Principles of Signals and Systems

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Lecture – 30

Example Problems in z –Transform – LTI System Output, Step/ Impulse Response of LTI System

Hello welcome to another module in this massive open online course. So, we are looking at example problems for the z transform let us continue this discussion.

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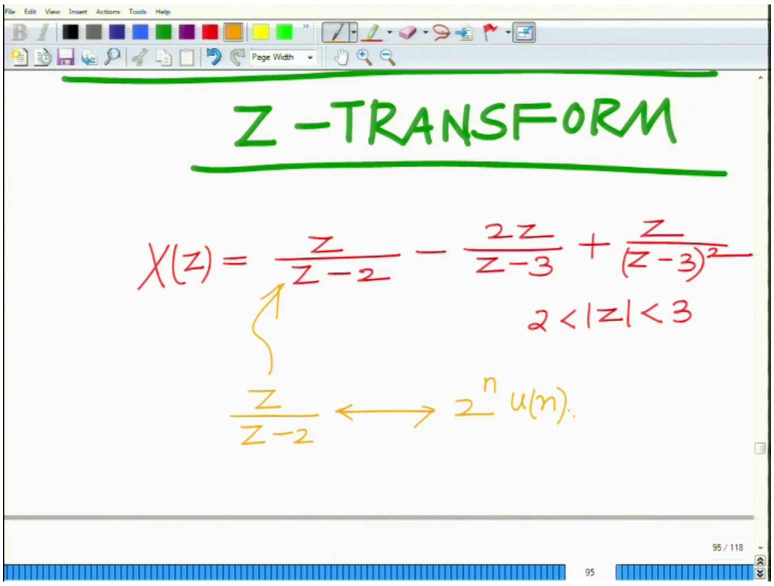

So, we are looking at example problems pertaining to the z transform and the properties of the z transform, and from our previous discussion what we have seen is that we are evaluating the inverse z transform, and we have seen that x z is basically given by this expression that is z over z minus 2 minus 2 z over z minus 3 plus z or z minus 3 square.

So, let me just write it down. So, x z in the problem that we have done so, far is z over z minus 2 minus, twice z over z minus 3 plus z over z minus 3 square. So, we have 2 less than magnitude of z less than 3. And now in this we know that for instance z over z minus 2 this corresponds to so, if you look at this term z or z minus 2, now remember now look at this the ROC corresponds to magnitude z magnitude z greater than 2. So, this is a right handed signal with respect to the pole at 2.

So, the inverse z transform of z over z minus 2 is 2 raise to n u n. So, this has inverse z transform 2 raise to n 2 raise to n u n.

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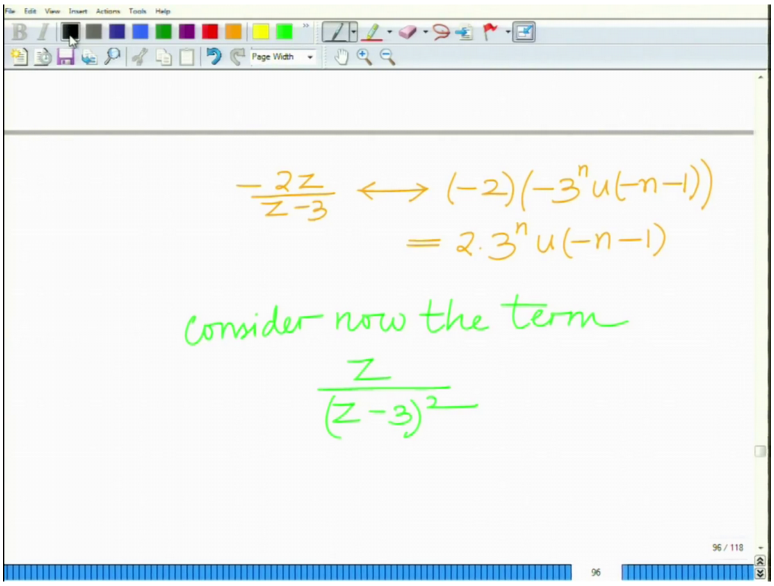

Now, minus 2 minus 2, z over z minus 3, now this has inverse transform. Now this is inverse d this is minus 2 now remember the ROC is magnitude z is less than 3. So, this is a left-handed signal with respect to the pole at 3.

So, the inverse z transform of this is minus 2 times minus 3 raise to n u minus n minus 1. So, this is equal to basically twice into 3 is to n u minus n minus 1, since the ROC is basically magnitude z less than 3 that is 2 less than magnitude z less than 3 correct the inverse z transform of z over z minus 3 is minus 3 raise to n u of minus n minus 1. Now, we come to the remaining term that is the term which is of the form z over z minus 3 square, consider now z over z minus 3 square.

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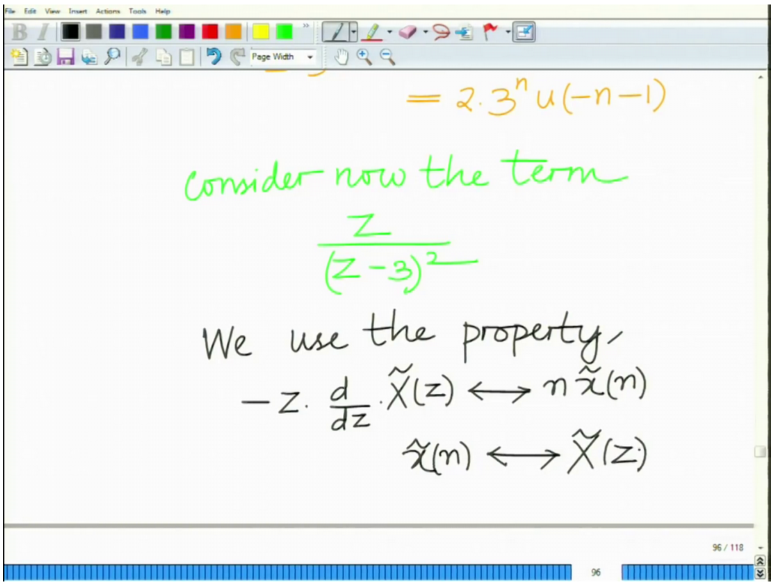

Consider now the term z over z minus 3 square, now to evaluate this we use the property to evaluate the inverse z transform, we use the property use the following property minus z d over z d z x tilde z that is if you differentiate the z transform, that corresponds to the sequence n x tilde of n that is the sequence or that is the signal n x tilde n as z transform minus z d over d z, that is the derivative of x tilde z x tilde z remember is z transform of x tilde n that is x tilde n has the z transform x tilde z.

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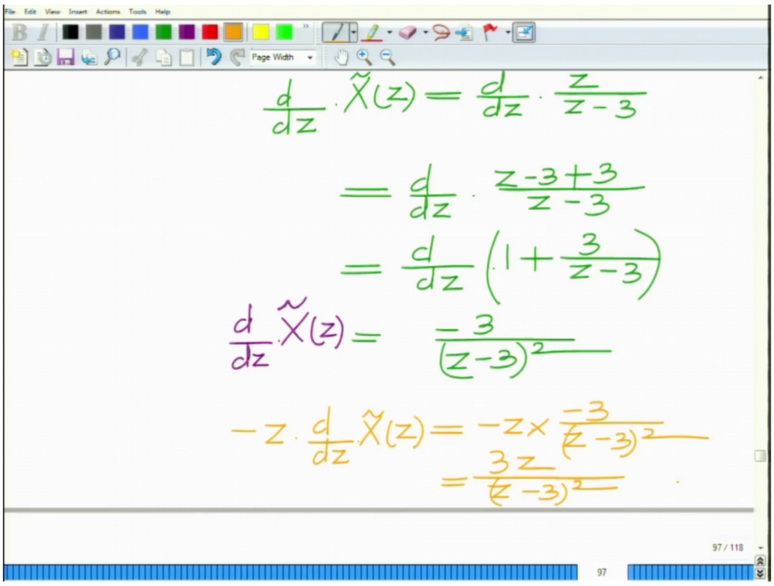

Now consider z over minus 3 now, let us say X tilde of z just differently noted by different notation equals z over z minus 3, now d of now differentiating this d over d z of X tilde z equals, the derivative of z over z minus 3 equals the derivative z minus 3 plus 3 by z minus 3 equals the derivative of 1 plus 3 over z minus 3. Now if you take the derivative of this you realize that this is basically nothing, but the derivative 1 is 0 what is remaining is minus 3 over z minus 3 whole square.

So, this is your basically d over d z of X tilde of z. Now minus z d over d z of X tilde z is minus z into minus 3 by z minus 3 whole square, this is basically 3 z by z minus 3 whole square.

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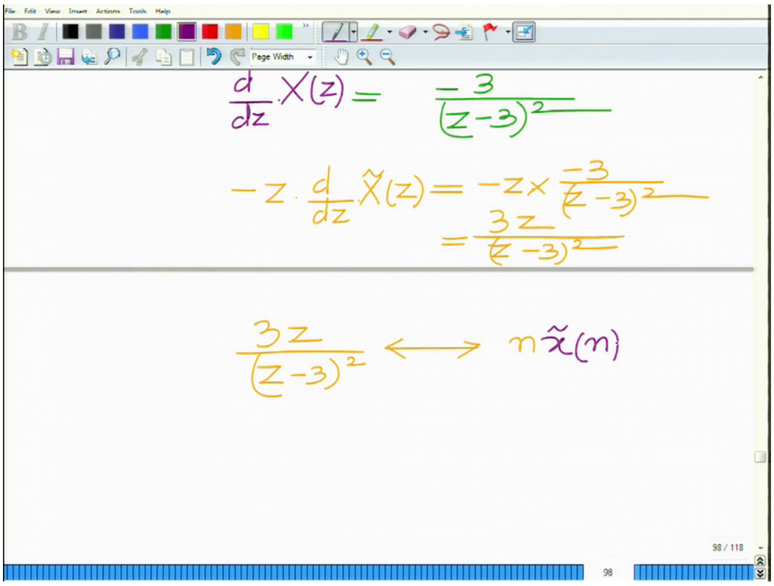

So, what we have is basically what we finally, have is basically 3 z by z minus 3 whole square, that is minus remember 3 z by z minus 3 whole square is this quantity minus z d over d z that is the derivative of X tilde z and therefore, this corresponds the inverse z transform of this would be this would be n x tilde n, where x tilde n is the inverse z transform of X tilde z.

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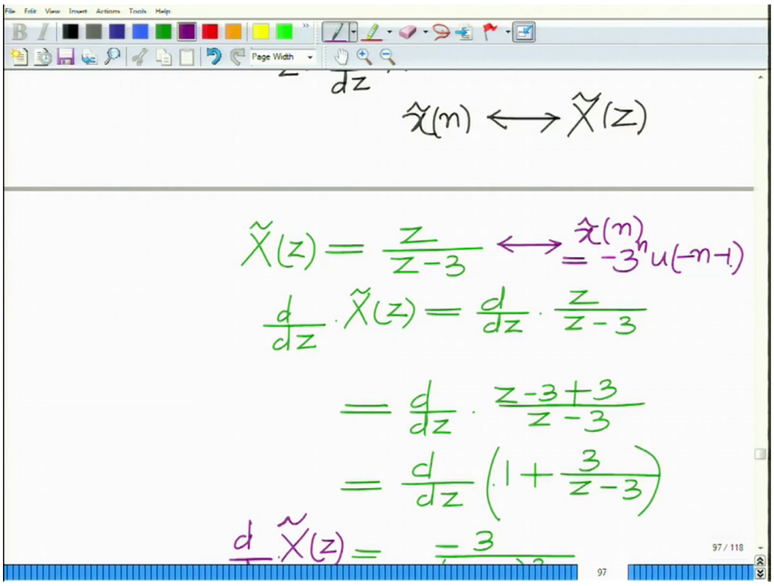

Remember this corresponds to x tilde n corresponding to z over z minus 3 this is equal to remember minus 3 raise to n u minus n minus 1. So, this will be n so, this will be n minus 3 raise to n u minus n minus 1 equals minus n 3 raise to n u of minus n minus 1, that is 3 z over z minus 3 square which implies that basically z over z minus 3 whole square will therefore, have the inverse z transform minus z bringing the 3 of the right side and dividing it minus n 3 raise to n minus 1 u minus n minus 1.

So, that is the inverse z transform of z over z minus 3 square that is minus n 3 raise to n minus 1 u of minus n minus 1, that is inverse z transform z over z minus 3 square corresponding to of course, the ROC 2 less than magnitude z less than 3.

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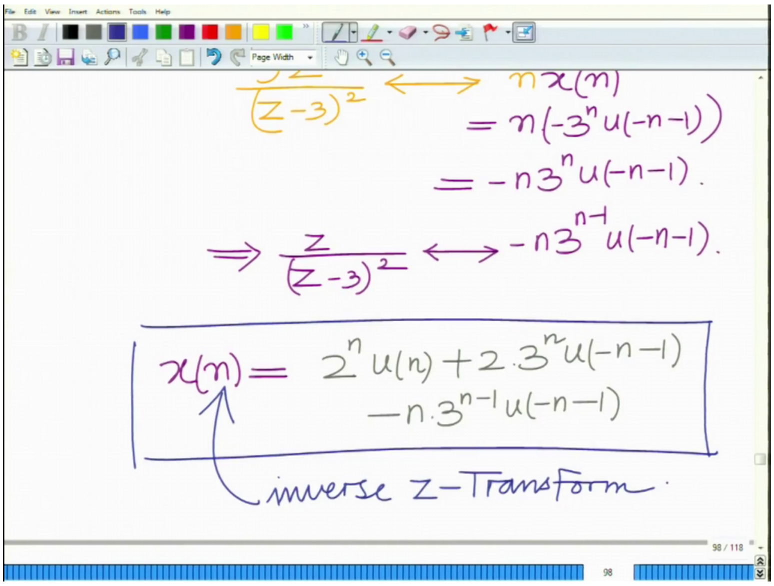

And therefore, the final inverse z transform basically we put together all the terms to get the final inverse z transform, that will be x n equals 2 to the ray to raise to n u n plus 2 into 3 raise to n u of minus n minus 1 minus n into 3 raise to n minus 1 u of minus n minus 1. This is the inverse z transform of the given function. So, this is the inverse z transform.

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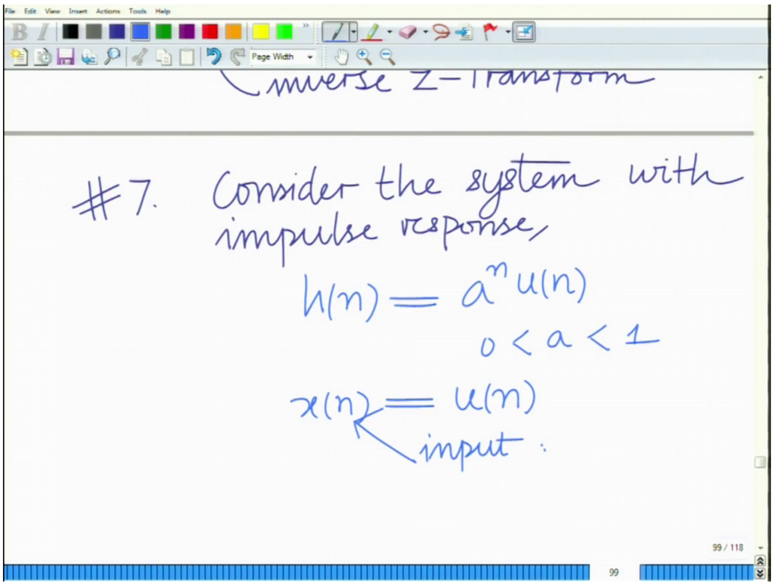

So, we have constructed the inverse z transform of this rational function remember we started with the rational function of z, which has a multiple pole multiple pole corresponding the basis which has a pole of multiplicity greater than 1 pole of multiplicity 2 at z equal to 3 all right, and that is basically the inverse z transform which we have basically derived using the first constructing the partial fraction expansion, and then constructing the inverse z transform of each of the terms in the partial fraction expansion ok.

Let us look at another example, now consider the system with impulse response h n equals a n u n is positive and less than 1, now consider the input x n that is the unit step function. Now this is given as the input. So, this is basically the input.

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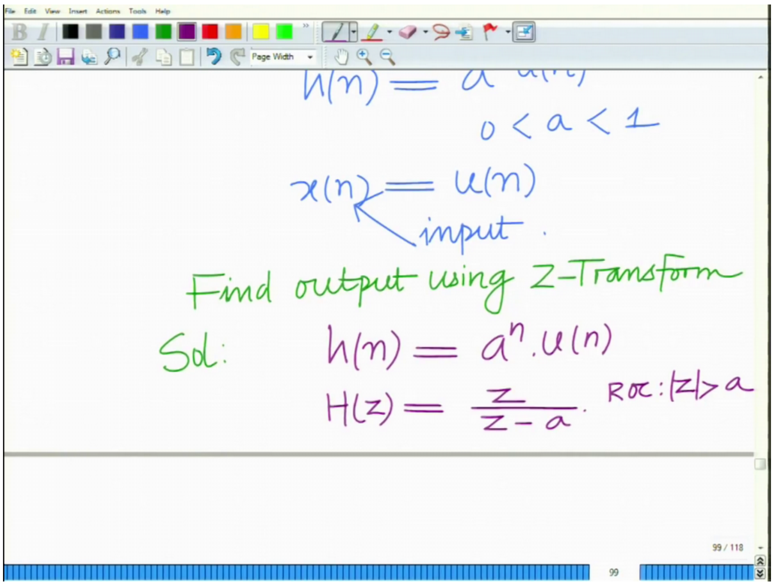

Now, we have to find the output find the output using the z transform. Now this can be solved as follows let us look at the solution.

So, we have the z transform remember h of n equals a n u n the z transform of this is H of z equals z over z minus a the ROC is magnitude of z greater than a all right this z transform of the input a n u n which is a right sided signal the z transform of this is z over z minus a.

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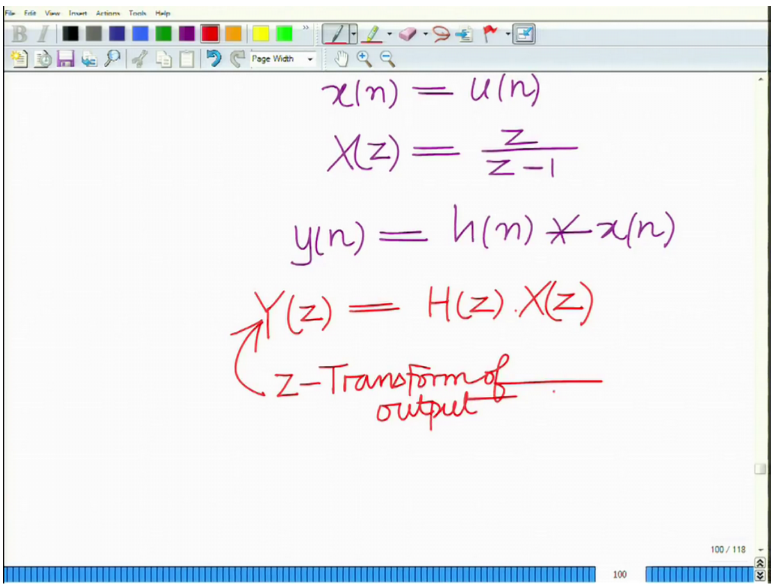

Now, let us look at x n x n equals simply u n and therefore, the z transform of this is H z equals well z over z minus 1, now y z remember z transform the output.

Now, remember we know y n is simply the convolution h n convolve with x n, which implies the z transform Y z equals H z into X z this is the z transform this is z transform of the output.

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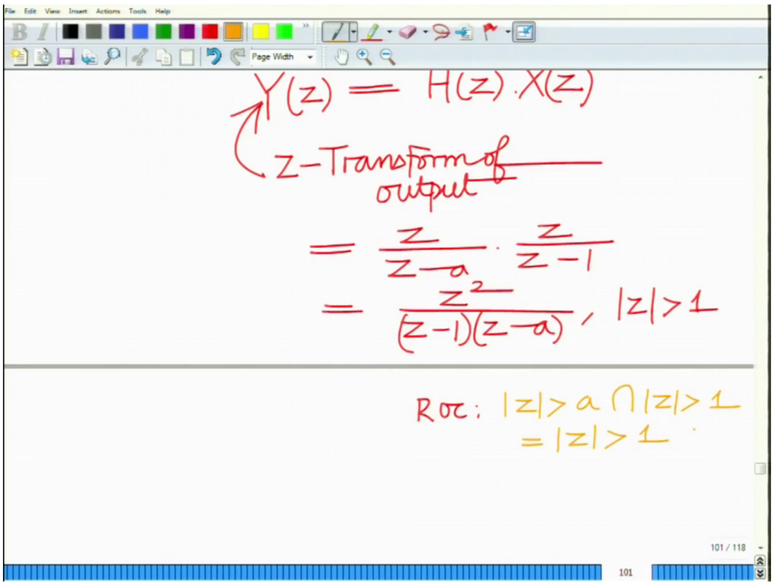

Which is x z times x z so, this is basically your z over z minus a times z over z minus 1, this is z square divided by z minus 1 into z minus a and the z transform of this now remember the ROC of this is basically magnitude of z greater than 1 previously we had magnitude of z greater than a so, you combine both these things. Now since a is less than 1.

So, now the ROC basically has to include both has to be basically both, magnitude z greater than 1 and magnitude z greater than a. Now since a is less than 1 correct the magnitude z so, this has to be the intersection of these 2 regions. So, the z transform of the net ROC will simply be magnitude of z is greater than 1. So, the net ROC is simply magnitude of z. So, you can also see this the ROC is intersection of magnitude z greater than a intersection with magnitude z greater than 1 which is equal to magnitude of z greater than 1 since a is less than 1 ok.

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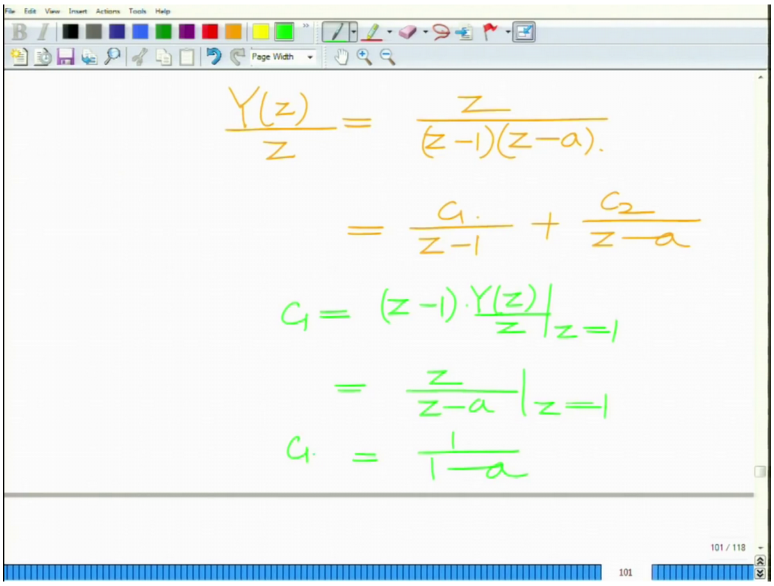

Now, if you look at Y z, now we have Y z equals z square by z minus 1 into z minus a. So, Y z over z that is equal to z over z minus 1 into z minus a which I can express as naturally I can express this as C 1 by z minus 1 plus C 2 by z minus a, now what is C 1 the coefficient C 1 is when z minus 1 into Y z over z evaluated at z equal to 1 which is basically z over z minus a evaluated at z equal to 1 that is 1 by 1 minus a. So, C 1 is basically 1 by 1 minus a.

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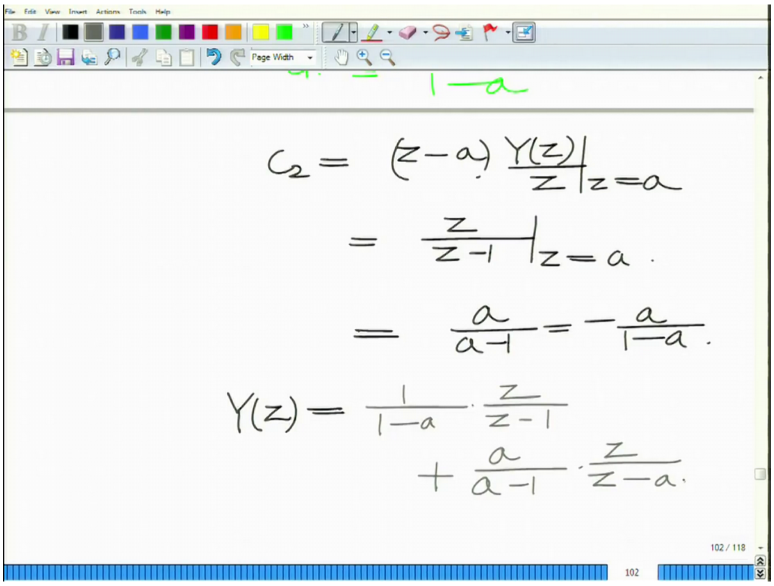

Now, how about C 2 is a well basically coefficient corresponding to the term z minus a correct so, that is z minus a times y z over z evaluated at z equal to a. So, this is basically you are what this is basically your z over z minus 1 evaluated at z equal to a which is basically you are a over a minus 1 or this is basically minus a over 1 minus a and therefore, what we have is basically we have y z equals now, C 1 is basically your 1 over 1 minus a so, this is 1 over 1 minus a z over z minus 1 plus a over a minus 1 into z over z minus a.

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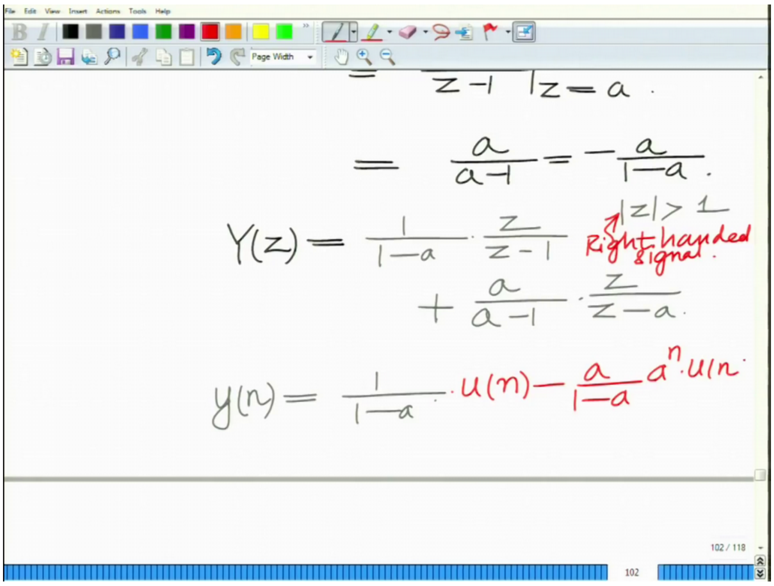

Now, 1 can take the inverse z transform you get y n equals 1 over 1 minus a now of course, consider remember the ROC is still magnitude of z greater than 1 which means this is a right handed signal there is a right handed signal. So, this is 1 over 1 minus a z over z minus 1 inverse transform inverse z transform is u n minus a over 1 minus a inverse z transform of z over z minus a is a n u n.

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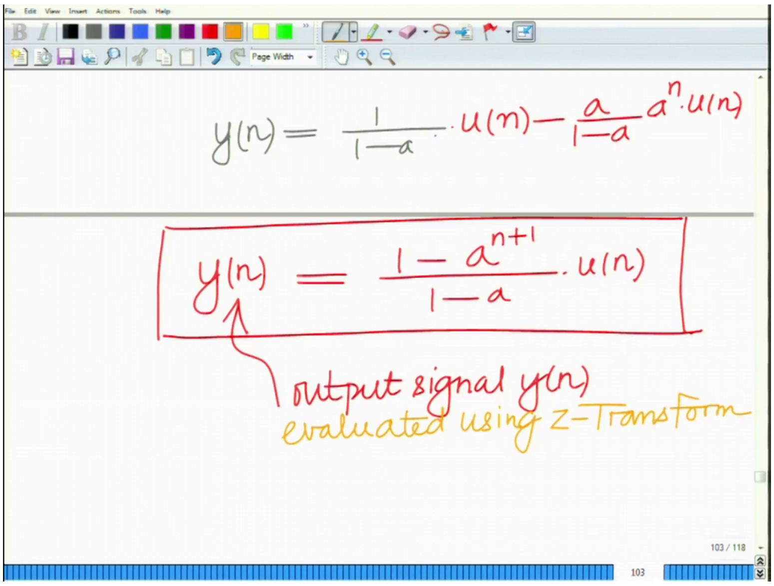

And therefore, combine these 2 what you get is 1 minus a raised to n plus 1 divided by 1 minus a into a u n. This is therefore, the output which we are now evaluated by using the z transform, this is the output signal y n which we have evaluated remember using the z transform technique all right.

So, we have given an input signal we were given the impulse response, which is I think if you look at the impulse response the impulse response is a raise to n u n correct, and you are given the unit step signal as the input that is x n equals u n and we are asked to find what is the output, and we have demonstrated using the z transform technique that is that the output is 1 minus a raise to n plus 1 over 1 minus a times u n. So, that is the output of this system all right.

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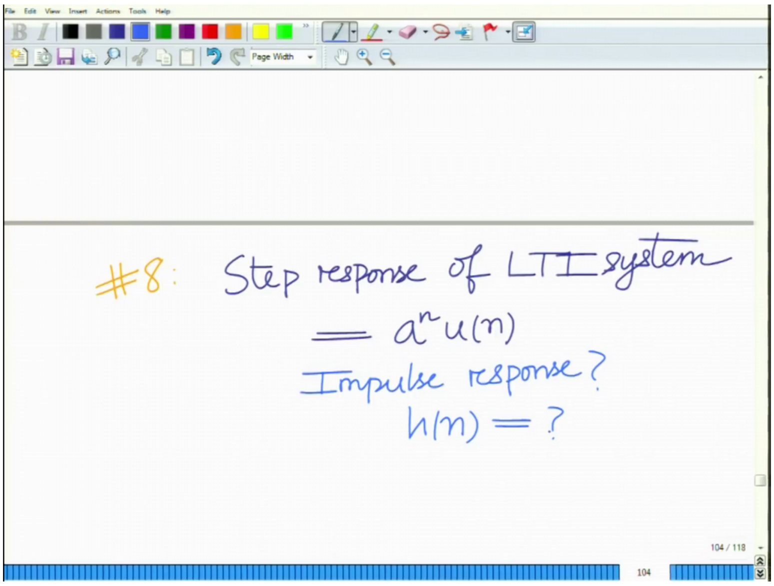

Let us continue to another example, in the example number 8 the step response now here, we are given the step response an LTI system equals a raise to n u n, now what we have to find is basically we have to find what is the impulse response of this system that is what is h of n for this LTI system.

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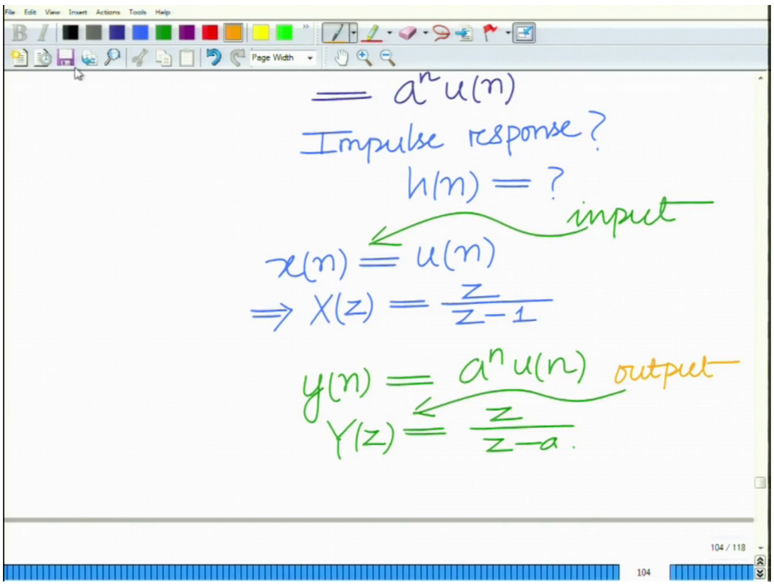

Now, remember what we know is if the input is x n we are given the unit step response, that is if the input is x n which implies X z equals z over z minus 1 correct magnitude of z is greater than 1 that is the ROC correct, this is the input. The output is y n equals a raise to n u n y z equals z over this is the output. Now, therefore, the transfer function H of z remember this is given as Y z over X z ok.

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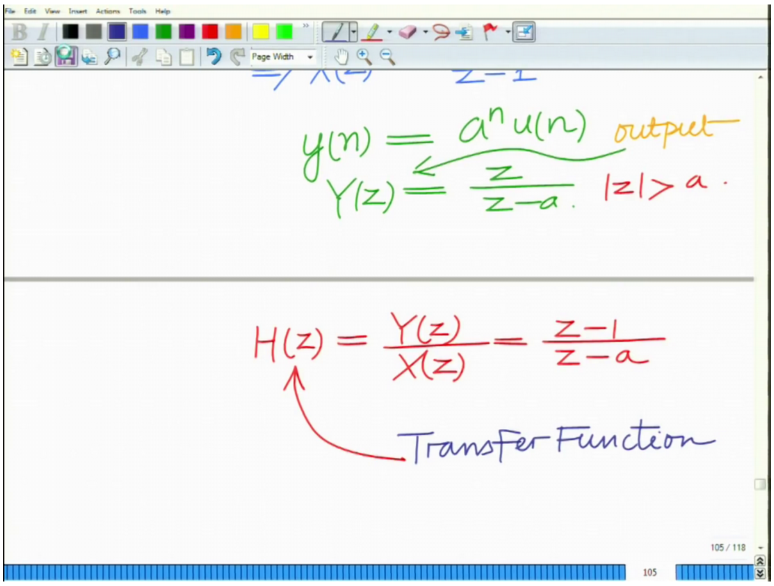

So, this is the ROC of course, again ROC is magnitude z greater than a now the ROC of both will be magnitude once again ROC will be magnitude of z greater than 1 since a is less than 1. And therefore this is Y z or X z and Y z over X z you can clearly see this is z minus 1 divided by z minus a, and this is basically your H of z this is the transfer function remember what is this, this is the transfer function for the LTI system.

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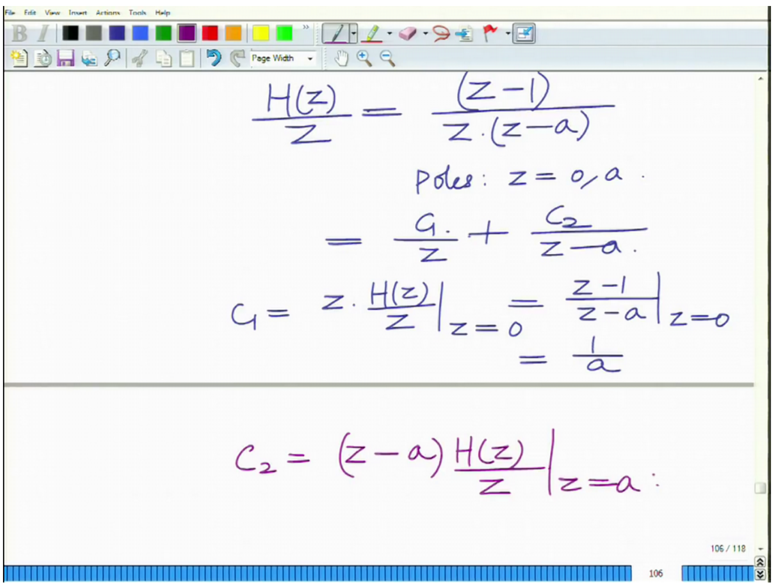

And therefore, now I can if I look at H of z over z that can be expressed as z minus 1 over z into z. So, there are 2 poles z equal to 0 comma a naturally. So, I can express this as C 1 over z plus C 2 over z minus a, now the coefficient C 1 corresponding to z is C 1 equals z times it is z over z evaluated at z equal to 0 which is basically z minus 1 over z minus a evaluated at z equal to 0 which is basically your 1 over a. Now C 2 equals z minus a into H z over z evaluated at z equals a.

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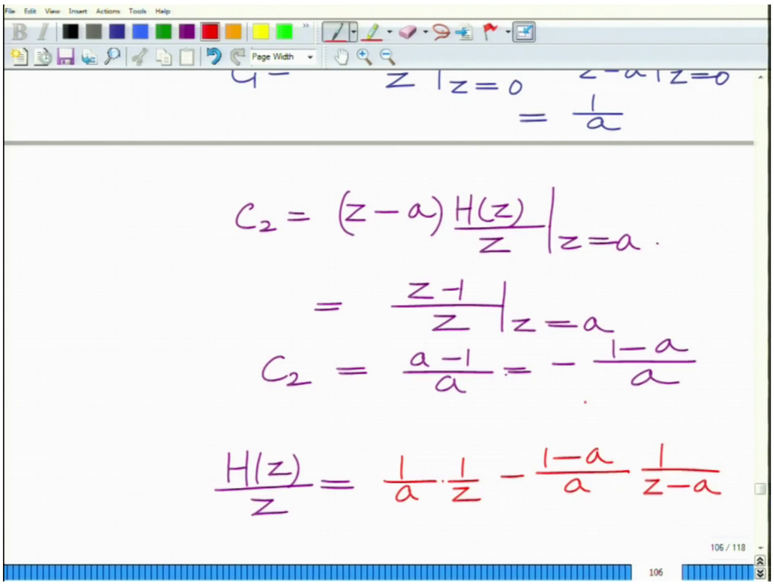

Which is basically you are now, this will be z minus 1 over z evaluated at z equal to a which is a minus 1 over a or which is basically also minus of 1 minus a over a that is your coefficient C. So, therefore, what we have is now H z over z is well 1 over a into 1 or z minus 1 minus a over a into 1 over z minus a.

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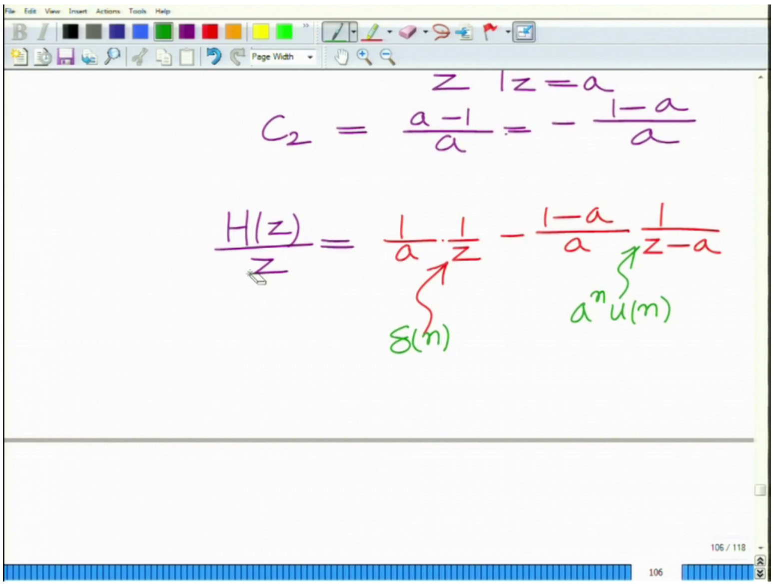

Now, for a right handed signal we know now we know 1 over z this impulse transform, now this impulse response this is corresponds to the delta function inverse z transform is delta n 1 over z minus a this is a raise to n u n and therefore, and now this is basically now of course, this is H of z over this is H of z over z correct now, let me just correct this.

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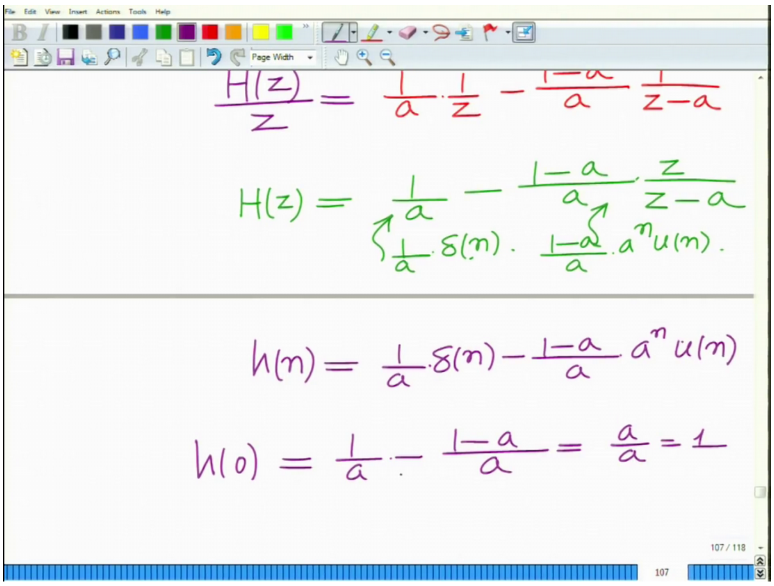

So, now we consider H of z equals 1 over a minus 1 minus a divided by z over sigma. So, 1 over a now this corresponds to 1 over inverse z transform is 1 over a times delta n, and the inverse z transform of this is 1 minus a divided by a, a raise to n u n. So, therefore, what we have is h of n. Now if you take the inverse z transform h of n is 1 over a delta n minus 1 minus a minus 1 minus a over a a raise to n u n ok.

Now, let us evaluate h of 0 h of 0 is basically delta of 0 is 1 1 over a minus 1 minus a over a, a raise to n a raised to 0 is 1. So, this is 1 minus so this is basically simply a over a equals 1. So, h of 0 is 1 and h of any n greater than 1 equals.

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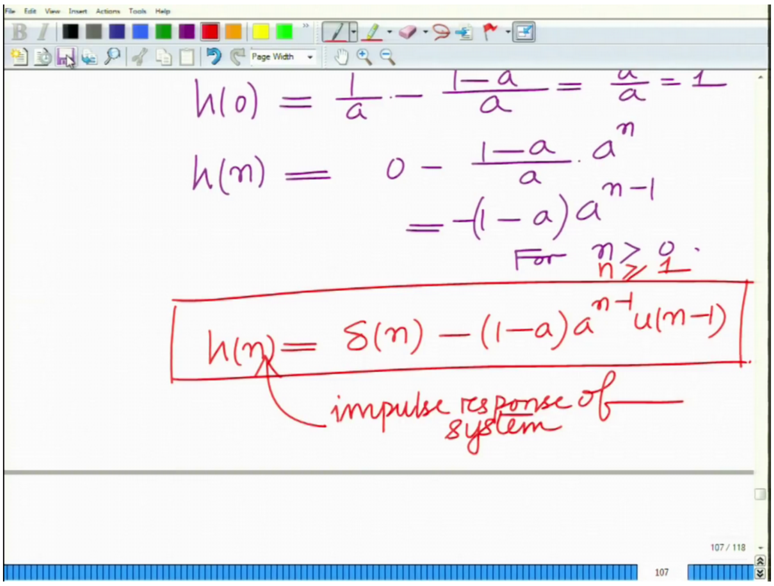

Of course delta n for n greater than 0 is 0 for any n greater than 0 H of n is simply 0 minus 1 over a over a a raised to n a raised to n, which is basically minus 1 over a a raised to n minus 1. Now this is for n greater than 0.

And therefore, combining both we have naturally h of n at n equal to 0 it is delta n delta n minus 1 over a into e raised to n minus 1 u n minus 1, that is this is for n greater than 0 or you can say n greater than or equal to 1. So, this is 1 minus 1 over minus 1 minus a raise to n minus 1 u n minus 1. So this is the impulse response of the system all right.

So, we have started with the system in which the step response is given correct, the step response that is the response to the step function u n is a raise to n u n, and we are asked to find the impulse response we found the impulse response of this system using the z transform technique, we have shown that this impulse response is delta n minus 1 minus a into a raise to n minus 1 u n minus 1 all right. So, this is the impulse response h n of the system all right.

So, we will stop here and look at other examples and continue with other aspects of the subsequent modules.

Thank you very much.