Principles of Signals and Systems
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Lecture - 41
Fourier Transform - Ideal Band Pass and Band Stop Filters, Non-Ideal Low-Pass Filter, 3 dB Bandwidth

Hello, welcome to another module in this massive open online course. So, we are looking at the Fourier transform and in particular looking at the frequency response of ideal filters. So, let us continue that discussion.

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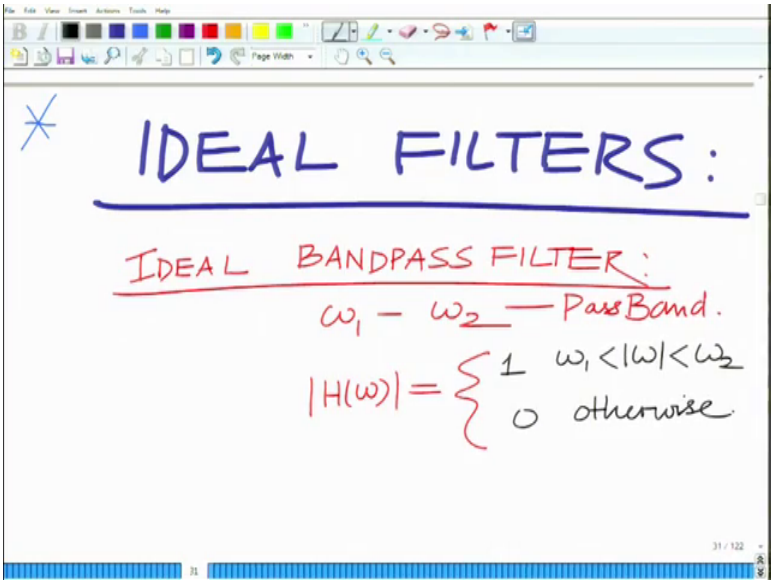

We have looked at the ideal low pass and high pass filters, we also want to look at what we are going to look at next are the ideal band pass and band stop filters ok.

So, let us continue our discussion on the concept of ideal filters ok. And we want to look at the notion of an ideal, of an ideal band pass. An ideal band pass filter as the name implies allows only frequency components of a signal within a certain band to pass through and suppresses of blocks all other frequency components which do not lie in this band, all right.

So, let us say the band is omega 1 to omega 2 we can call this as the pass band then we have magnitude for this band pass filter we have magnitude H of omega equals 1 for omega 1 less than more omega magnitude omega less than omega 2 and 0 otherwise ok. So, it allows only frequency components which lie within this band either omega 1 to omega 2 or minus omega 1 minus omega 2 to minus omega 1 to pass through.

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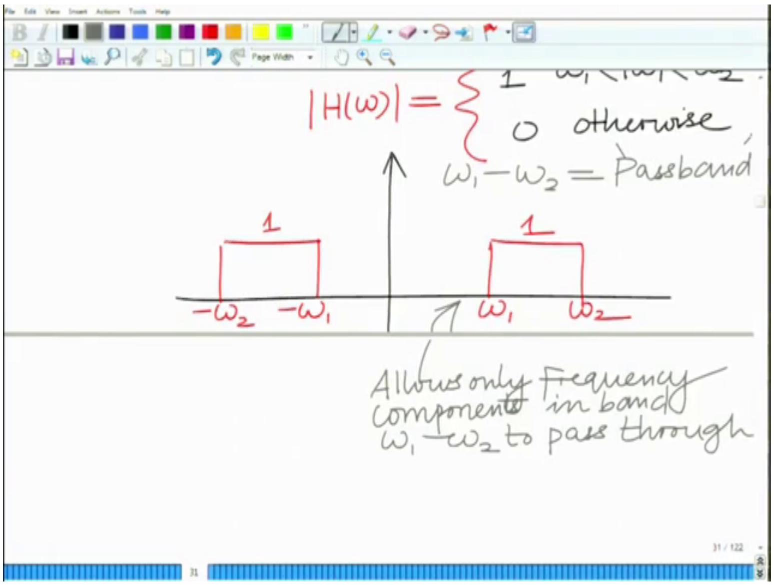

So, we have this band which is either omega 1 to omega 2. So, it has a gain of 1 or minus omega 2 to minus omega 1.

So, allows, so what this does is it basically allows only band of frequency allows only frequency components, only frequency components in the band omega 1 to omega 2 to pass through. And this is also known as the pass band that is omega 1 to omega 2 this is also known as the pass band ok.

And this is a band this is an ideal pass ideal band pass filter and the opposite analogously we have an ideal band stop filter which basically stops all the blocks all the frequency components belonging to that band and passes or allows undistorted transmission of all the other frequency components which do not lie in this band, all right.

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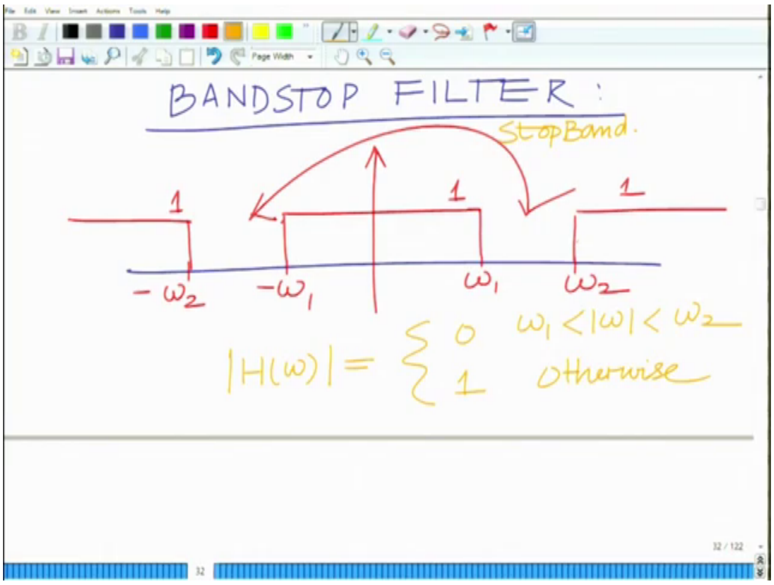

So, we have also a band stop and the band stop filter you have well, you have let us say omega 1 to omega 2 is the stop band then you have all the frequencies in this band which are blocked. And that is gain is 0 in this band and all other frequencies pass without distortion. So, these are the stop bands.

So, omega 1 to omega 2 is the stop band. So, the band stop filter is magnitude H of omega. So, its characterized by the frequency response magnitude H of omega equals 0 omega 1 less than mod omega less than omega 2 and this is 1 otherwise ok. So, this is 1 and this is 1 otherwise. So, these are the stop bands all right.

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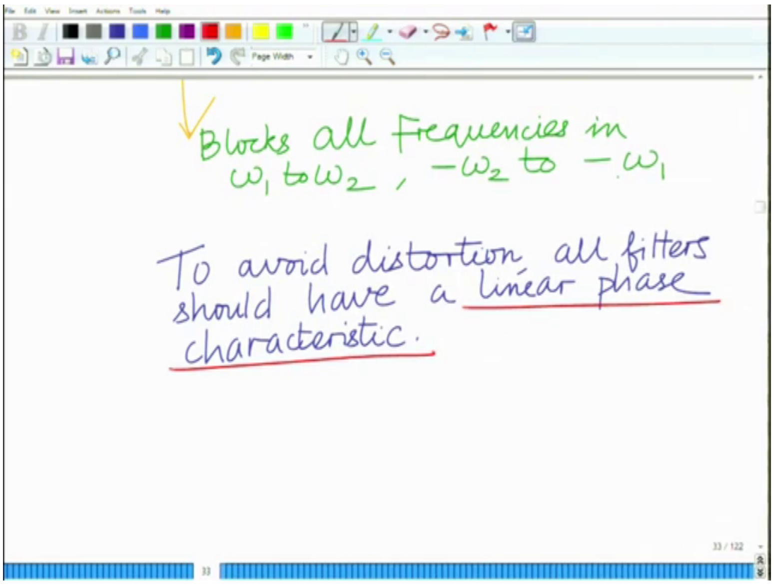

So, this basically blocks all frequencies that belong to the band omega 1 to omega 2 or minus omega 2 to omega. So, blocks all frequencies omega 1 to omega 2 minus omega 2 to minus omega 1, ok, so it all blocks all the frequencies in these bands ok.

And further we have only talked about the magnitude response, but we would not talk to the phase response , but we know that to avoid distortion for a distortion less response the phase response has to be linear ok. So, although we have not specified on the phase response you can note that the phase response has to be linear to avoid distortion ok.

So, we can say to avoid distortions all these filters all these ideals filters should have should have a linear phase characteristic, this should have a linear phase characteristics. Remember for a distortionless; a distortion less LTI system simply attenuates and delays the signal corresponding to that is if the input is x t the output is k times k x t minus t d td is the delay k is the scaling factor all right. So, the phase response of that is the magnitude response is k the phase response is minus j that is minus omega t d that is the phase which has a linear characteristic in omega all right. So, that is basically that basically characterizes a distortions, ok.

So, now so far we have looked at ideal filters, but of course, it is difficult to design such ideal filters as I said which such sharp cut offs. So, naturally we have to look at rely on non filters that are non ideal to a certain extent. So, let us start look at also look at was one of the basic non ideal filter its one of the basic low pass non ideal low pass filter which is formed by a simple RC circuit, ok.

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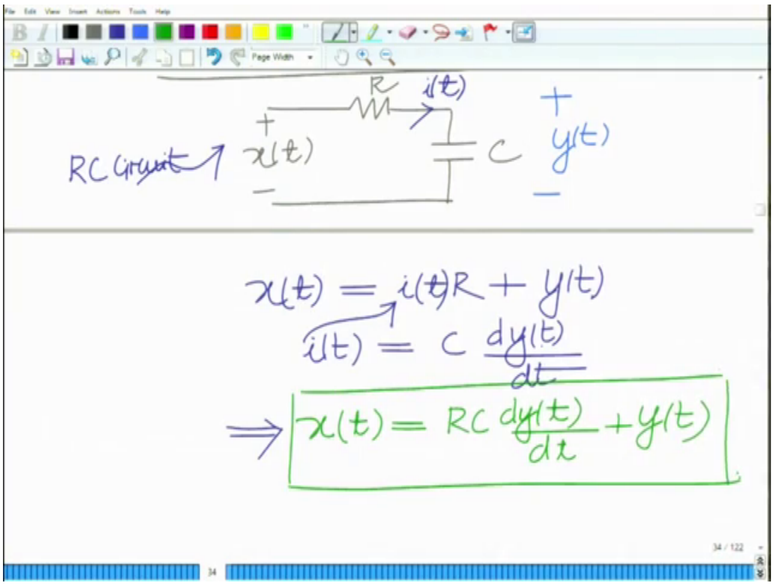

So, we want to look at a non ideal frequency selective filter, let us look at a non ideal low pass filter which you can say is simply given by this RC circuit, the simplest. So, let us say simplest low pass filter let us say x t is the input voltage y t is the output voltage let us look at the relation between x t and y t. So, this is an RC circuit a simple.

And we have let us say if we have this if I call this if we denote this current by i t then we have x t, the voltage is voltage drop across the resistance which is i t times R plus the voltage drop across the capacitance which is y t.

But we know the current across the capacitor is c times the derivative of the voltage across the capacitor that is a dy t or dt which basically implies now, substituting this expression for i t over here we have the expression x t equals RC dy t over dt plus y t. This is a constant coefficient differential. You can see this is the constant coefficient differential equation which characterizes the LTI system ok.

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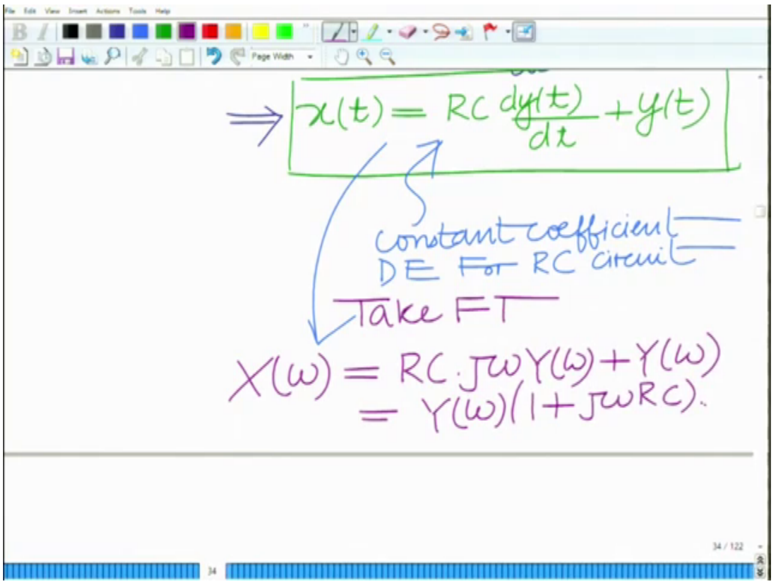

This is a constant coefficient differential equation for the LTIs over the RC circuit. Now, if you take the Fourier transform take the Fourier transform then we have X of omega equals RC, remember the Fourier transform the derivative is J omega times the Fourier transform of the signals of J omega times y omega plus y omega which is well y omega times 1 plus J omega RC ok.

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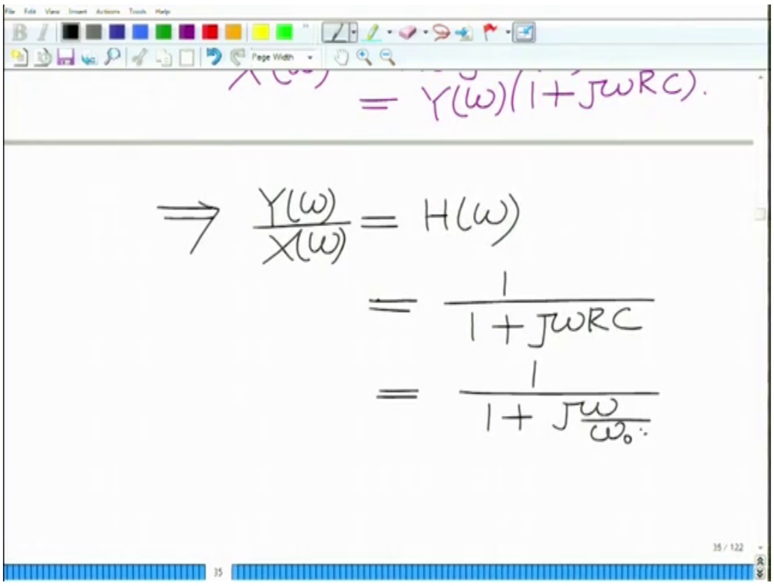

And now, therefore, if you look at y omega over x omega, so which is nothing, but the frequency response that is basically you can readily see that this is given as 1 over 1 plus

J omega RC, 1 over 1 plus J omega RC which is basically I can also write this as 1 over 1 plus J times omega over omega naught where omega naught equals. Now, you can see omega naught equals 1 over RC ok.

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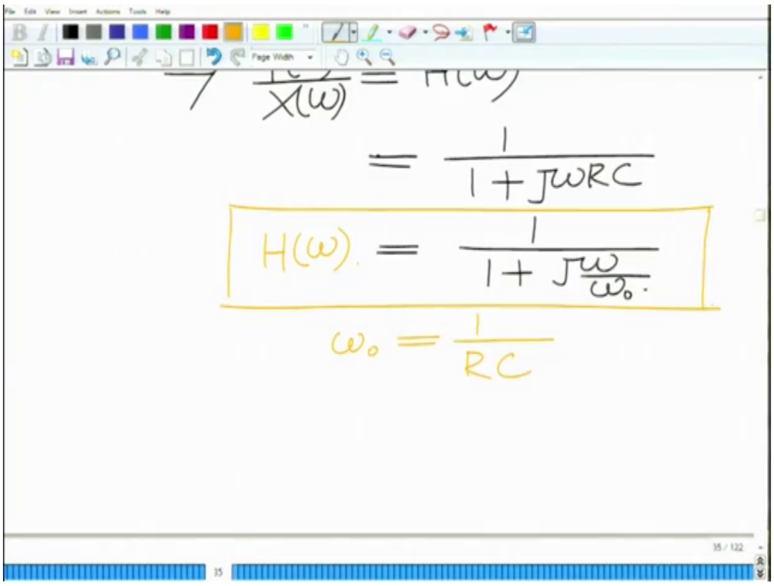

So, for an RC circuit I can represent the frequency response is 1 over 1 plus J omega or omega naught where omega naught equals 1 over omega naught equals 1 over RC ok. So, basically that is the omega naught is 1 over RC ok.

Now, if you look at the magnitude. Now, let us look at the magnitude response of this the magnitude of this is 1 over 1 plus or 1 over 1 plus omega over omega or 1 over 1 plus omega over omega naught square under root that is the magnitude response of this.

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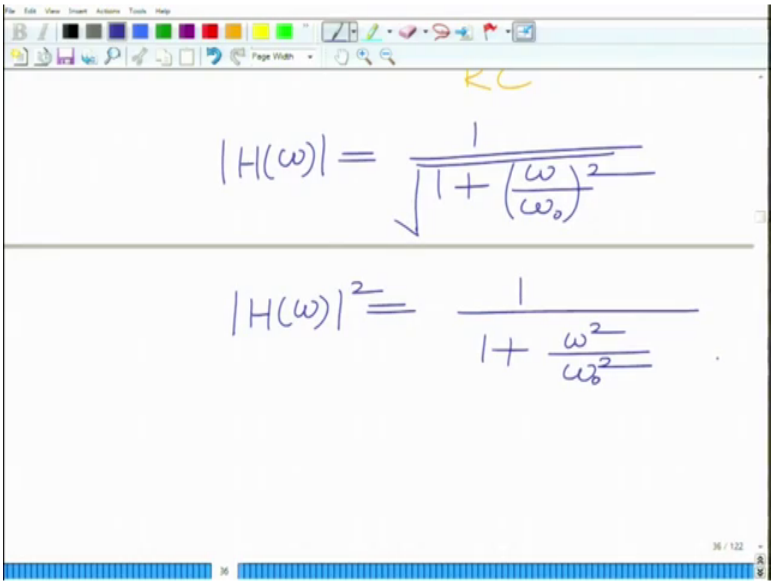

And therefore, magnitude H of omega square if you take the square of the magnitude response that is nothing, but 1 over 1 plus omega square divided by omega naught square.

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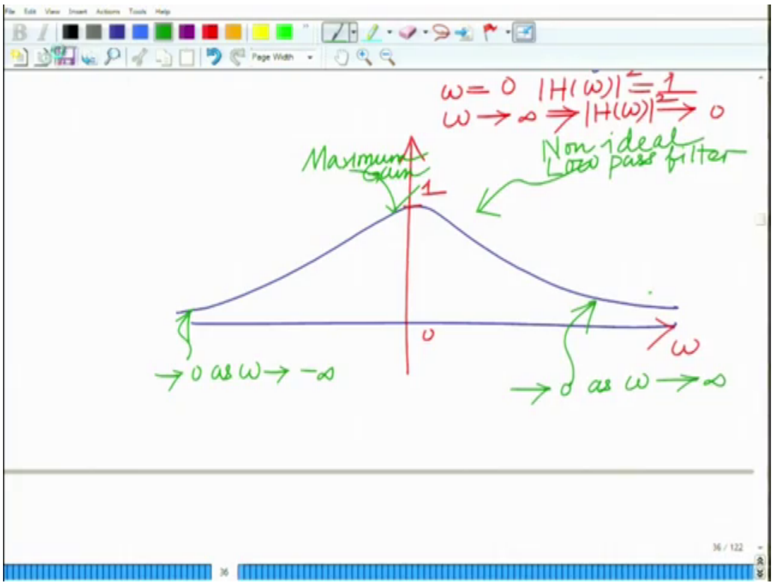

And if you look at the magnitude response you will observe that if you look at the magnitude response it is easy to see something, that is at omega equal to 0 magnitude H of omega square equals 1, omega tends to infinity implies omega square tends to infinity. So, 1 plus omega square over omega naught square tends to infinity. So, 1 over 1 plus omega square over omega naught tends square tends to 0. So, magnitude H of omega square tends to 0 and therefore, this is 1 at omega equal to 0 and as omega tends to infinity it tends monotonically to 0 similarly as omega tends to minus infinity tends to 0. So, you can basically see this has peak at omega equal to 0 it has a maximum gain of unity and both sides all right it decays to that is omega tends to infinity omega tends to minus infinity it decays to the gain of the filter it decays to 0.

So, this is clearly a low pass filter and non ideal low pass filter because it does not have any sharp edges, but it smoothly decaying to 0 the gain is decaying to 0 as omega tends to either minus infinity or infinity. So, this is basically your non ideal low pass filter.

This is a non ideal low pass filter and tends to 0 as omega tends to infinity and again here also tends to 0 as omega tends to minus infinity and this is basically the maximum gain which is unity ok.

And of course, what we would like to do is we would like to we would like to what we would like to do is we would like to characterize the bandwidth of this filter, but because it does not have any sharp edges we cannot clearly characterize what is the cutoff frequency all right what is the stop band and what is the pass band as we did for the ideal filters.

Therefore, you would like to develop measures in matrix to characterize the bandwidth of this filter, one such metric is what is known as the 3 dB bandwidth ok, the way to characterize the bandwidth of this filter the pass band and the stop band is what is known as the 3 dB bandwidth.

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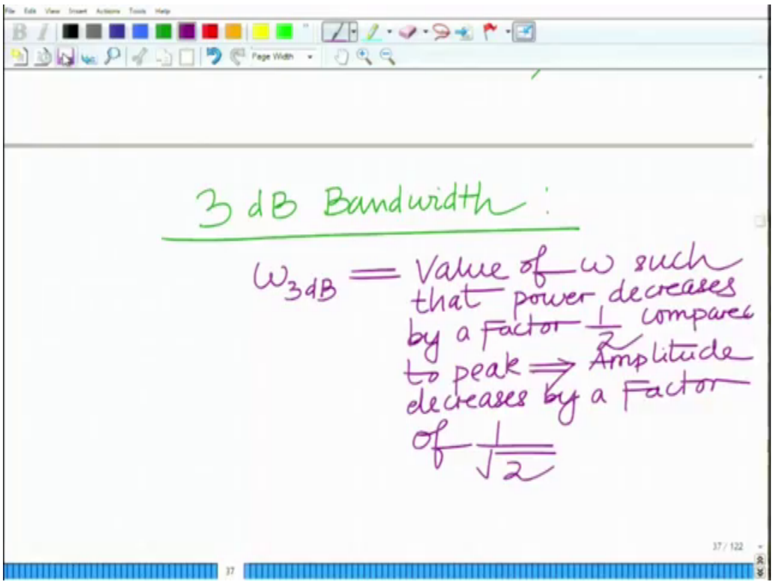

So, we want to define the 3 dB, we want to define the 3 dB bandwidth of the filter omega 3 dB the technical definition of this is value of omega such that, such that. And power the power of the filter you can say the power decreases by a factor of half power decreases by a factor of half when compared to the peak implies the amplitude decreases by a factor of 1 over root 2.

So, this can be understood as follows what we want to show is that if at the peak the amplitude all right the amplitude of the transfer function of the amplitude of the frequency response is at the peak if the amplitude of the frequency response is let us say k all right. Then at the 3 dB point the amplitude decreases by a factor of root 2 that is its k over root 2 and therefore, the amplitude decreases by a factor of square root of 2 the power which is the square of the amplitude decreases is basically a factor of half in relation to the power at the peak, ok.

So, basically what this implies is that if you look at the and of course, for the previous low pass filter you can see the peak occurs at 0 and in fact, it is the gain is unity. So, we have magnitude H of 0 equals 1, which implies at the 3 dB point we must have magnitude H of omega 3 dB equals 1 over square root of 2 which implies magnitude H of omega 3 dB square equals half.

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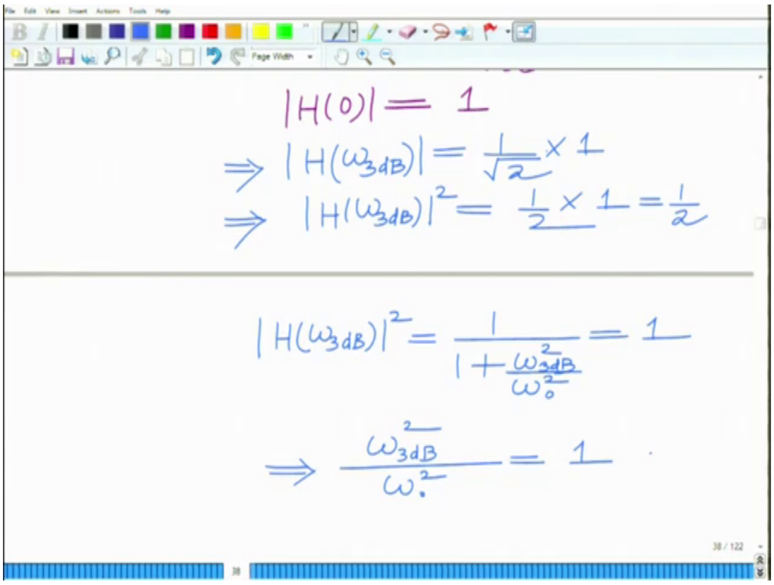

Basically it is all related to the amplitude at the peak. So, this is you can say half 1 over root 2 into 1. So, this is half into 1 which is nothing, but half. And now, you can see magnitude H of omega 3 dB square equals 1 over 1 plus omega square by omega naught square which is equal to 1, omega naught square omega 3 dB which implies omega 3 dB square where omega naught square equals 1.

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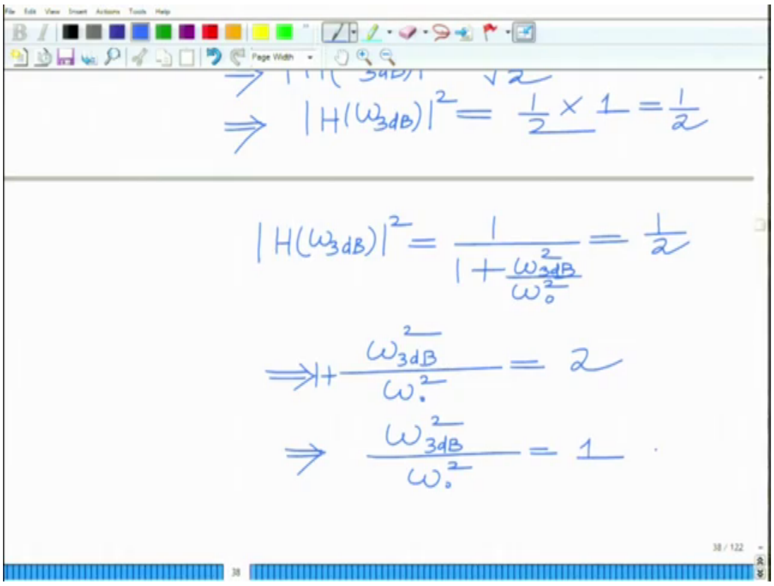

I am sorry this is equal to this is equal to half which implies omega naught square by omega 3 dB square which implies omega 1 plus omega 3 dB square by omega naught square equals 2; which implies omega 3 dB square omega naught square equals one which implies very simply that omega 3 dB equals omega naught which is nothing, but this is equal to remember this is equal to 1 over RC correct.

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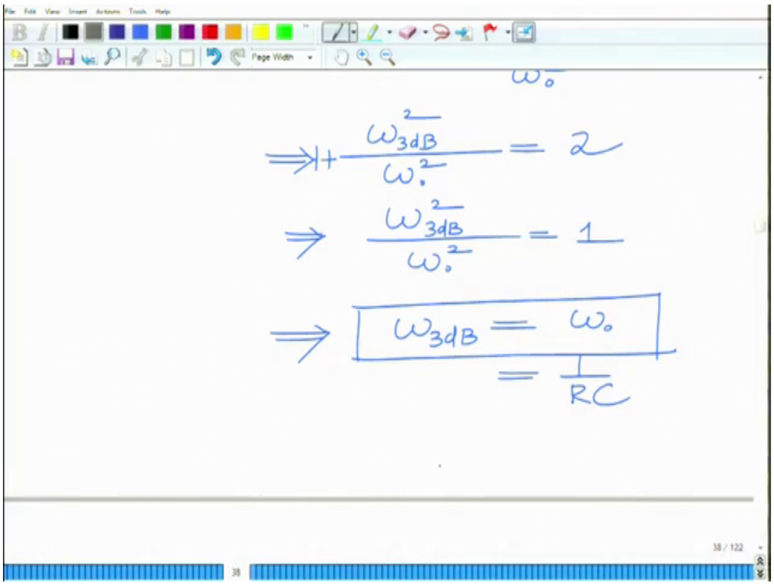

So, omega 3 dB equals omega naught which is 1 over RC. So, 3 dB bandwidth of this low pass filter you can think of this as an approximate cutoff frequency of this low pass filter is defined as that is one of the ways to define the crocket of frequency of this non ideal low pass filter is one over RC, where R is of course, that tends C is the capacitance ok.

And why is this known as 3 dB and the relation is obvious because if you look at the power that is magnitude H 3 dB square equals half. So, if you look at the decibel value of this that is 10 log to the base 10 magnitude 3 dB square or 20 log to the base 10 times the magnitude that is equal to 20 or 10 log to the base 10 of half which is equal to minus 3 dB you can say this is approximately minus 3 dB.

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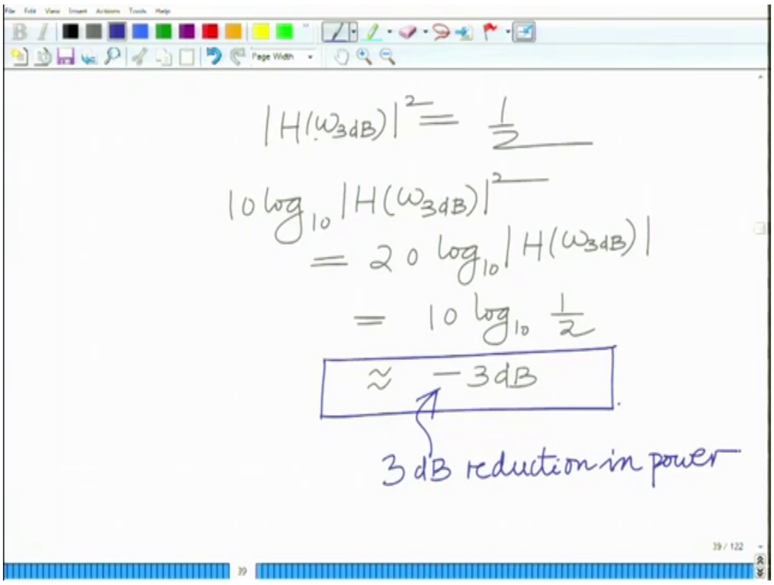

So, the 3 dB magnitude the 3 dB the 3 dB point at the 3 dB point the output power of the signal is attenuated by 3 dB correct, at the 0 frequency remember the gain is one the magnit the magnitude is one the power is also one all right. At the 3 dB point there is a 3 dB frequency the magnitude decreases by a factor of root 2 that is 1 over root. So, the power decreases by a factor of half.

So, in decibel values the power decreases by a factor of 3 dB or minus 3 dB. So, therefore, is known as the 3 dB frequency. So, 3 dB reduction in this, so 3 dB reduction in power, so this gives you a 3 dB reduction in power hence it is known as the 3 dB point. And the 3 dB bandwidth is 1 over RC that is what we have seen ok. So, the 3 dB bandwidth of this RC circuit of this non ideal low pass filter is 1 over RC ok.

Let us look at a few other aspects. Now, we can also define the signal bandwidth.

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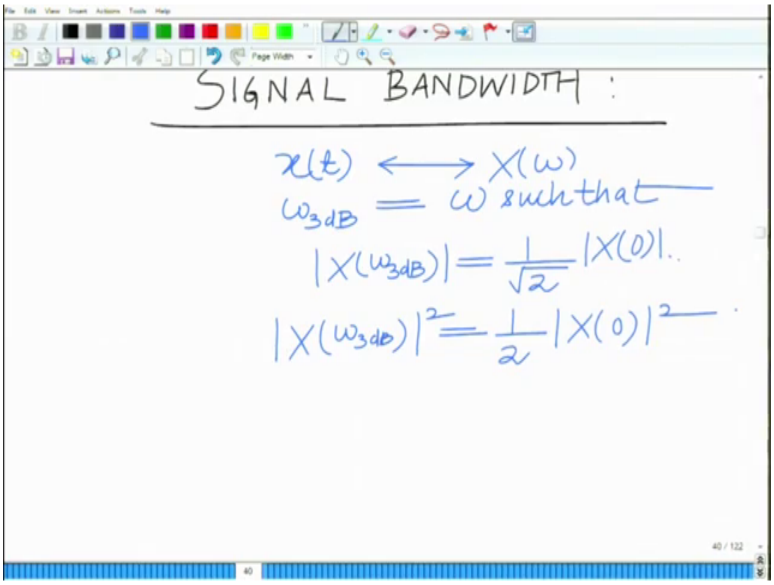

So, far we have defined the bandwidth of a filter, we can also use the same definition to define the bandwidth of a filter, the bandwidth of a filter to define the bandwidth of a filter. Let us say of a signal let us say we have a signal x t that has a Fourier transform X of omega 3 dB bandwidth omega 3 dB equals the frequency omega.

Such that we have magnitude X omega or magnitude X omega at the 3 dB point equals 1 over root 2 of course, magnitude x of 0 which means magnitude X of omega 3 dB square which recall is the energy spectral density of the signal equals half magnitude X of 0 square.

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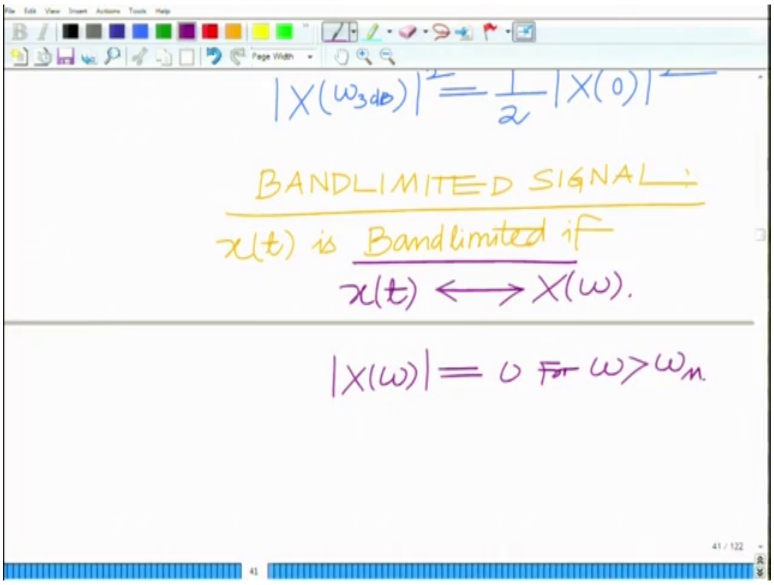

And also there is the notion of what is known as a band limited signal. I think something important a notion of a x t is a band limited signal x t is a band limited signal if let us say we have the Fourier transform of x t which is X of omega x t is band limited if magnitude of X of omega equals 0 for omega greater than omega. So, omega M is the maximum frequency and this plays an important role we will talk more about this later. So, this is termed as the maximum frequency.

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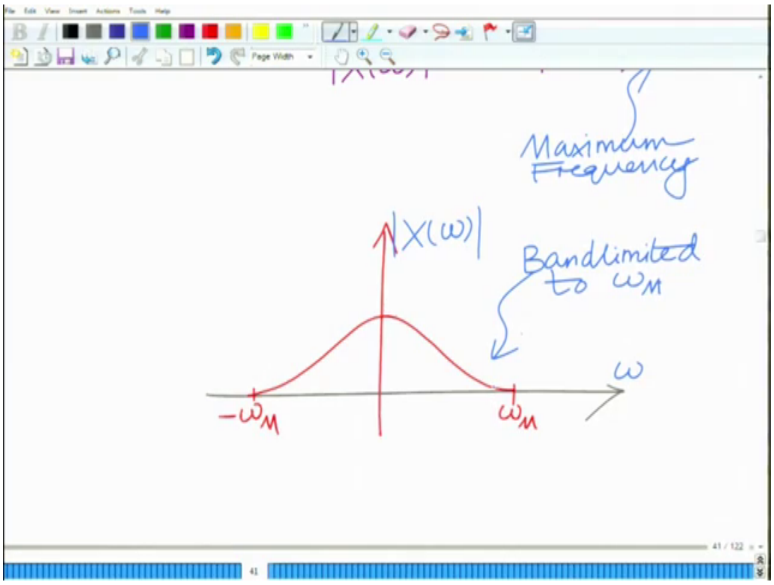

So, all the frequency components in the spectrum X of omega for omega larger than omega m that is omega larger than omega M or omega smaller than minus omega M are 0 ok. And you can visualize this as follows if you have the omega axis then let us say ok. So, this is your omega M, this is your minus omega M. So, this is band limited to the signal is this is your, let us say this is the magnitude spectrum and this is the this is band limited to this is band limited to omega M ok. Band limited signals have an important role to play in sampling and so on, all right.

So, this is basically more or less summarizes what we wanted to talk about the Fourier transform we have looked at the various aspects of the Fourier analysis of continuous time signals starting with first a periodic signals. Looking then also at a periodic signals, the application of Fourier transform in the analysis of LTI systems ideal filters and now, also non ideal filters and how to characterize the bandwidth of a non ideal filter all right.

So, I will stop here and look at other aspects in the subsequent modules.

Thank you.