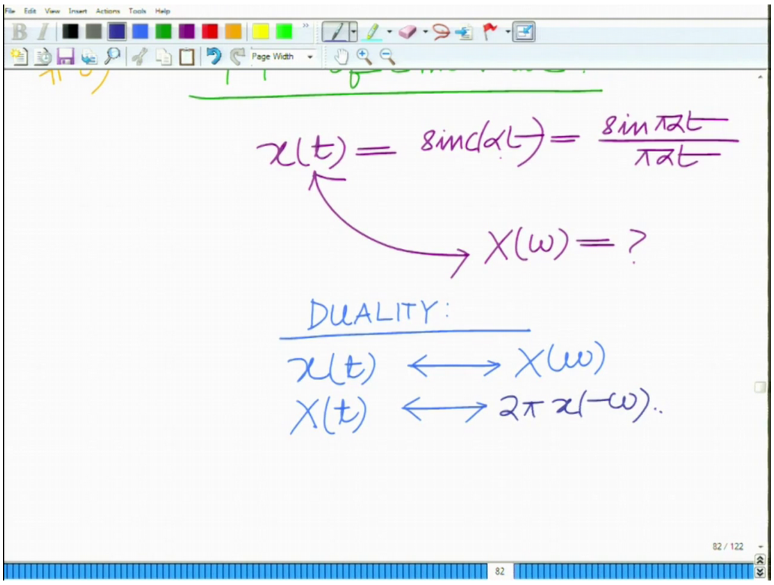

The duality principle all right which states that if x t has Fourier transform x of omega then capital X t that is capital X of t, must have Fourier transform we have already seen this 2 pi x of minus omega this follows from the duality principle.

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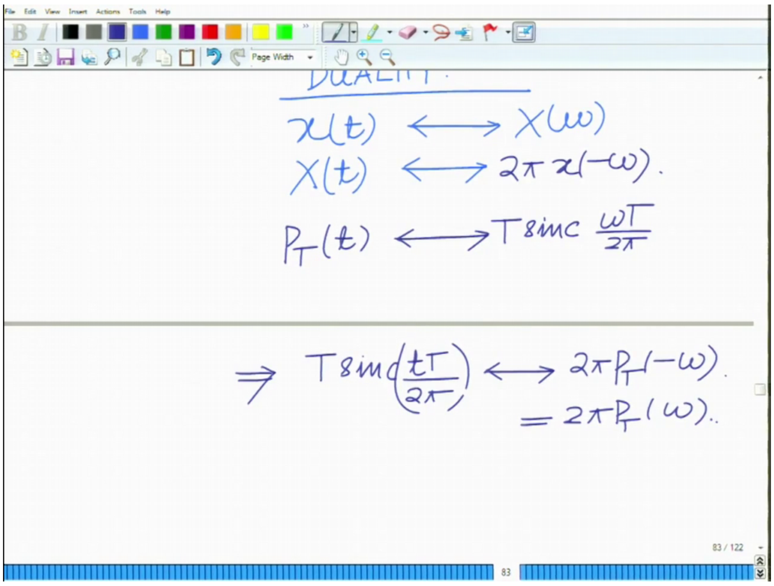

So, now we have P T remember from the previous problem, P T t has the Fourier transform T sinc omega T over 2 pi. Now using the duality it follows that if we replace a omega by T that is T sinc T times T divided by 2 pi, this has the Fourier transform P of T or 2 pi, this has the Fourier transform 2 pi 2 pi P T of minus omega. Now look at this is the pulse, right the pulse is an even function correct we have seen as the pulse is centered at 0, right it is symmetric about 0. So, P T of minus omega is same as P T of omega, so this is equal to twice 2 pi P T of omega since pulse is an even function.

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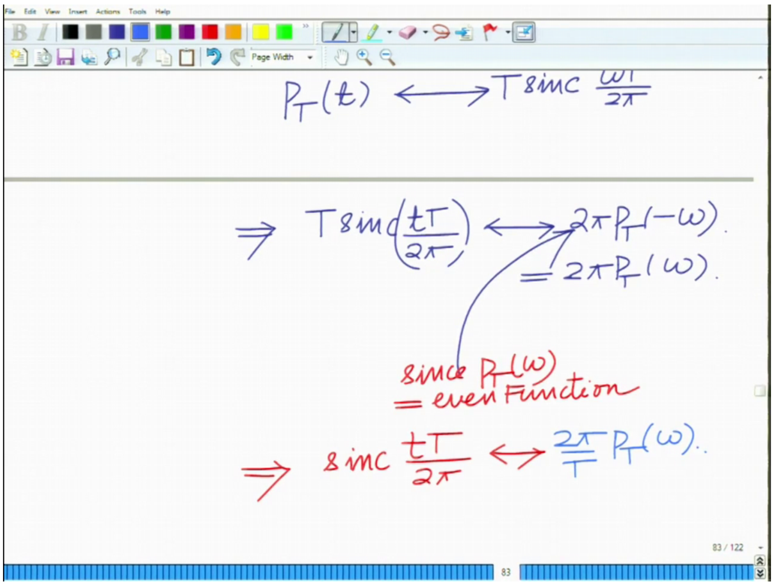

Since your square pulse is since or since P T of omega equals is an even function ok, and this implies that if you look at sinc t times T by 2 pi, that has the Fourier transform which you take to T into the other side that has the Fourier transform 2 pi over T P T omega now said.

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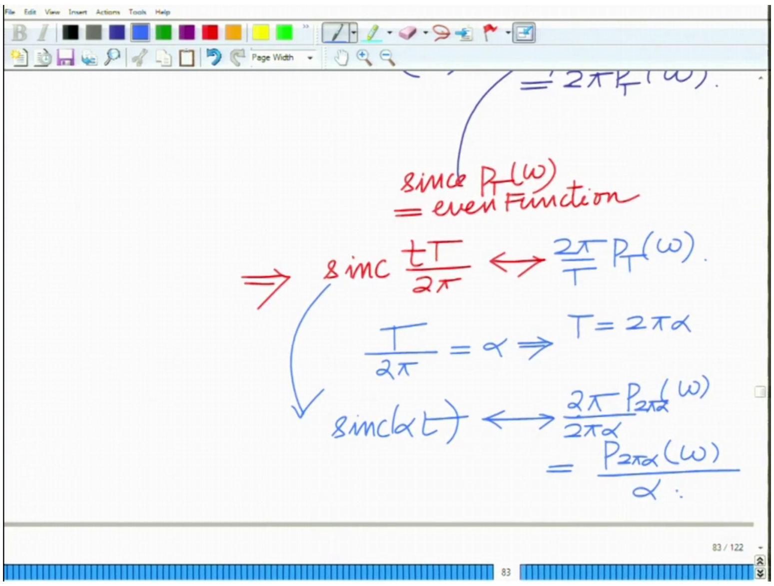

t over 2 pi equals alpha implies T equals 2 pi alpha and therefore, from here you will have sinc of alpha t, has the Fourier transform which is 2 pi over t. So, this will be your, I am sorry I am setting T pi T over 2 pi equals alpha. So, this will be T equals, so you will have 2 pi over T will be 2 pi over 2 pi alpha and P of T is 2 pi alpha 2 pi alpha omega which is nothing, but P of 2 pi alpha divided by alpha.

Now, if you look at the P of 2 pi alpha that is nothing, but pulse of width 2 pi alpha centered at 0 and height 1, now you have 1 over alpha times. So, the height will be 1 over alpha this is pulse of a 1 over alpha P 2 alpha omega is a height is a pulse of height 1 over alpha and frequency domain centered at 0 and of width 2 pi alpha therefore, its plans from minus pi alpha to pi alpha ok. So, therefore, this will be if you realize that you can see.

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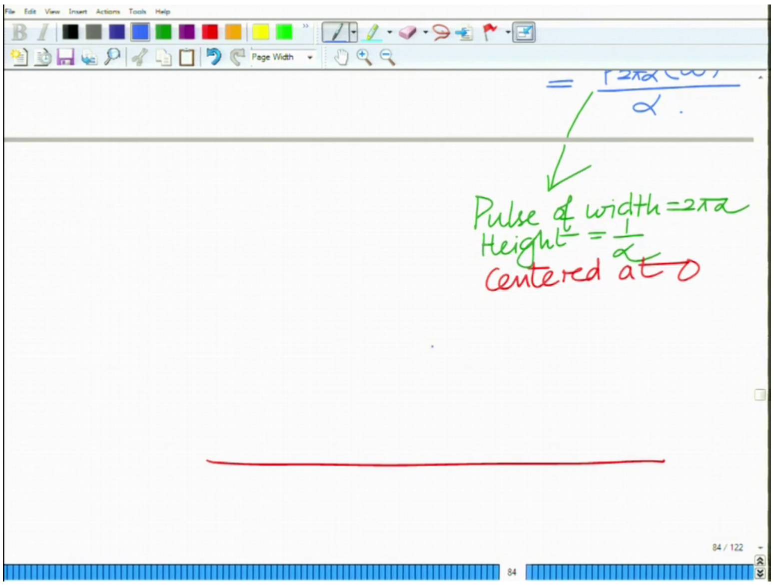

This is this is pulse of width 2 pi alpha, height equals 1 over alpha centered at 0. So, therefore, this is going to be something that is that looks like this just going to draw it.

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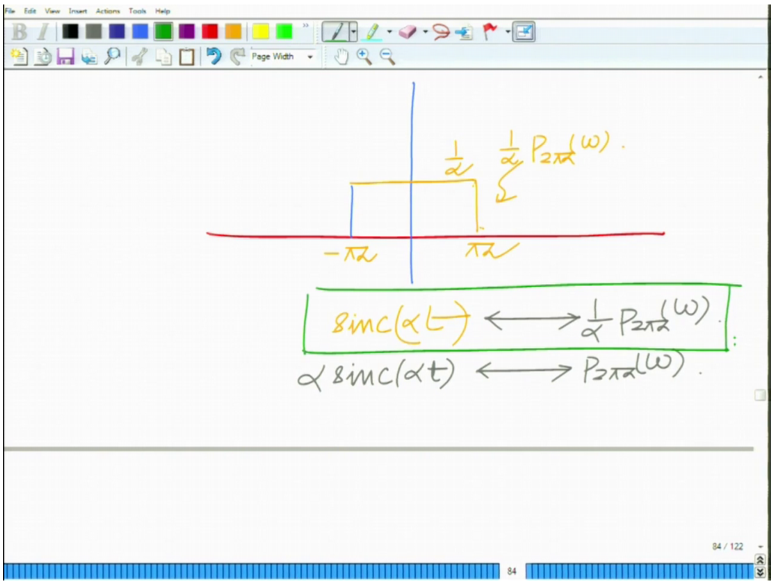

This is from minus pi alpha 2 pi alpha height 1 over alpha, and this is your P of 2 pi alpha 1 over alpha and this is the Fourier transform that is of alpha this is the Fourier transform of sinc alpha T.

So, sinc alpha T which is mode be set out to find the Fourier transform of has the Fourier transform 1 over alpha P 2 pi alpha P 2 pi alpha omega and therefore, this also implies that if you bring the alpha to the left hand side alpha sinc alpha T has the Fourier transform P 2 pi alpha times omega. That is basically what we have over here and another simple manipulation if you look at, so if you set alpha equals and if you set alpha.

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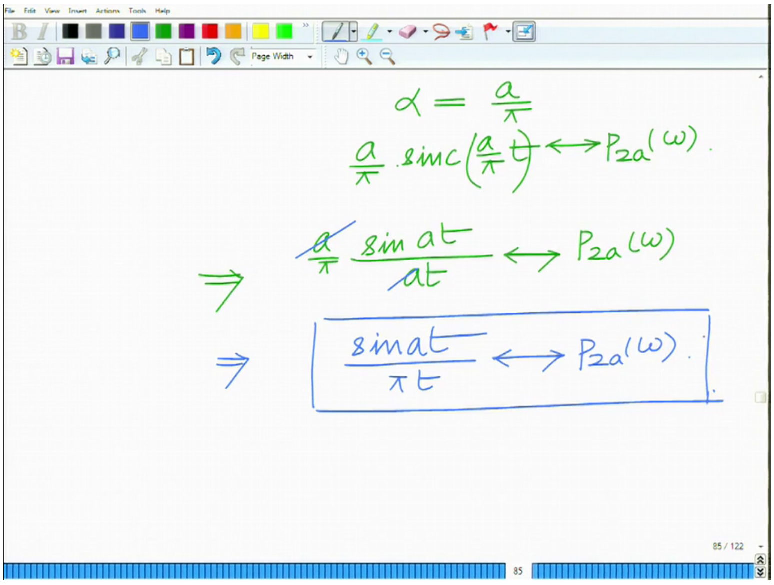

Equals a over pi, this gives a over pi sinc a over pi t is your P of 2 pi alpha alpha equals a over pi. So, this is p of 2 a omega now remember sinc of x is sin of pi x divided by pi x. So, this will be your this implies that a over pi times sin pi over pi t that is sin a t divided by pi a over pi t that is sin of pi over.

So, that is a t that has the Fourier transform, P of 2 a of omega which implies that which implies that now you have a is cancelling. So, sin a t by t equal to which implies that sin a t by pi t has the Fourier transform that is P of 2 a of omega, so sin of pi t by pi a t and this is basically.

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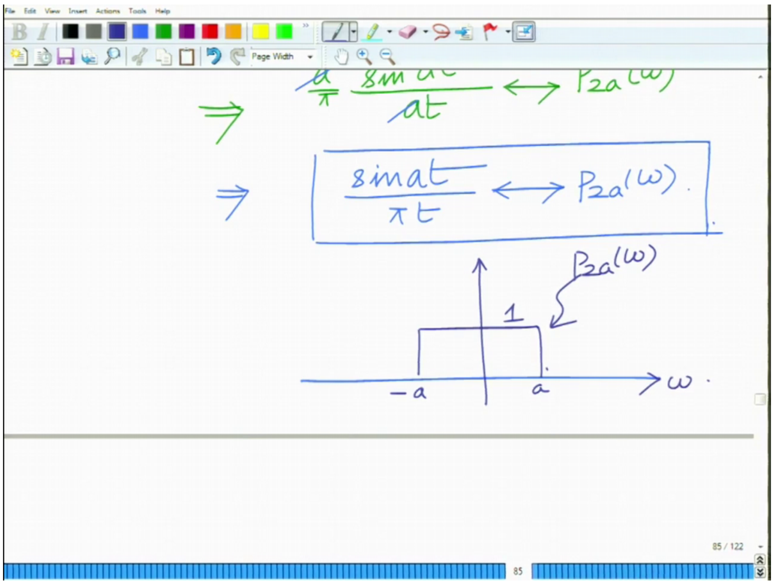

You can see this is pi pulse of height 1 width 2 a, this is your P of 2 a of omega and this has the. So, sin a t over pi t has the Fourier transform of 2 a omega, which is the pulse of width 2 a that is from minus a centered at 0 from minus a to a and of height 1 in a frequency domain, in the angular frequency domain omega domain all right . So, with we will stop here and continue with other problems in the Fourier analysis for continuous time signals in the subsequent modules.

Thank you very much.