Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 23
Laplace Transform – RL Circuit Problem, Unilateral Laplace Transform, RC Circuit with Initial Conditions

Hello, welcome to another module in this massive open online course. So, we are looking at example problems for the Laplace transform. Let us do few more or. In fact, one or two more, and then we will go on to a new topic.

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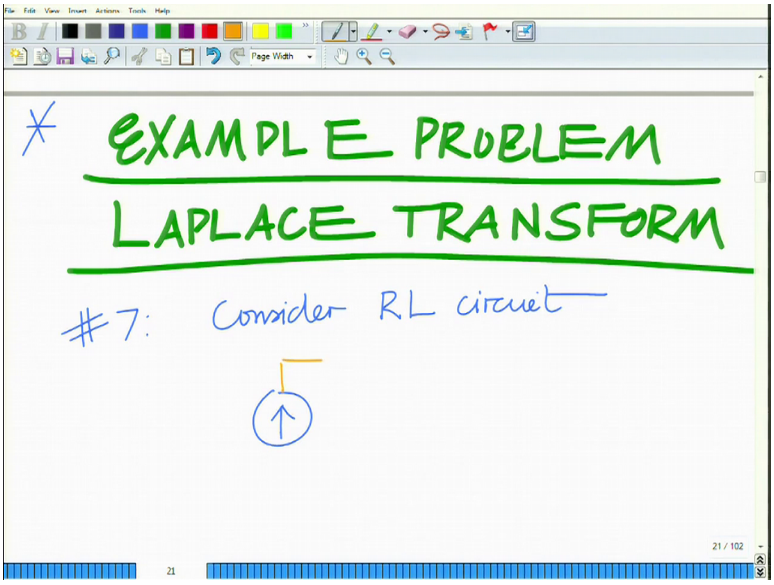

So, I want to start with another example problem or the Laplace transform, and the problem is the following. So, this is role number 7, consider the RL circuit that is given below. So, I have a current source which is connected in parallel with a resistance R and inductor L correct.

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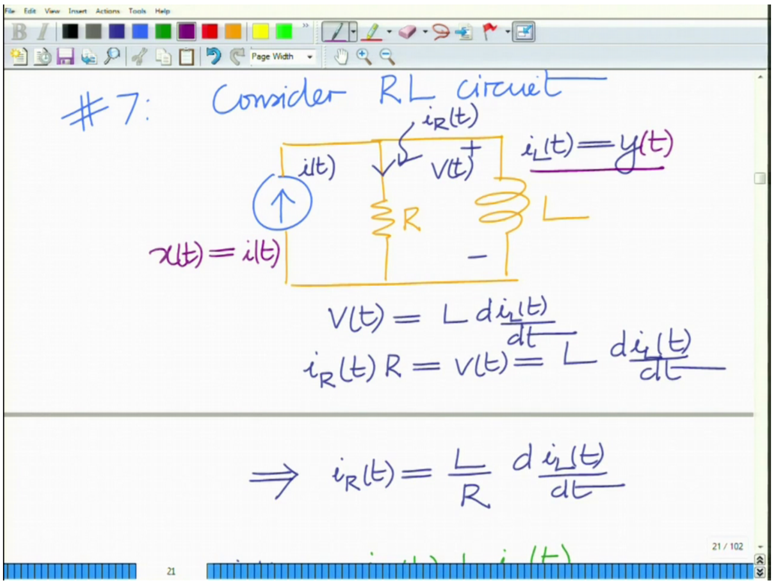

And if we call this as output current il t equals y. And if we call this as i t then we have vt equals L d il t d by dt ok.

And if you call the current through the resistance as iRt, we have therefore, by ohms law iRt into R equals v t equals L d il t by dt which implies iRt equals L over R d il t over dt, and further we have the it current equals iR t plus il t.

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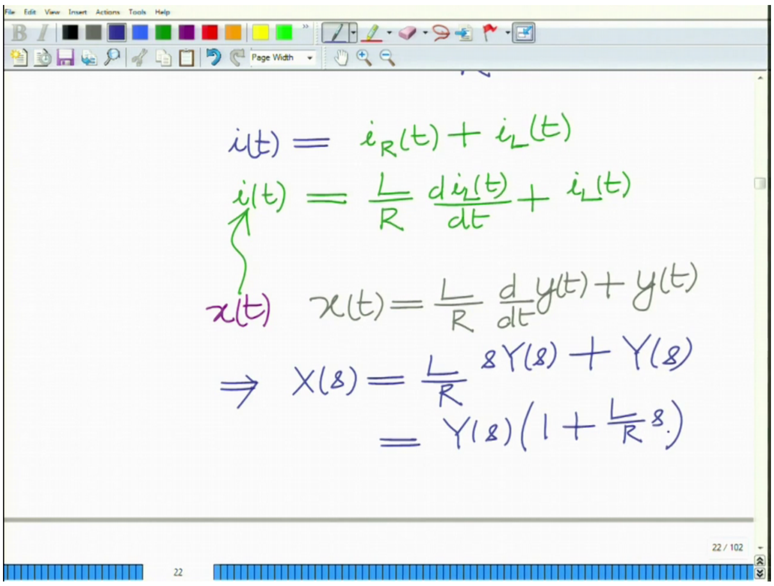

This is current through the resistance iR t plus current through inductor il t which is equal to L by R d il t by dt plus il t ok.

So, this is my input it. So, this is the input output equation. So, if we denote il t by x t this is my input x t. So, I have xt equals the input which is it. And of course, you already said that ilt is the output that is y of t. So, the input is the source current that is i t and the output is the inductor current, output yt is the inductor current that is i l t ok.

And. Now, therefore, we have xt. Writing this in terms of x t yt, we will have xt equals L by R d over dt by t plus y t. Now taking the Laplace transform we have X s equals L over R. Remember Laplace transform d over d t y t is s Ys this is Y s plus the Laplace transform of Y t which is Ys, which is equal to. Well Y s into 1 plus L over R into s, which means the transfer function. Remember the transfer function, input output transfer function is defined as the ratio of output Laplace transform to input Laplace transform; that is Y is over X s.

Therefore H s the Laplace transform of the transfer, all the transfer function is basically Laplace transform.

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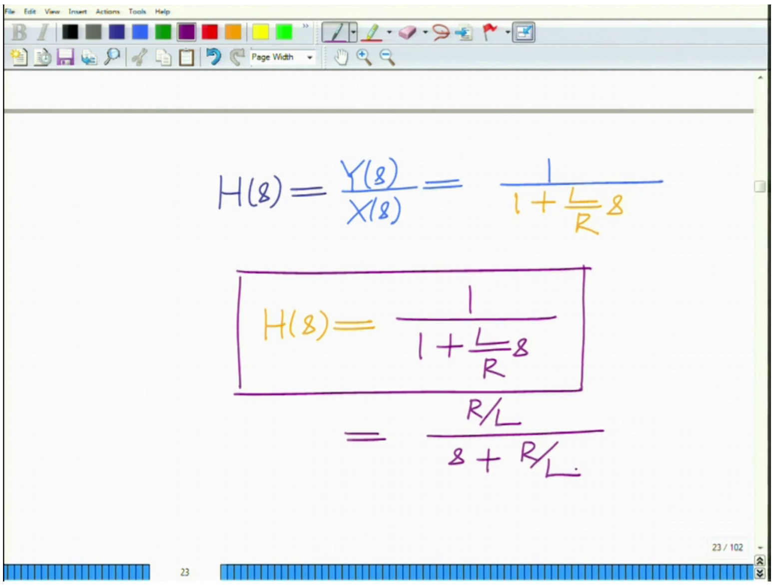

The impulse response or basically the transfer function is basically Y is by X s equals 1 over 1 plus L by R s. So, we have Hs equals 1 over 1 plus L by R times s, and observe the pole of the system, but this can also be written as R over L divided by s plus R over L. So, the pole, it has a one pole P 1 equals minus R over L.

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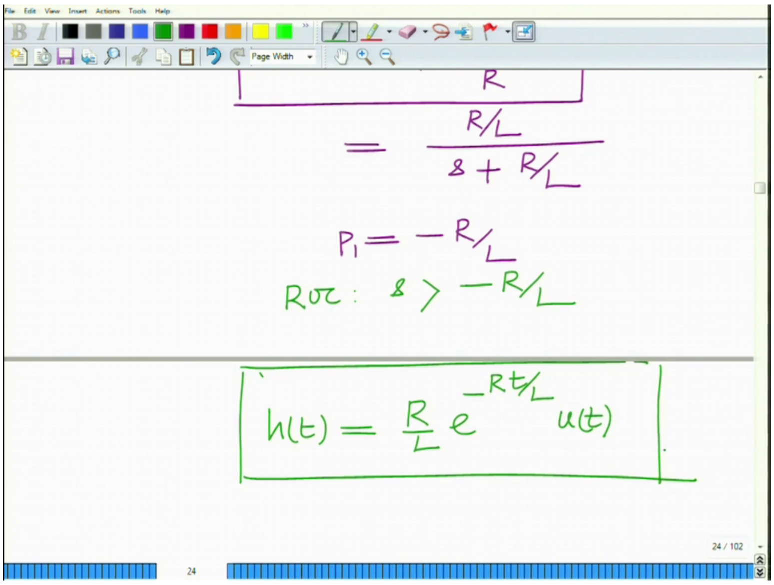

Therefore Roc, now let us assume a right handed signal. Therefore, Roc is basically. If Roc is basically for right handed signal, Roc is basically s greater than minus R by L, and the corresponding inverse Laplace transform; that is Ht, you can see is basically that is R over L times e raise to minus Rt over L ut. So, this is the. So, this is the impulse response.

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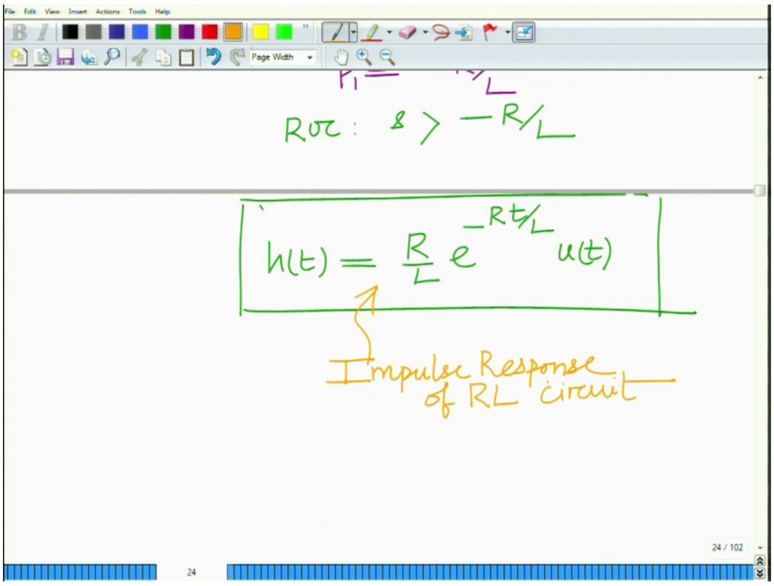

This is the impulse response of the Rl circuit. So, basically we have found the impulse response of this Rl circuit, which is R over L e raise to minus R t over L times u t, assuming that it is a right handed signal and the Roc is real part of s greater than minus R over L all right.

Now, let us proceed on to a different topic which is something that we are going to discuss briefly, and something we did not discuss before; that is the concept of the unilateral Laplace transform. Remember what we have looked at so far, is the concept of bilateral Laplace transform.

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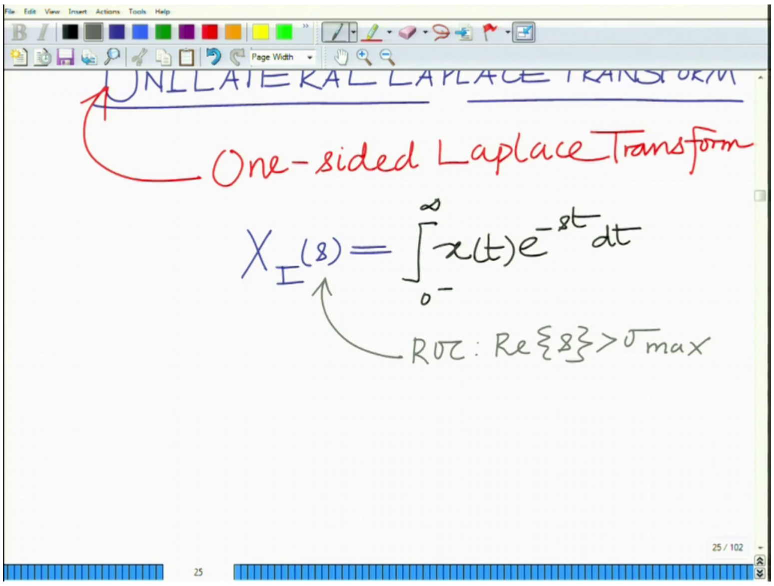

The unilateral Laplace transform, the unilateral Laplace transform or one sided Laplace transform can be defined as follows that is X I of s equals 0 minus to infinity xt e raise to minus st e raise to minus st dt and therefore.

Now, the Roc. Now you look at this, we are clearly considering a right handed signal there. So, for Roc is of the form when you are looking at the unilateral Roc we are considering a right handed signal. So, Roc will be of the form real part of s greater than sigma max. Now typically this is useful for determining the response of a causal system to a causal input with non zero initial conditions all right.

So, the unilateral Laplace transform or the one sided Laplace transform, as we are going to illustrate, it is basically very useful in scenarios where you have a causal system with a causal input and nonzero initial conditions. So, this is useful for the unilateral Laplace transform, useful for a causal system with a causal input.

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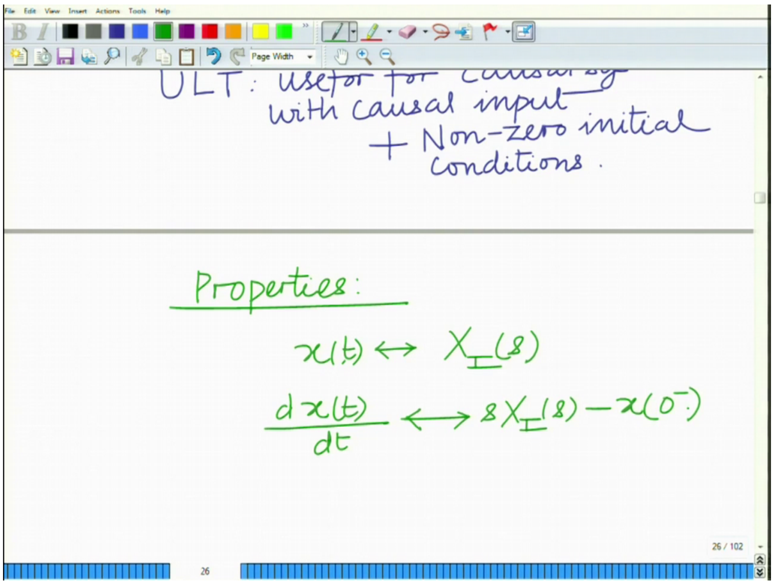

Remember causal input is non zero only for t greater than or equal to 0 plus where you have nonzero plus where you have nonzero initial conditions all right.

Now, some of the properties of this unilateral transformer as follows; slightly different from those of the bilateral Laplace transform. So, let is say the signal xt has unilateral Laplace transform X I of s, then dx t over dt. The derivative has a Laplace transform s X I of s minus the value of the signal x at 0 minus. So, this is the derivative of s. Remember previously for the bilateral, we simply had the Laplace transform of the derivative of the signal is s Xs.

But here we have s X I of s, where X I of s is a unilateral Laplace transform minus x of 0 minus there is a value of this signal at 0 minus further.

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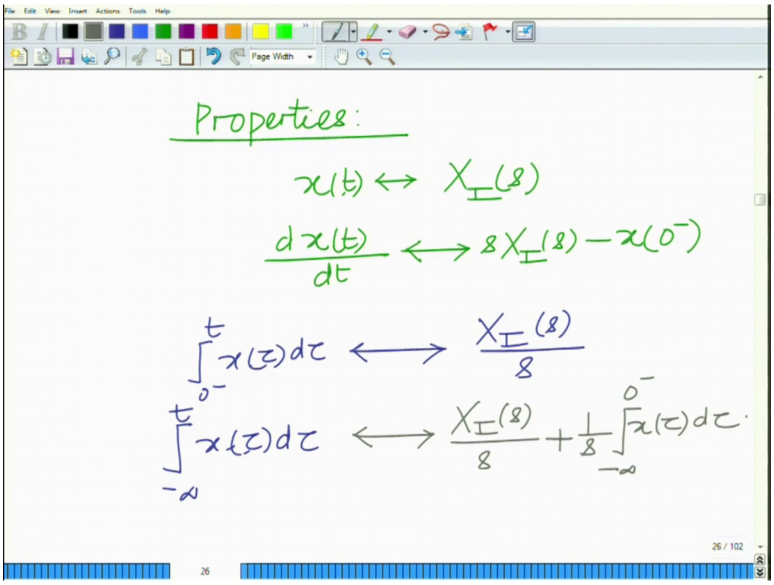

If you consider integral 0 minus, integral 0 minus to t x tau d tau; this will have the Laplace transform X I of s divided by s and integral minus infinity to t x tau d, tau will have the Laplace transform X I of s over s plus 1 over s integral minus infinity to 0 minus x tau d tau. So, these are some of the properties of the unilateral transform.

Let us look at an example to understand this better.

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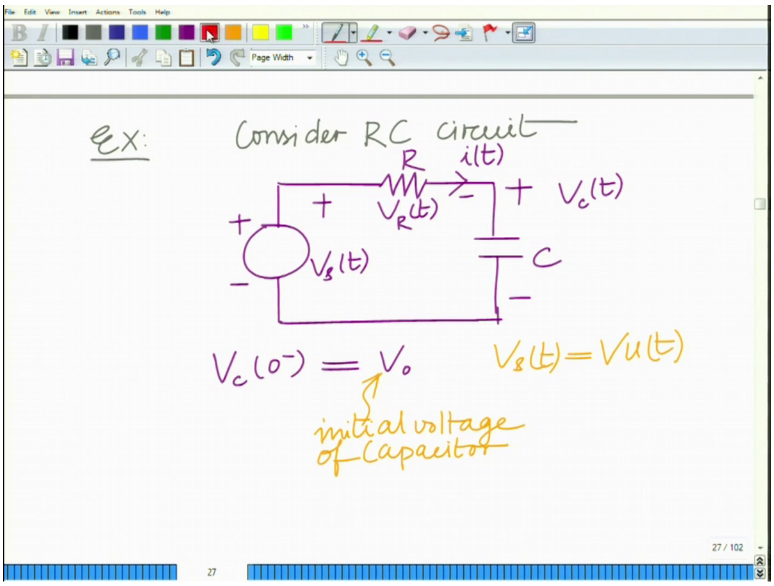

Let us consider the RC circuit below. Consider the RC circuit where you have a voltage source connected across a resistance, and a capacitance and series. So, you have this, let us say is the Res, the voltage Vc t is the voltage across the capacitor V Rt is the voltage across the resistance. Let us say we assume a current of i t, let us say the source voltage is given by V s t ok.

Now, we are given the initial condition. We see 0 minus equals V naught, or remember this is the initial condition, this is the initial voltage of capacitor. Further we are given that the source voltage equals V times u t that is The source voltage is as follows.

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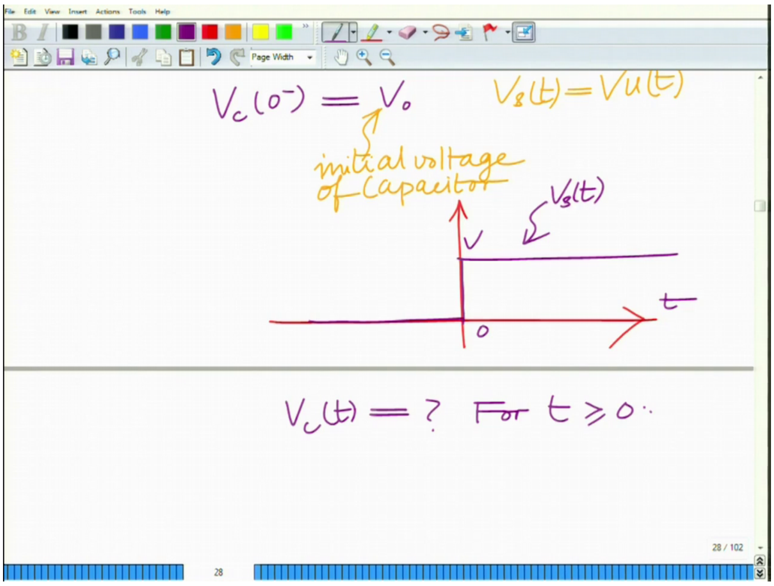

So, we have a step change. You have a step change at t equal to 0. We have step change to V. So, this is basically your Vs t the step function at time t equal to 0, it becomes weak. So, its V times u t, and the initial capacitor voltage; that is Vc t at t equal to 0 minus is V naught, the initial cap initial voltage across the capacitor is V naught ok.

Now, we are required to find what is the capacitor voltage for time t greater than or equal to 0. How does the capacitor voltage vary, change as for time t greater than or equal to 0. So, we are required to find what is Vc t equal to for time t greater than equal to 0. Now taking the unit

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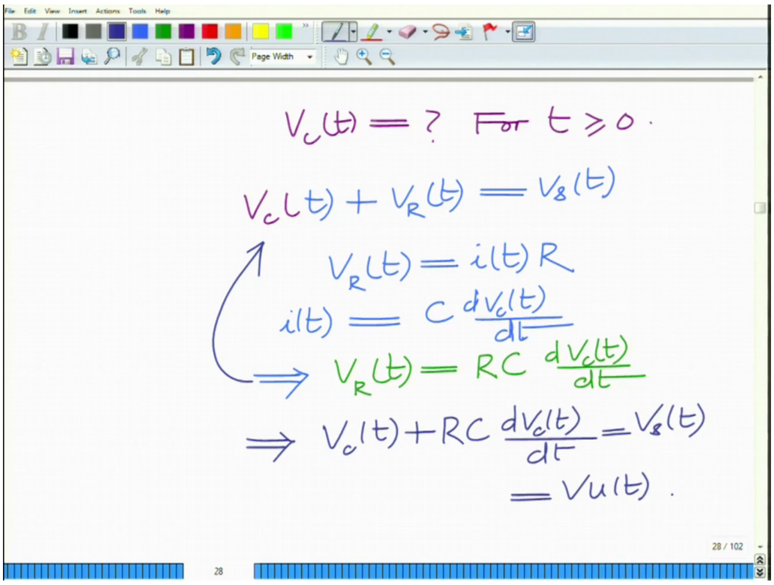

Now what we can write this as from the system; now voltage across the capacitor. Now you can clearly see the voltage across the capacitor plus voltage across the resistance, is equal to the source voltage.

Now, you can see the voltage across the resistance equals i times the resistance, correct i t times the resistance, but i t equals we have C dv dv c t dv c t or dt; that is c times the derivative of the voltage across the capacitor that is a current, which implies the voltage across the resistance equals R C d v ct over dt, which basically now implies substituting this here, we have V ct plus R dv ct or dt equals Vst which is given to be V times u t.

Now, we take the one sided Laplace transform on both the side, because we have a system with nonzero initial conditions. So, we are using the one side or the unilateral Laplace transform. So, taking the unilateral Laplace transform.

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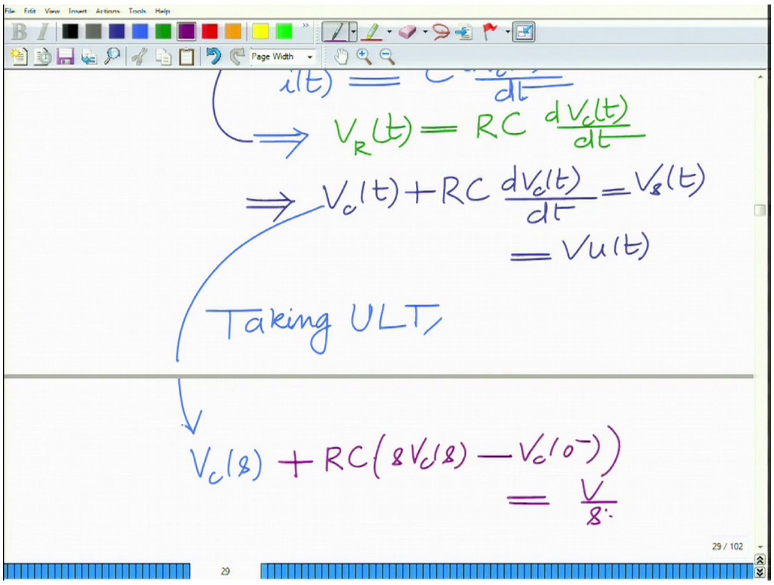

So, taking the unilateral Laplace transform. The unilateral Laplace transform of Vc t, lets say is Vcs plus the unilateral Laplace transform dv c t dt. As we have seen before is s Vcs minus Vc 0 minus which is V naught equals the Laplace transform of a Vu t which is V over s. This implies.

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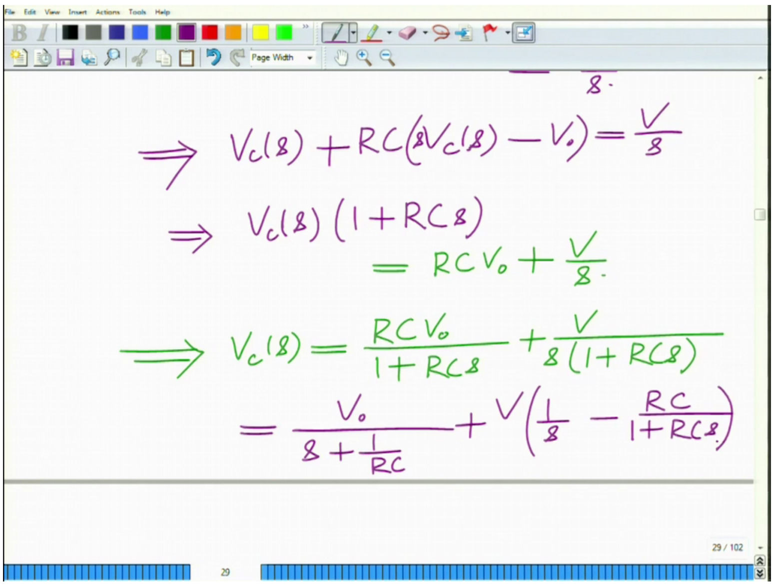

That we have Vcs plus RC Vc s minus V naught equals V over s which implies Vcs into this is s Vcs, Vcs into 1 plus s RC or 1 plus RC s. Lets write it that 1 plus Rc s Vc s into

1 plus Rc s equals R C into V naught plus V by s implies Vcs equals RC V naught by 1 plus RC s plus V by s into one plus RC s.

Now, further simplifying this; this is simply V naught over s plus 1 by RC plus V into 1 over s minus RC over 1 plus RCs correct. So, you have this equals.

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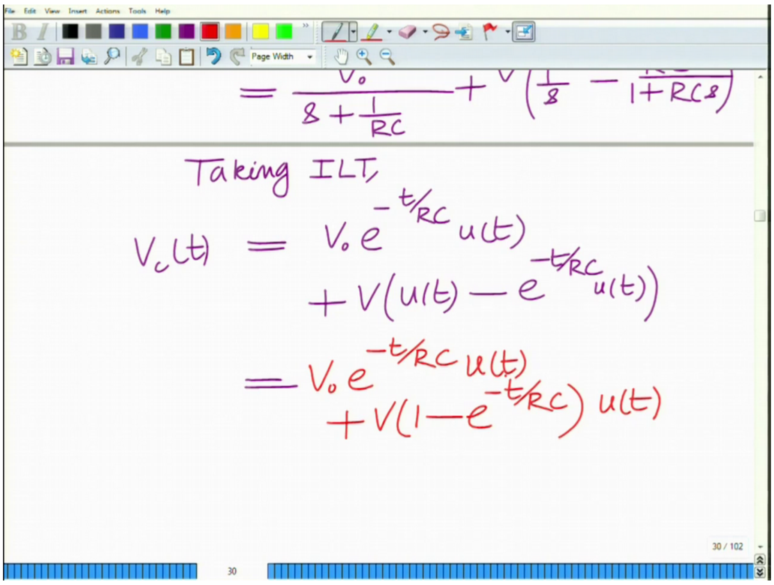

Now, again which if you take the inverse Laplace transform you get Vc t. Now the inverse Laplace transform is V naught over s plus RC is V naught e power minus t over RC into u t plus V into inverse Laplace transform 1 over s is ut minus inverse Laplace transform of R C over 1 plus RCs is simply e raise to again similar to before e raise to minus R over C 1 minus e into e raise to minus t over RC ut.

So, Vc t equals this, which is equals to second taking inverse already i taking the inverse Laplace transform. So, this is V naught e raise to minus t over RC plus V 1 minus e raise to minus t over RC into u t, which means for t greater than or equal to 0 Vc t equals.

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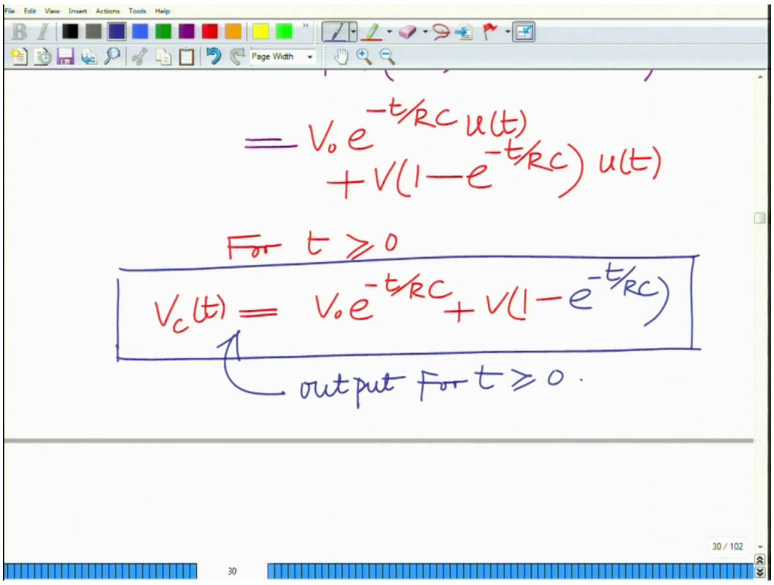

We have Vc t equals V naught e raise to minus t over RC plus V into 1 minus e raise to minus t over RC for t greater than or equal to 0.

Remember this holds only for t. So, this is output for t greater than or equal to 0 and 0. Therefore, using the inverse Laplace transform from the differential equation of the system, using the concept of the unilateral Laplace transform or basically the one sided Laplace transform, we have been able to find the output for t greater than or equal to 0 for a system with nonzero initial conditions, all right.

So, in this module what we have seen is, we have seen different problems as well as the concept of the unilateral Laplace transform. So, we will stop here and continue in subsequent modules.

Thank you very much.