Principles of Signals and Systems
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Lecture – 02
Signal Classification – Deterministic/Random, Even/ Odd, Periodic Signals

Hello welcome to another module in this massive open online course. So, we are looking at a classification of signals various types of signals. And let us continue our discussion on this; let us look at a different classification of signals that is analog and digital signals.

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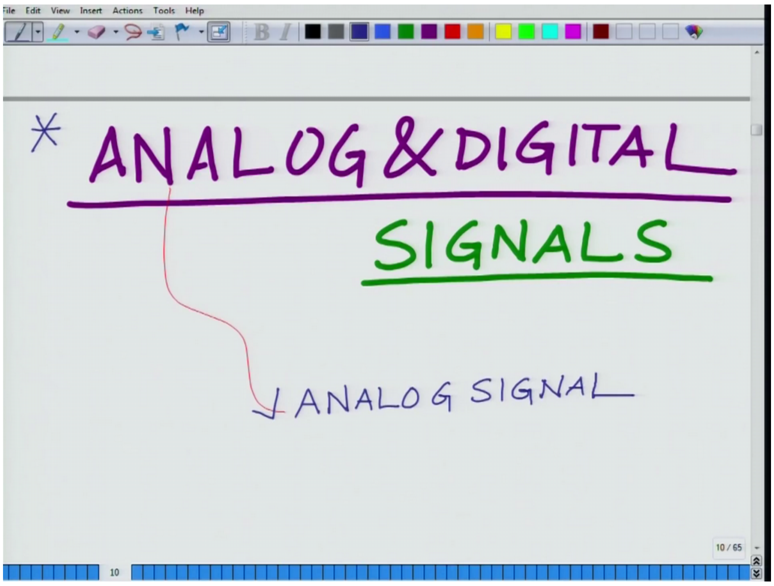

So, what we want to look at now is basically analog and digital signals, so analog signals are basically continuous time signals correct these are continuous time signals which can take all possible values over a continuous interval ok.

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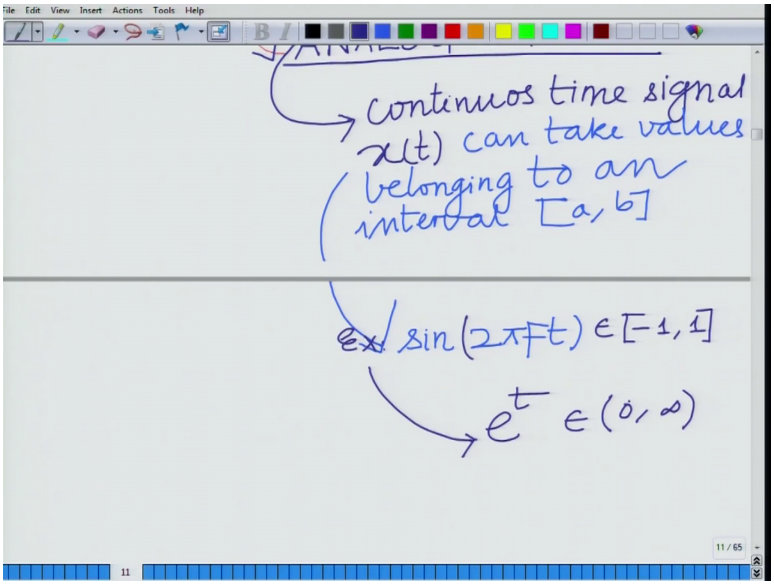

So, an analog signal this is a continuous time signal x t correct, x t which can take values that at its belongs to a comma b, that is it can take values or let us say it take any value belonging to an interval a comma b. Alright it can take values over a continuous interval. So, such a signal which is continuous time system is termed as an analog signal for instance.

We have several examples of analog signals classic examples is again sine 2 pi Ft correct for instance, we can have sine 2 pi Ft which is sinusoidal signal, which is needless to say this is an analog signal. And it takes all values that belong to the interval 0 to 1 or rather minus to 1. So, you can take all values belonging to the internal minus 1 to 1 for instance another such analog signal under that example. So, this is one example another such example is your correct.

So, sine t so, sine 2 pi Ft it belongs to we can take values all values belonging to minus 1 to 1 correct, and another such example is e to the power of t, which can take all values belonging to well basically e to the power of t, and we can take the all positive value. So, basically at from t equal to minus infinity to infinity to this can take all values basically from 0 to infinity of course, at t equal to minus infinity as t tends to minus infinity and this tends to 0 and e to the power of t it looks something like this.

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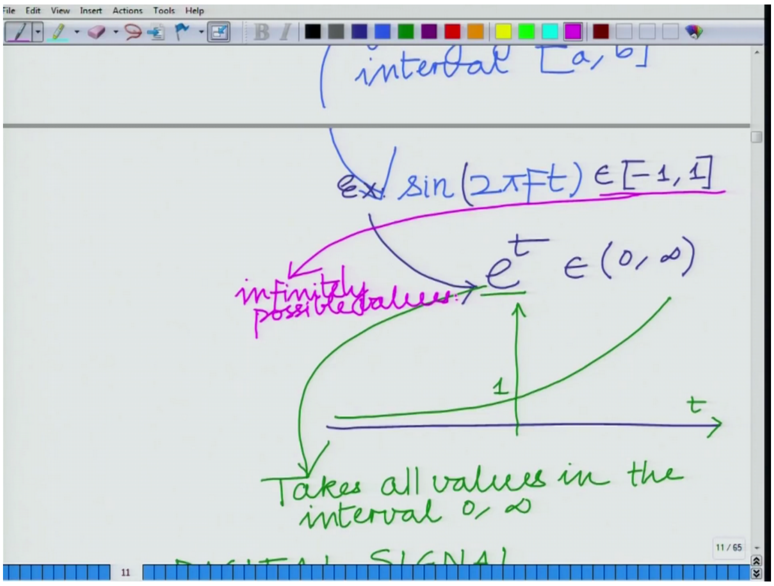

Because all of you are familiar with e to the power t and that looks something like this at t equal to 0 this is equal to 1 and as t tends to infinity t tends to infinity as t tends to minus infinity it tends to 0. So, you can see it takes all value, so e to the power t takes all values in the interval 0 to infinity it is a continuous time signal, and takes all values in 0 to it takes all values in the interval 0 to infinity ok.

And on the other hand we have also a digital signal. A digital signal is a discrete time signal; discrete time signal, which we have seen is already defined as discrete time instants and can take only values that belong to a discrete side in addition to being discrete time signal at digital signal takes values only belonging to a that is a can take only one of possible finites set of values.

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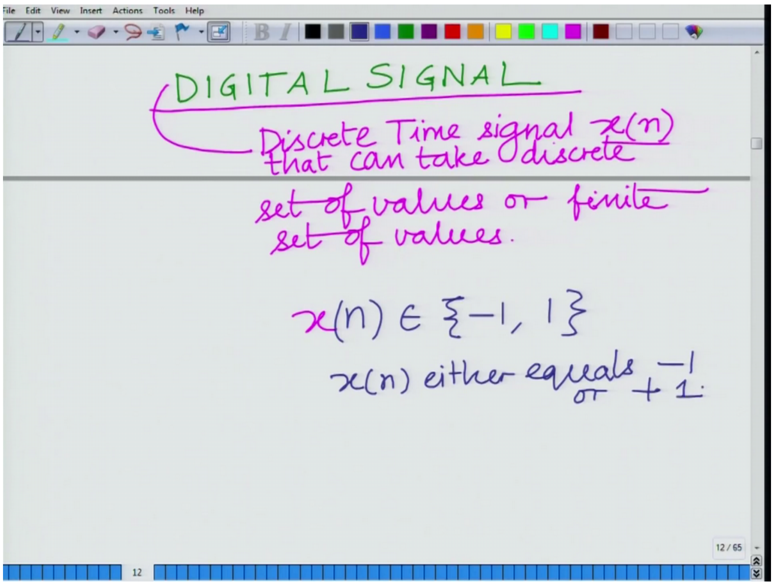

A digital signal, this is a discrete signal or a discrete time signal x n, discrete time signal that can take values that can take only a discrete set of values or basically a finite possible values are the possible values, it can take or finite for instance if you can take. If you look at sine t it take you can take any sine 2 pi Ft you can take any value in minus 1 to 1. So, it can take values from a infinite, so these are infinitely possible values. So, these are basically you can take values from a infinite set.

So, it is an infinitely possible values, however discrete signal can take a digital signal can take values only from a finite set or discrete set, or discrete set of values or basically it can take only a finite it is possible values cannot the set of all possible values has to be the size of the set of all possible values has to be finite for instance x n can either be typical examples x n is either minus 1 or plus 1 for instance. if you have discrete time signal x n such that x n is either minus 1 remember this is not an interval that is x n either equal to so, it can take only values from a discrete set or you can take only a finite number of values that is 2 possible values.

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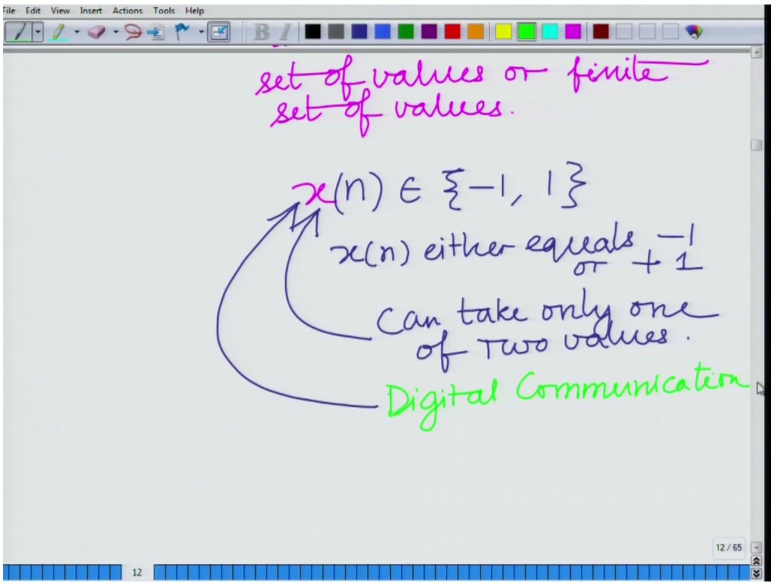

You can take only 2 possible values only 1 of 2 values, and this frequently occurs for instance and communication this is termed as a binary phase shift Keene systems, where transmitted digital symbol can either be minus 1 or plus 1 voltage level can be minus 1 volts or plus 1 volts to indicate the information symbol 0 or 1 for instances occurs in digital communication this is known as a digital constellation. So, this occurs a classic example of this would be in a digital communication system. This is termed as a digital constellation and so.

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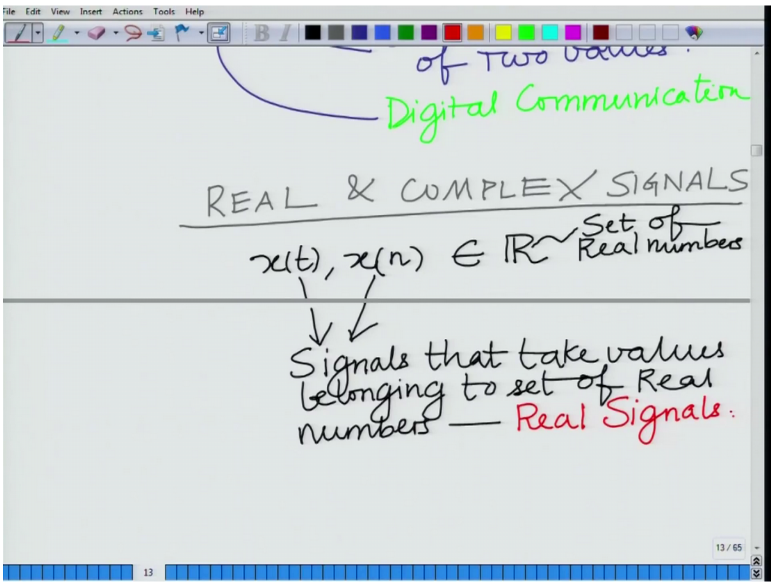

Now, yet another classification of signals can be real and complex signals ok, now this is obvious a real signal either it continuous time or discrete time signal. If it belongs if the values belong to the set of real numbers that is you can take only real values it is termed as a real signal, either can be either a continuous time signal or a discrete time signal.

If it takes values belonging to the set of real numbers or set of real values it can take it is a real valued signal belonging to the set of real numbers. This is termed as such signals are termed as that is signals that take values belonging to set of real numbers these are termed as real these are termed as real signals.

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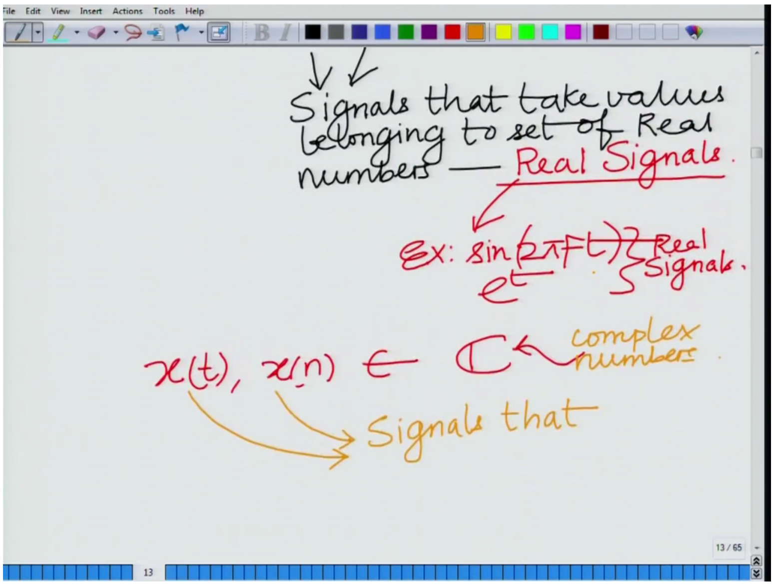

On the other hand for instance and again examples are abundant for instance again you go back to the previous signals that is sine 2 pi Ft or e to the power t both of these signals, these are needless say these take only real values. So, these are real signals.

On the other hand if you look at other examples such as for instance, on the other hand if you take other signals such as x t or discrete time signal x n, which can take values belonging to the set of complex number, so this is a set of complex numbers. So, when these take that is signals that take values belonging to the set of complex number.

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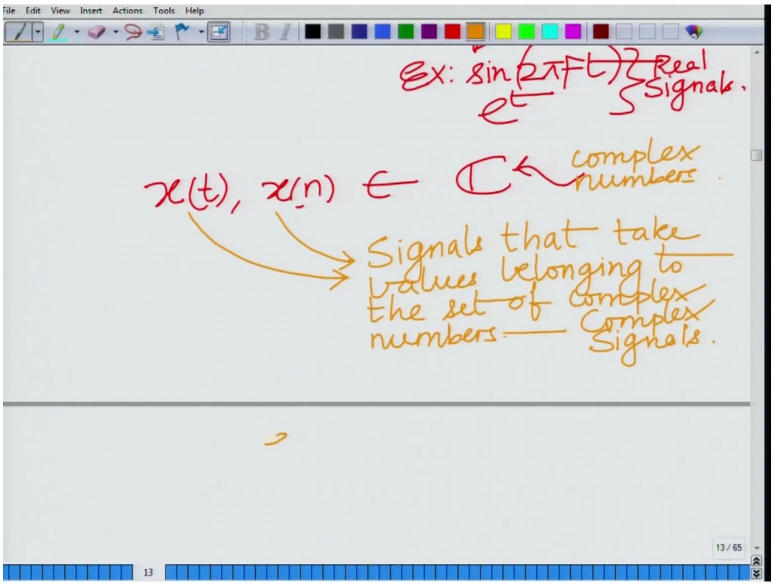

That takes complex value signals that take values complex numbers these are termed as complex signals ok.

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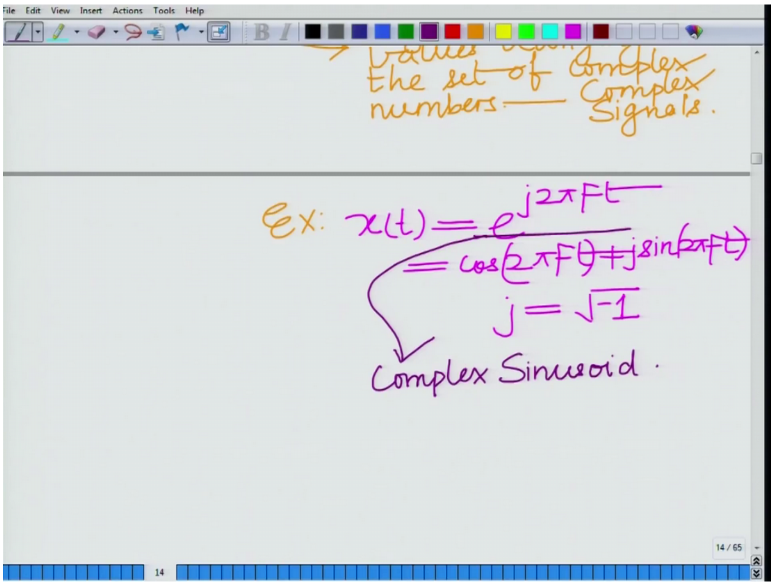

For instance the classic example of a complex signal is x t equals e to the power of j 2 pi Ft which can also be written as cosine 2 pi Ft plus j sin 2 pi Ft with j equals the imaginary number square root of minus 1 this is a complex number correct this is termed as a also termed as a complex sinusoid, cosine j 2 pi t plus j sine 2 pi Ft this is also termed as a complex sinusoid, these are such signals are basically needless to say these are complex signals, which can complex value we can take values belonging to set of complex number. So, we have real signals or complex signal.

And complex signals although they do not occur at practice is in the sense that these are not these do not exist in nature that can be they are very useful in the representation of signals in fact, again once again if you go back into the analysis communication systems. All communication signals can be analyzed or represented as complex signals right assuming in face and quadrature components of a communication systems.

So, complex signals have a great utility I mean significant utility in the analysis right in the study of signals and also analysis, carrying out analysis in various areas all right to understand different concept and carry out the analysis in various areas of electronics and communication engineering all right. So, these are also an important class of this is also represents an important class of signals ok.

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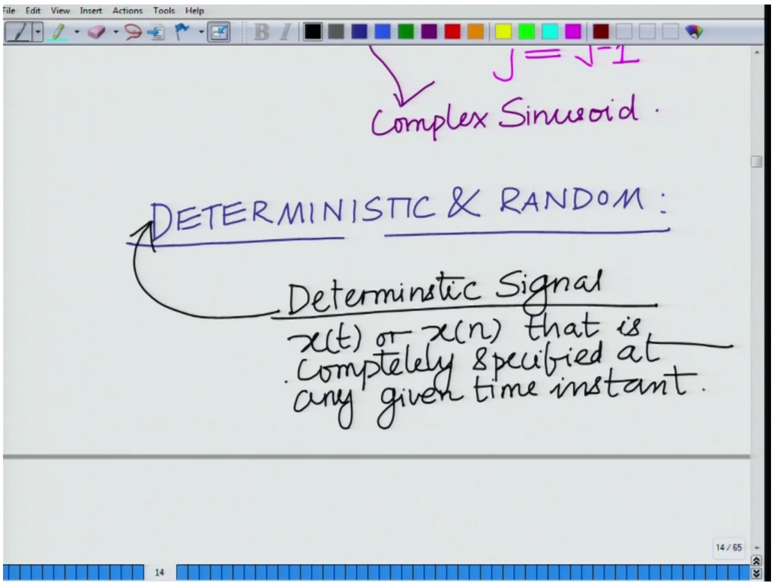

And yet another important class classification of signals is deterministic and random signals. This is another very important that is deterministic; deterministic and random, random signals these can also be either continuous time or discrete time signals. A deterministic signal a is completely specified its deterministic as the English word implies it is determined, this implies x t or x n that is completely specified deterministically at any given time instant, again we go back and take the example for instance.

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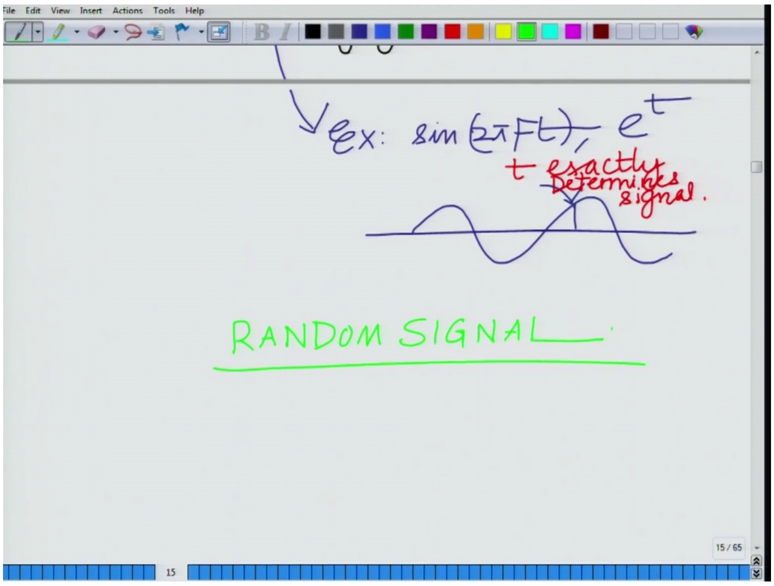

It is given by a formula for instance sine 2 pi Ft e to the power of t these are deterministic signals in the sense that at a given time instant t there is no ambiguity, they are completely specified by a certain formula given a time instant one can exactly determine what is the signal for instance. If you take a look at sine 2 pi Ft given a time instant I can exactly determine correct time instant t exactly determines the signal.

However this is not the case in a random signal; in a random signal, as the name implies is random in nature it take random values at various time instance. So, a random signal again either continuous time or discrete time takes random value at different time instant.

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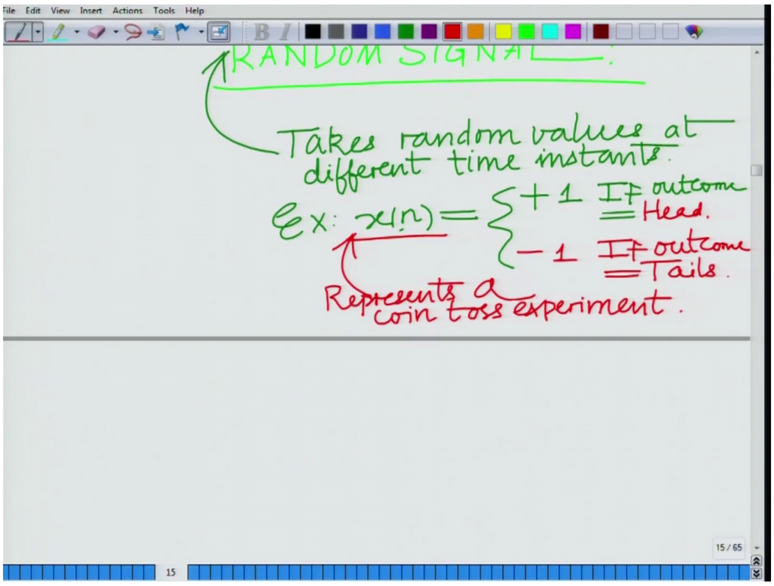

So, this takes random values at different time instants takes random values at different time instance. Hence it is not completely determine ahead of for instance, example x of n.

So, let us say your signal your signal in the outcome of coin toss experiment. So, of its outcome is a heads it signaled by plus 1 if the outcome is tails it is signaled by minus 1. So, signal x n equals plus 1 if outcome equals heads or if outcome is heads equals minus 1, if outcome equals tails. At each so this is basically representing a coin toss experiment it represents a coin toss experiment what this means is if at every instant of time you tossing a coin if the outcome is head, your representing it minus 1, if the outcome is tail you if the outcome is head, you representing it by plus 1 if the outcome is tail you representing it by a minus 1 and therefore, since the outcome of the coin toss experiment is random the signal itself is random minute this is a discrete time random signal right.

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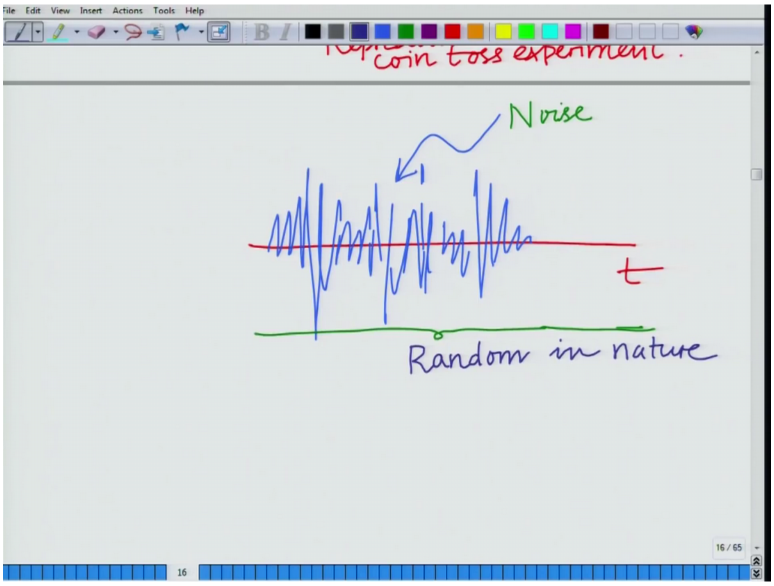

Classic example of a continuous time random signal is noise for instance. Let us say you have t correct and some kind of a signal which look like this. This is termed as noise, which is random in nature observe that this is random in nature and it (Refer Time: 20:11) have and it causes significant of interference correct, this causes the noise basically interferes with the signal the desired signal the information bearing signal. So, noise sometimes also not considered as a signal, but by going by or definition. So, it is basically a random in nature.

Random in nature and it is fundamentally important to understand the properties and behavior of noise to characterize to analyze the behavior of several systems. So, as important as it is to understand the signal, because the noise limits the performance of a system is as important to also understand the properties and behavior of noise to completely characterize the performance and behavior of a system.

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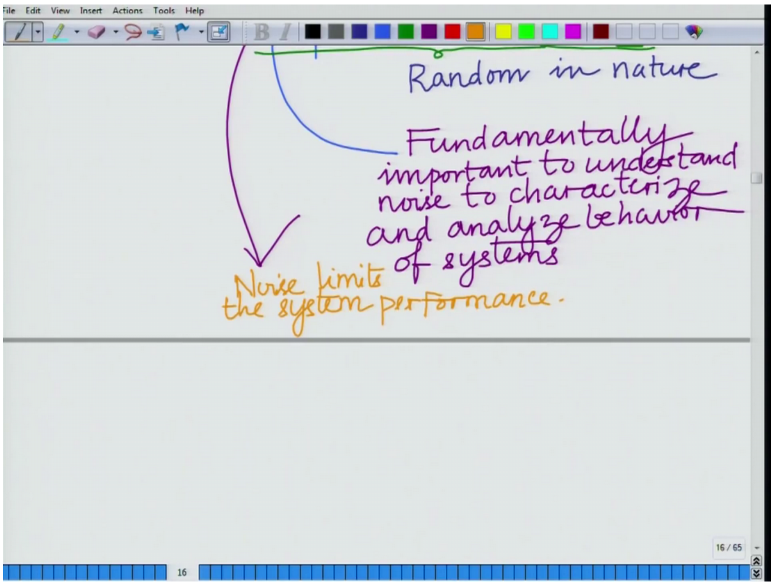

So, I will leave it at that because the analysis of is basically falls in the domain of analysis of random signals, correct and it is fundamentally important to understand noise.

To characterize correct and analyze behavior of systems, that is because noise limits the performance of systems correct, that has to be something that is remember noise limits the performance of systems; noise limits the system performance, and it is fundamental importance in analysis performance analysis characterization of the behavior of any system any practical system.

Because noise is an inevitable component or whenever we analyze signal, there is also an underline noise component that is present its power may be less or it is the relative power level; power level relative to the signal might be varying, but it is an its underlying component and it is fundamentally important characterize this the properties and behavior of noise fully understand the behavior of the system.

And for instance have once such again going back to our example of communication systems the behavior, and analysis of noise is fundamentally important to characterize the performance alright. The quality of information transfer the accuracy with information can be transferred for instance from a base station to the mobile, and that your carrying to analyze the quality alright to analyze the bit rate at which to analyze the quality, with which the information the accuracy, with which the information or the information symbols can be received its very important to understand and characterize the noise properties of the system. So, it is important in analysis of systems.

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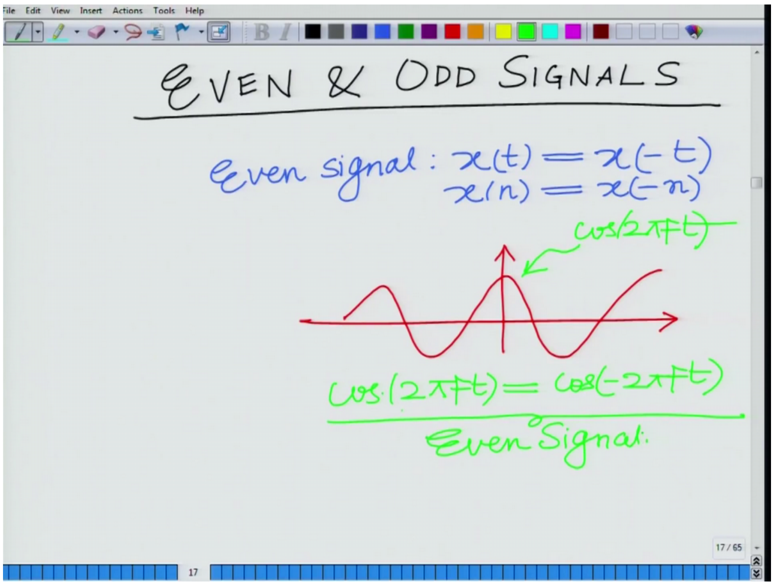

Again another characterization which come in hand is in several analysis is even and odd signals, and even signal as most of you are must be familiar similar to an even function is x t equals satisfies x t equals x of minus t or for a discrete time signal x n equals x of minus n for instance you have a classic example, that is your cosine 2 pi F c t for instance let me just draw it. So, this your cosine 2 pi F c t or cosine 2 pi F c t this is an example you can see that cosine 2 pi Ft equals cosine minus 2 pi Ft.

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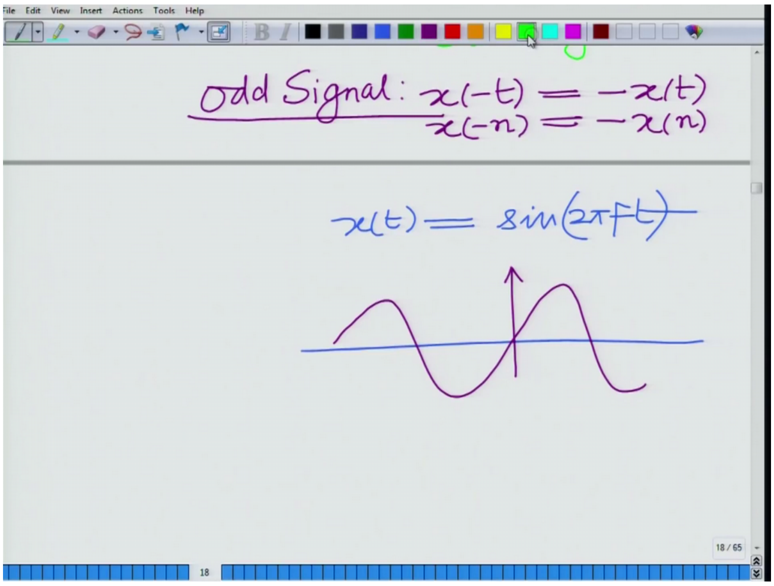

And this is an example of an classic example of an even signal. On the other hand an odd signal is x satisfies x t equals minus x of t or x n equals sorry minus x of minus t or minus x of minus n. So, or I am sorry x of yeah x of x of minus t equals minus x of t. So, let me write it in slightly different way I think that will be much more x of minus t equals minus of x if t and x of minus n equals minus of x of n I think that is slightly better way to write the definition of odd signal.

And again another classic example of an odd signal is x of t equals sine of 2 pi sine of 2 pi Ft for instance, you can see here this is the sine signal correct sine of 2 pi Ft and you can see that basically this is an odd signal for instance you can clearly see here.

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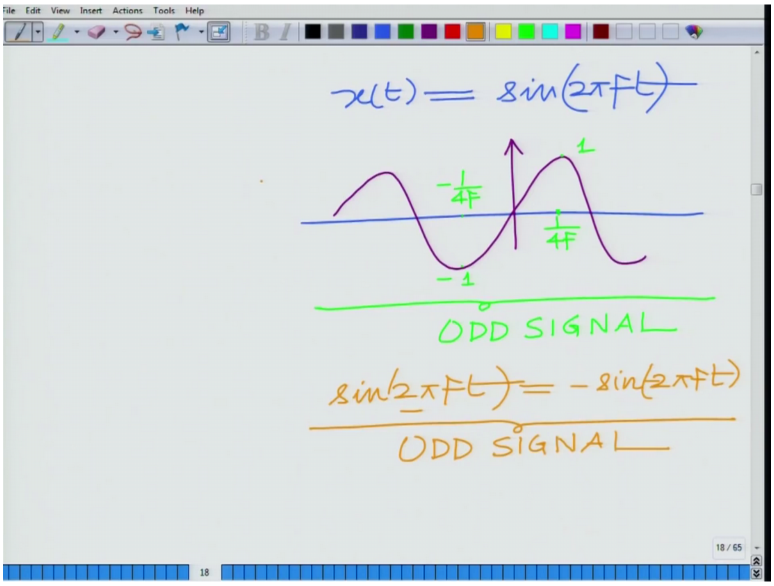

And here for instance this is the value where t equal to this is the value for pi by 2. So, t must be equal to t must be equal to 1 by 4 f, and this is the value correspondingly at minus 1 by 4 f and you can see this is basically 1, and at minus 1 by 4 f it is minus 1. So, this satisfies basically sine of 2 pi Ft equals minus of sine of minus 2 pi Ft. So, this is basically your, this is the classic example of an odd signal.

Again comes in handy is concept of even and odd signals comes in handy when analyzing the properties of the behavior of signals. So, even signal has even symmetry that is symmetric about 0 x of t is x of minus t, and odd signal as odd symmetry that is x of minus t is minus of x of t ok.

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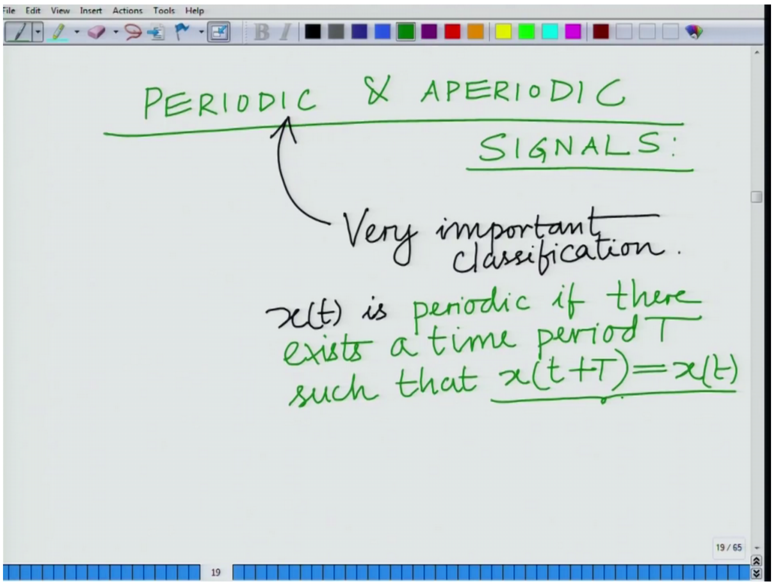

And another very important classification of signals is periodic versus aperiodic signal. So, we have periodic versus aperiodic signals. So, periodic and aperiodic signal if t is the period ok.

So, this is an important classification again, this is very important classification this is a very important classification x t is periodic if there exist t, or if there exist a time period t, such that x of t plus t is equal to x of t, this holds for all t not a particular t, for all t that is for instance.

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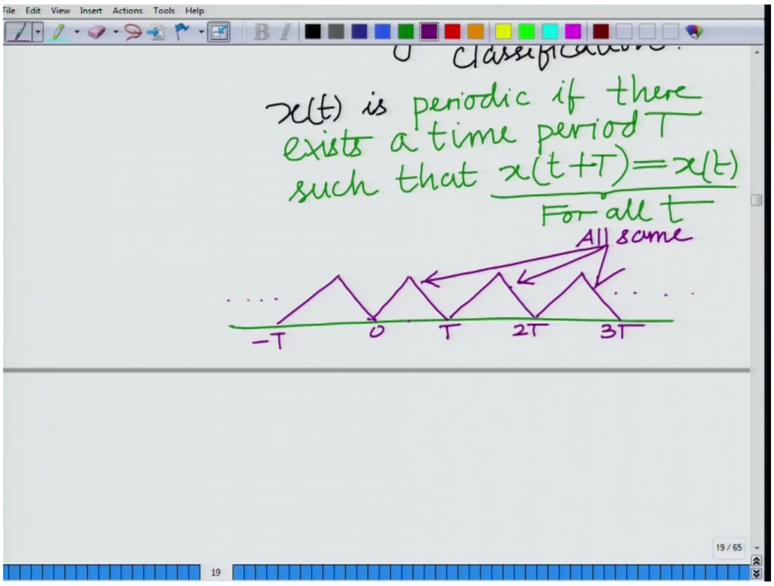

You take a look at this, that is there exist time period capital t, such that if you look at x of t plus t shift like look at x of t, plus t at a shift of capital t right it is equal to x of t, and this is true for all t this is true for all time for instance. Let us consider a periodic triangular way ok.

So, this this is 0 this is t this is 2, 3 t minus t and so on. And you can see all the values for instance if you look at this particular value and look at t later, so these are all the same that is for any t that is if you look at values t apart capital t apart they are all the same alright. So, for any small t x of plus x of t is equal to x of t plus capital t, where t is known as the period for odd and this holds for all time instance t ok.

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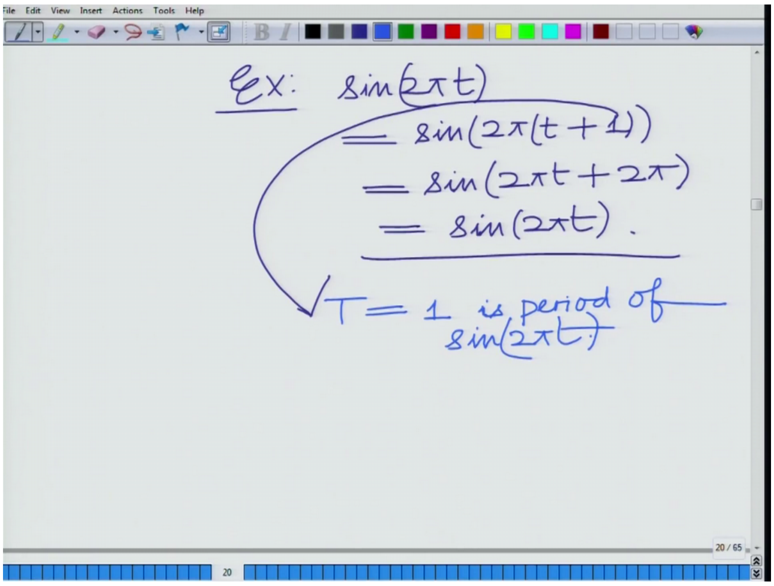

Such a for and a classic example again is the sine correct, we all know that the sine is the periodic signal sine 2 pi Ft you can see that this is equal to sine 2 pi Ft plus the period, here is going to be well the period here is going to be 1 over F correct, which is equal to sine or just let us just make it sine 2 pi t.

So, this is sine equals sine 2 pi t plus 1 is equal to sine 2 pi t plus 2 pi sine of theta plus 2 pi sine of theta. So, this is again equal to sine of 2 pi t. So, this is a period of sine 2 pi t t equal to 1 t equal to capital L. Now there is a notion of fundamental time period what is for all periodic signals there is a notion of what is known as a fundamental period.

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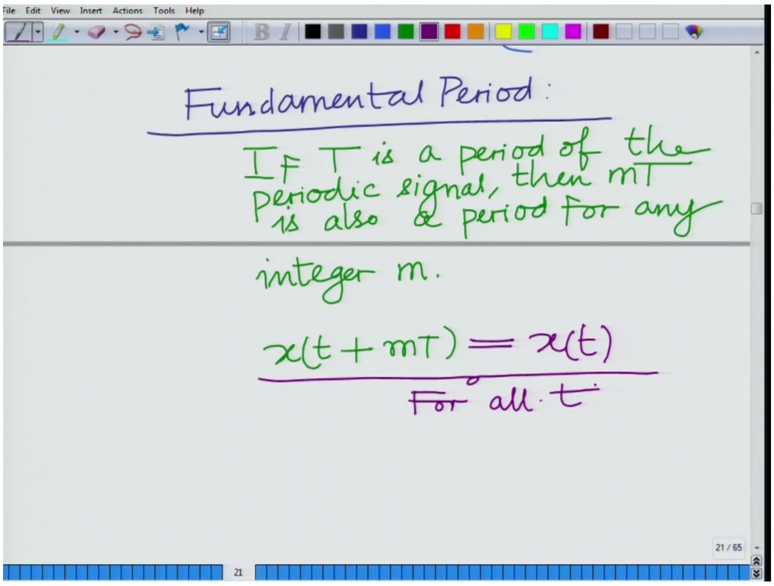

Now, what is the fundamental time period or the fundamental period, now first realize that if t is a period of the periodic signal, them m T is also a period, where m is any integer is also a period correct yes or no. Because extent is also period for any integer m, and the reason this is true correct for instance we have x of t plus m T equals basically x of t plus or we will have x of t plus m T equals x of t correct, and this also holds for all t correct, because if it is because its periodic with t then its periodic with 2 t. As well for instance here you can see if you look at point and 2 t apart it has to be identical ok.

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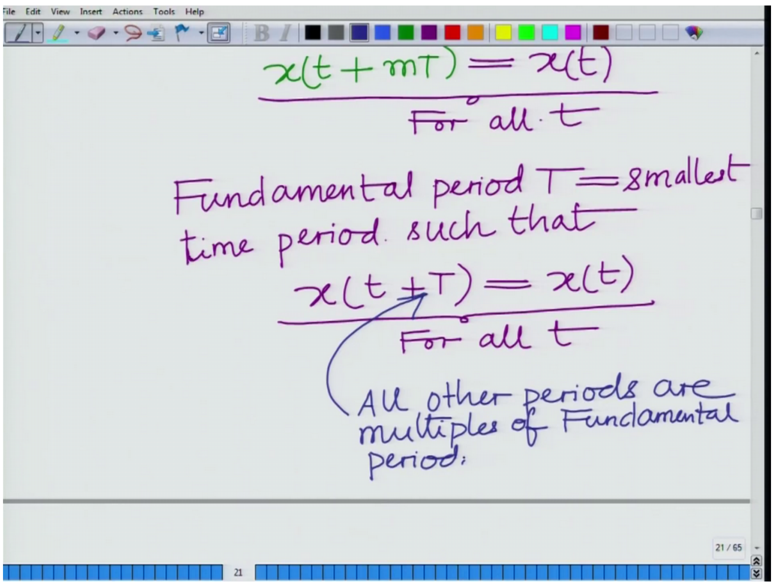

So now, therefore the fundamental period is the smallest positive integer therefore, now fundamental period t is equal to the smallest positive integer, equals smallest positive well it is a smallest positive it is a smallest positive number not integer. Let us put it this way, so is the smallest time period, such that x of t plus t equals x of t holds for all t. Such that this holds for all t, for instance and all other periods are basically multiples of this fundamental period all other periods of this periodic signal are basically multiples of this fundamental period. So, all other periods.

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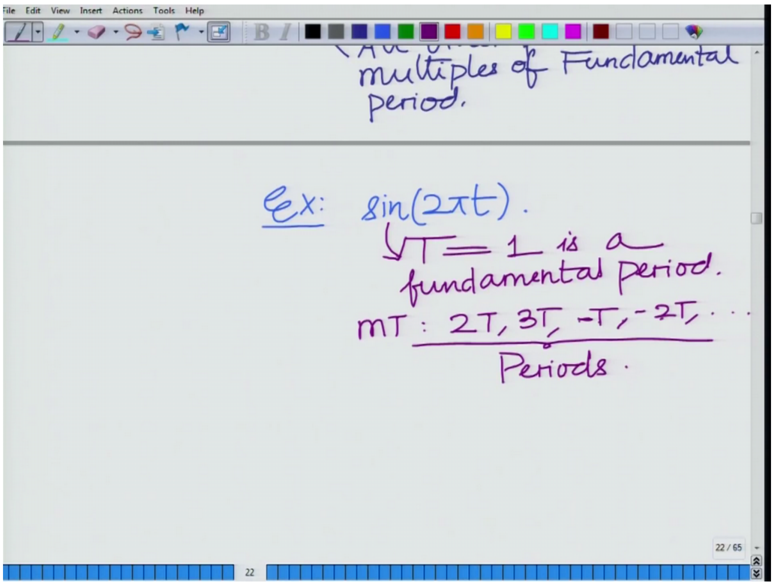

All other time periods are multiples of you are all other periods for example, again let us go back to our example sine 2 pi t you can show that T equal to 1 is a fundamental period or T equal to 1 is a for this signal, T equal to 1 is a fundamental period.

And therefore, it follows that any m T for instance such as any m T, such as 2T 3T etcetera. Now these are also periods and, in fact for that matter also minus T, minus 2 T and so on these are also valid periods, because x of sine of T for instance 2 T for instance here T equal to 1. So, 2 3 minus 1 minus 2 etcetera are also periods, because it because sine of 2 pi t minus 1 or sine of 2 pi t plus 2 is also equal to sine of t .

So, any multiple of the fundamental period is also a period and the fundamental period is the smallest possible duration, such that x plus t plus capital T that is the fundamental period equals x of t for all time instance t ok. Now, the same can be defined for a discrete time signal again.

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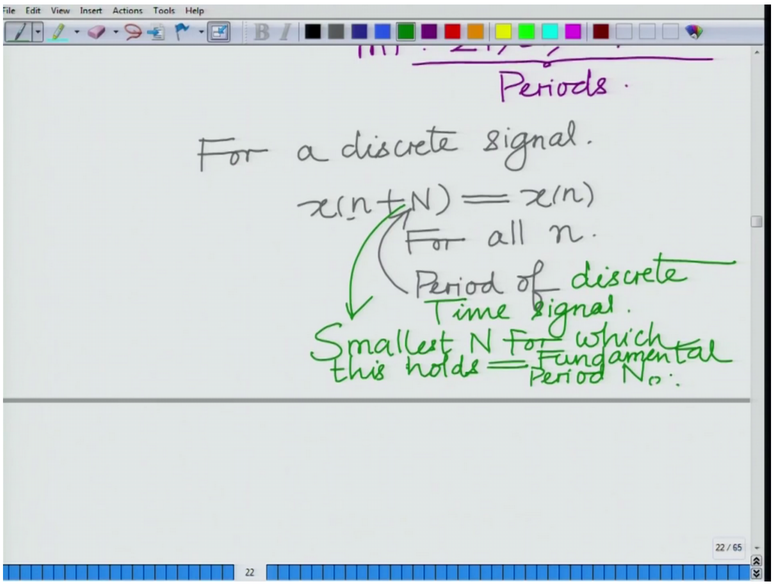

For a discrete signal; discrete periodic signal, we must have x of n plus N equals x of n and this must hold for all n this. And this is basically this is basically your this is known as the period of the discrete time signal, and this is the smallest N for which this holds is known as the fundamental period; smallest N for which this holds equals the fundamental period N naught, capital N naught. So, that is basically periodic signals continuous time periodic signals and discrete time periodic signals alright.

So, we have seen various classes of signals alright starting with I mean there are several classes of signals such as deterministic and random even and odd signals and several different classes of signals alright. So, you can go over these different classes and try to understand that better alright. So, we will stop here and continue with other aspects in the subsequent modules.

Thank you.