Principles of Signals and Systems
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Lecture – 18
Laplace Transform Properties – Time Shifting Property, Differentiation /Integration in Time

Hello, welcome to another module in this massive open online course. We are looking at the Laplace transform.

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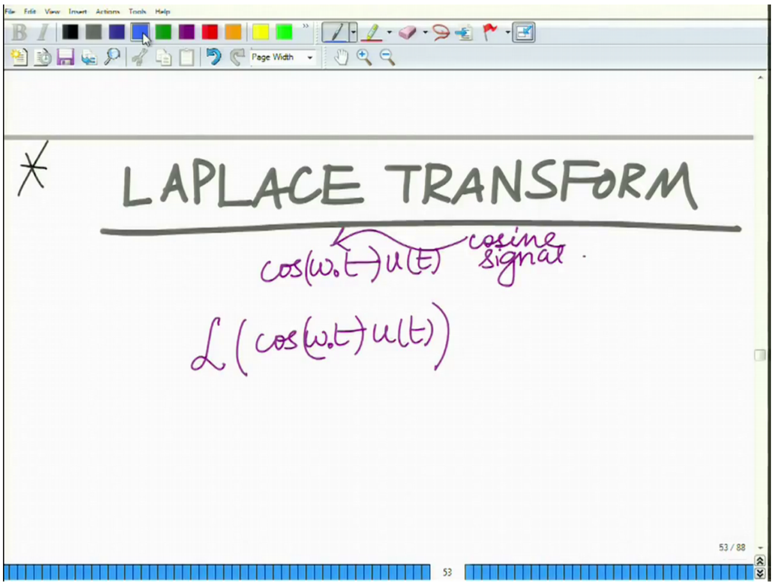

And, looking at the Laplace transform, the various properties of the Laplace transform, the concept of the region of convergence and the Laplace transform the impulse and the unit step functions. Let us now, look at the Laplace transform the cosine function. We have looked at the Laplace transform of the unit impulse and the unit step function.

Let us continue looking at the Laplace transform of some common functions, Laplace. Let us continue looking at the Laplace transform of some common functions or some common signal, let us look at the Laplace transform in particular of cosine signal. This is your cosine signal and the Laplace transform and cosine t u t cosine omega t times the unit step function. The Laplace transform of cosine omega naught t u t this is equal to integral because of the u t, it is nonzero only for t greater than equal to 0.

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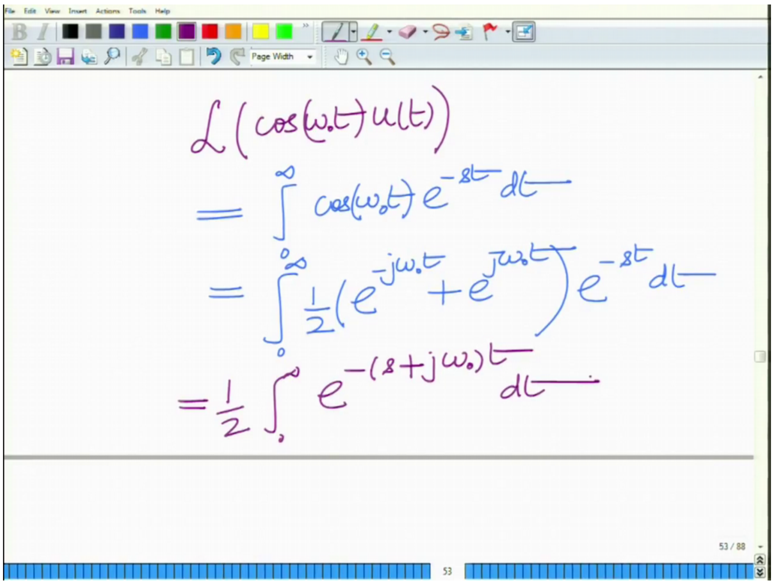

This is equal to 0 to infinity cosine omega naught t e power minus st dt, which I can write as cosine omega naught t. I can write as half e raise to minus j omega naught t plus e raise to j omega naught t, e raise to minus j omega naught t plus e raise to j omega naught t, e raise to minus st dt which is equal to well half integral 0 to infinity e raised to minus s plus j omega naught t dt.

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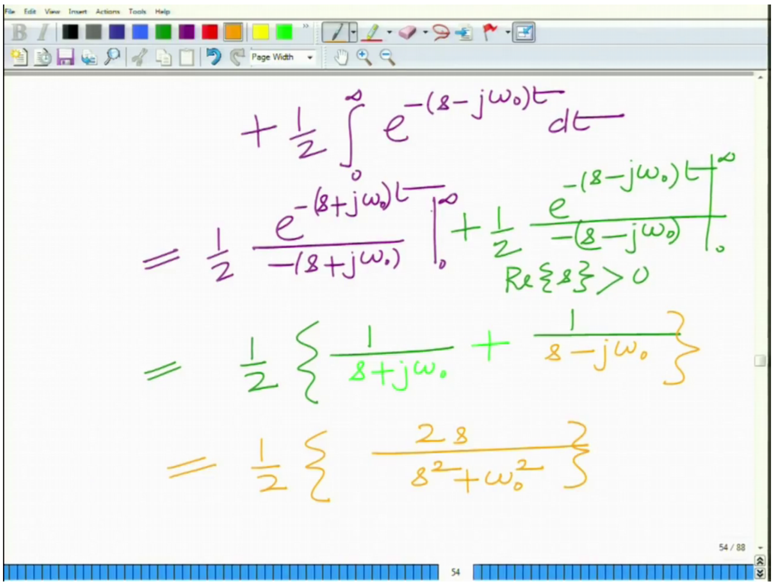

Plus half integral 0 to infinity, e raises to minus s minus j omega naught t dt. I have written it as the sum of two integrals. This is the first integral as well this is e raise to minus s plus j omega naught t divided by minus s plus j omega naught evaluated between the limit 0 to infinity, plus half e raised to minus s minus j omega naught t divided minus s minus j omega naught evaluated between the limits 0 to infinity. This is half of 1 over now, if you evaluate this between the limits 0 to infinity e raise to minus s plus j omega naught t evaluated as limit t tends to infinity tends to 0, but only if the real part of s is greater than 0, this is the region of convergence.

Similarly, e raised to minus s minus j omega naught t tends to 0 as t tends to infinity, only if s greater the real part of s is greater than 0. This is basically your ROC. This is your real part of s greater than 0 that is ROC and in the ROC this is the first term is 1 over s plus j omega naught plus 1 over s minus j omega naught which is equal to half 1 over s plus j omega naught plus 1 over s minus j omega naught that is 2s divided by s plus j omega naught times s minus j omega naught that is s square plus omega naught square.

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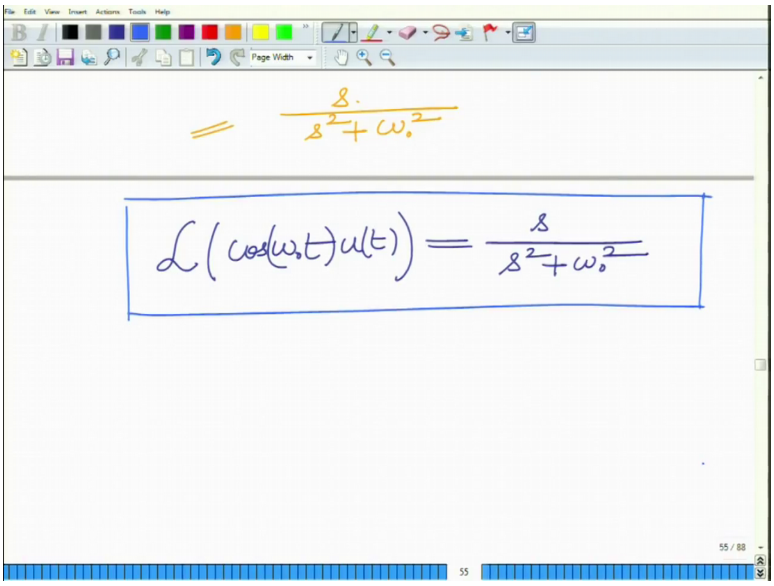

Which is equal to s by s square plus omega naught square. This is the Laplace transform. To summarize the Laplace transform of cosine omega naught t u t equals s square by s square plus omega naught square. I am sorry, s divided by s square plus omega naught square. This is the Laplace transform of the signal, cosine omega naught t u t.

Now, let us look at and similarly you can compute the Laplace transforms of some other commonly occurring signals; for instance, you can try to compute, we have computed the Laplace transform of cosine omega naught t u t. You can try computing the Laplace transform of sine omega naught t u t and similarly, you can compute the Laplace transform of several other commonly occurring signals.

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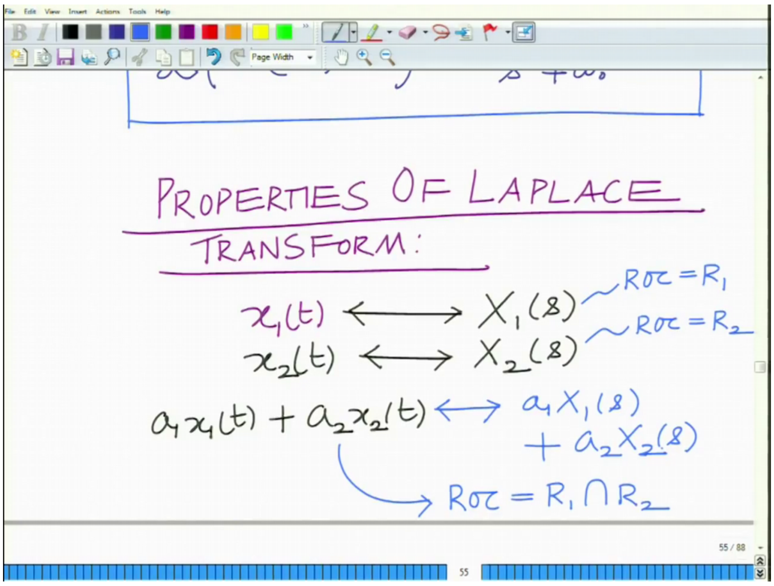

Now, let us look at some properties of Laplace transform some of the interesting aspects of Laplace. Laplace transform which indeed make it a very handy tool. Let us look at the properties of the Laplace transform some of the properties of the Laplace transform, for instance, if you have 2 signals x 1 t with Laplace transform X 1 s, x 2 t with Laplace transform X 2 s, then the Laplace transform is a linear transform. The Laplace transform of a 1 x 1 t plus a 2 x 2 t, this is a 1 X 1 s and you can easily show this is a 2 X 2 s because integral of a sum is the sum of the integrals and, but the only important point here is to look at the region of convergence because remember we said the Laplace transform is incomplete without specifying the region of convergence.

The region of convergence of the first one is Roc region of convergence is R 1 second one region of convergence is R 2, then the region of convergence of the linear combination will be the intersection. The ROC will be equal to R 1 intersection with R 2 because naturally both the Laplace transforms X 1 s and X 2 s have to exist. X 1 s exists only in the region of convergence that is defined for R 1 X 2 s is defined for R 2 and therefore, the region of convergence of the sum will be R 1 intersection R 2 that is the intersection of these 2 regions of convergence.

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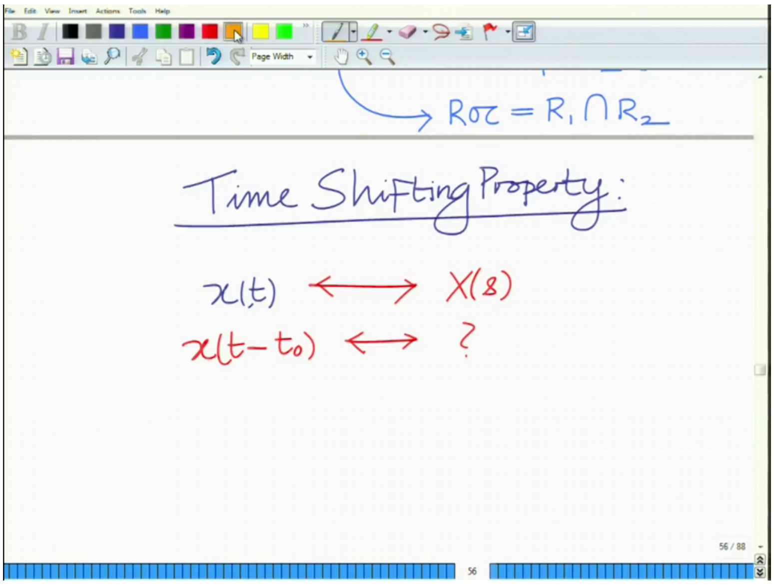

That is something important to keep in mind. Now, time shifting property, let us look at another important property which is the time shifting property. What happens when a signal is the time shifting property of the Laplace transform that is if Laplace transform of x t that is if x t has Laplace transform X s then, what can you say about the Laplace transform of x t minus t naught?

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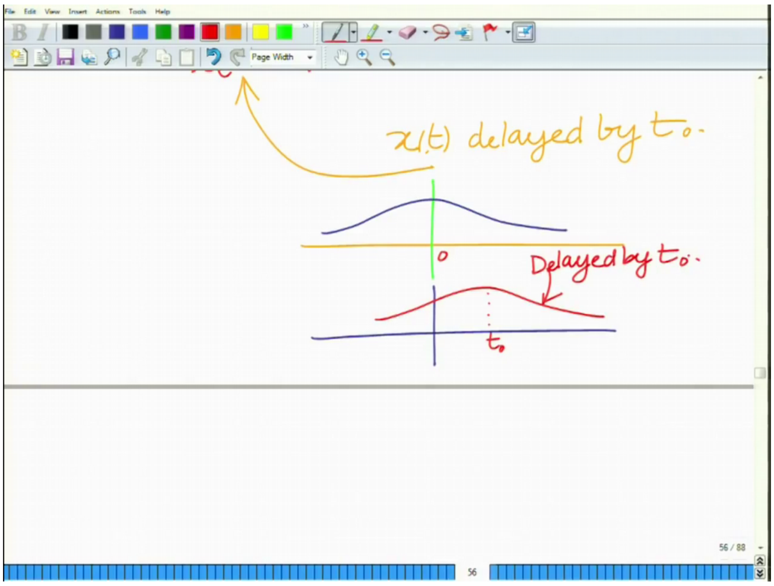

That is x t minus t naught is x t. Remember, this is x t delayed by this is a delayed version of x t, that is, if you have x t which is something like this, which is a signal that looks something like if x t is this signal this point is 0. If this is the signal x t then x t delayed by t naught will look something like this, that is, at this point is t naught.

So, this is shifted to the right by t naught.

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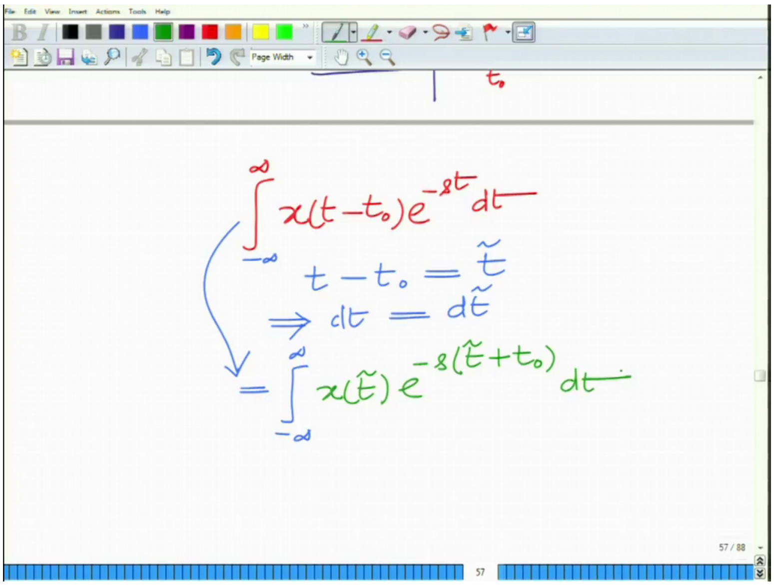

Now, what is a Laplace transform of the delayed signal that is x t minus t naught? Well, for that I can perform the integral x t minus t naught e raise to minus st dt, which is equal to, now I can set t minus t naught equals t tilde which implies dt equals dt tilde. This will become this integral will become equivalently integral 0 to well, t minus t naught or this integral is from minus infinity to infinity. This will also be from minus infinity to infinity this integral will be x of I can write it as in terms of t tilde x t tilde e raise to minus s t tilde plus t naught dt.

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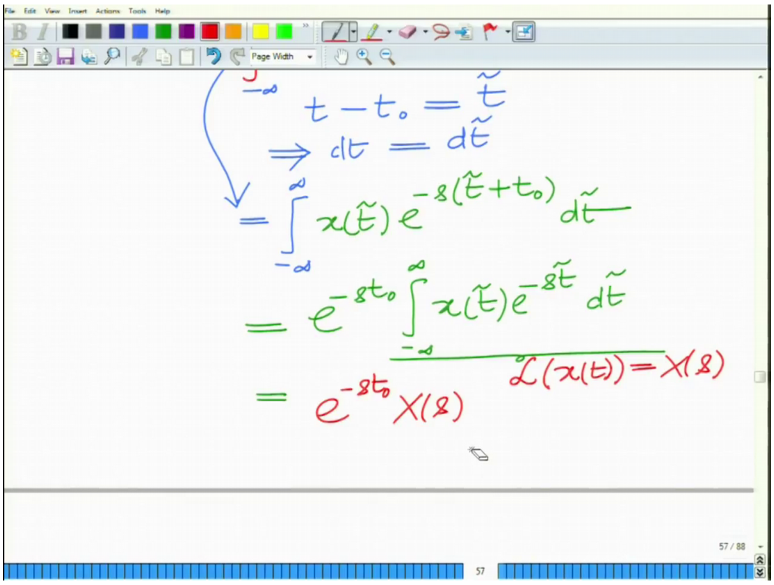

Which is, now I can bring out the e raised to minus st naught outside integral minus infinity to infinity x of t tilde e raised to minus st tilde d tilde and this we know, this is basically the Laplace transform.

This is the Laplace transform of x t which is equal to X s. This is equal to e raise to minus st naught times x t. Basically, the Laplace transform of the delayed signal x t minus t naught is e raised to minus st naught X s and you can see the region of convergence will be the same because we are directly using the Laplace transform x S. Whenever x s exists, e raise to minus st naught X s exists as long as s is a finite quantity.

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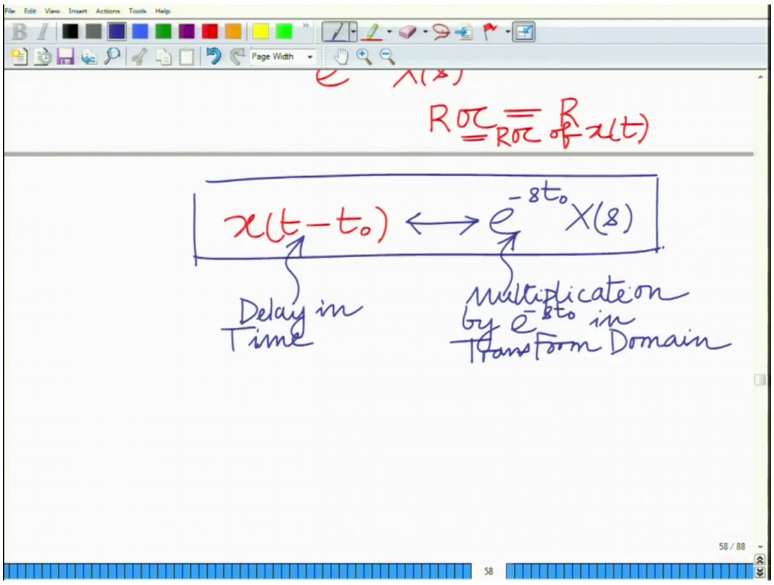

Basically, the region of convergence will be x A, region of convergence of Laplace transform of F t x t that is the region of convergence of x t or the Laplace transform of x t and therefore, we can say that x t minus t naught equals or not equals that is the Laplace transform of x t minus t naught is e power minus s t naught into, basically a delay in time domain, when we delay signal in time domain is equal to multiplication by e raise to minus st naught in the Laplace domain or in the s domain. Delay and this is an important property R raises to minus st naught in the transform domain. x t minus t naught has the Laplace transform, e raise to minus st naught X s.

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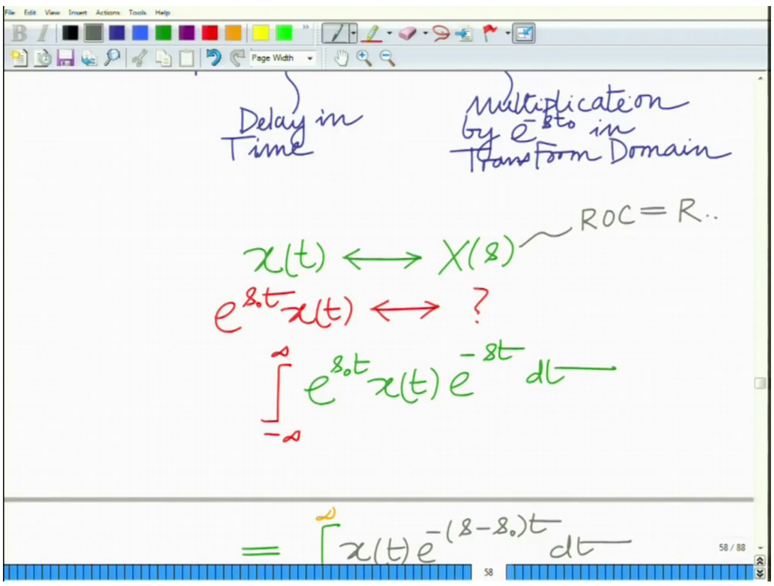

Now, let us look at another, what happens when you multiply in the time domain that is if you have x t, let us look at the other scenario or when you have x t whose Laplace transform is X s if you multiply this by e raise to s naught t, that is, if you multiply this by an exponential signal then what is the Laplace transform and to find this Laplace transform we can perform minus integral my integral minus infinity to infinity e raised to s naught t x of t e raised to minus st dt.

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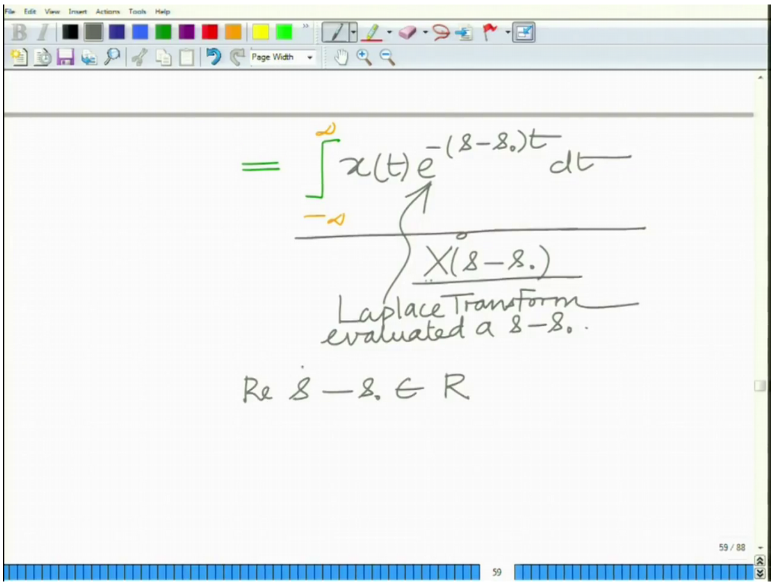

Which I can write as integral minus infinity to infinity x t e raised to minus s minus s naught t dt and if we look at this is the Laplace transform evaluated at s minus s naught. This is X of s minus s naught because look at this x of integral x of t e raised to minus s minus s naught t dt. This is a Laplace transform evaluated at s minus s naught that is X of s minus s naught. This is Laplace transform evaluated at s minus s naught; however, for the region of convergence now, look at this convergence if s minus s naught or real part of s correct real part of s let us put it this way this converges if this has ROC equals R.

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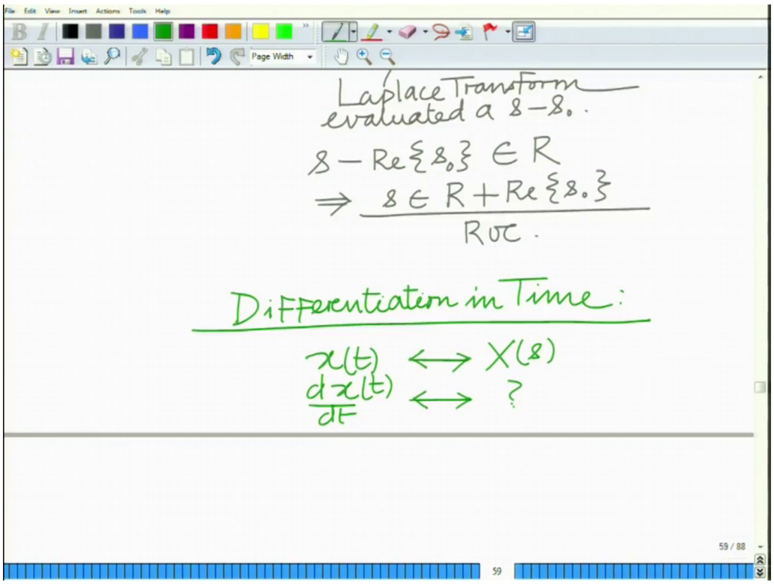

Then, we need for this to converge we need s minus or the real part of s minus or s minus real part of s naught belongs to R which implies s belongs to R plus real part of s naught. This converges if s minus the real part of s naught belongs to the region of convergence of x t therefore, the region of convergence of R raise to s naught t x t will be real part, that is, the region of convergence of x t plus the real part of s. This is the region of convergence. The previous region of convergence of x t plus real part of s naught, that is, the region of convergence of x t shifted by the real part of x, let us look at another property that is differentiation in time that is if x t has region of has X s that is x Laplace transform X s what can we say about d x t dt, what is the Laplace transform of this?

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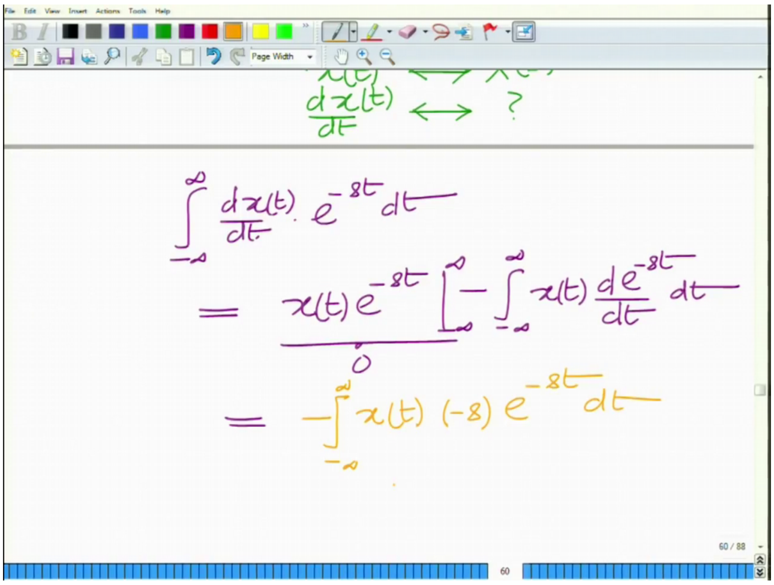

To evaluate the Laplace transform of this we have d x t or dt e raise to minus st dt which is now if you apply integration by parts integrate d x t or dt that gives x t multiplied by e raise to minus st x t minus integral 0 to infinity, I am sorry, or integral minus infinity to infinity of course, this has to be evaluated between the limits 0 to infinity minus integral 0 to infinity x of t in integration by parts I have to differentiate de raise to minus st or dt dt.

Now, if you look at this x t e raised to minus st evaluated between the limits, I am sorry, minus infinity to infinity we can assume that the signal x t such that x t evaluated at e raised to minus st x t evaluated from minus infinity to infinity that is x t into e raised to minus st evaluated at both infinity and minus infinity is 0. For instance, if x t is a signal is right signal that exists only for t greater than 0, then x t evaluated at x t raised to minus st evaluated equal to minus infinity is 0 and x t similarly x t into e raised to minus s t. Since, e raised to minus st is decreasing.

We can assume that x t into e raised to minus st evaluated at t equal to infinity is also 0, term is 0 and we are left with the second term. That gives us integral minus infinity to infinity x t derivative of e raised to minus st dt, which is x t, derivative of e raised to minus s t is basically minus s, e raised to minus to minus st dt.

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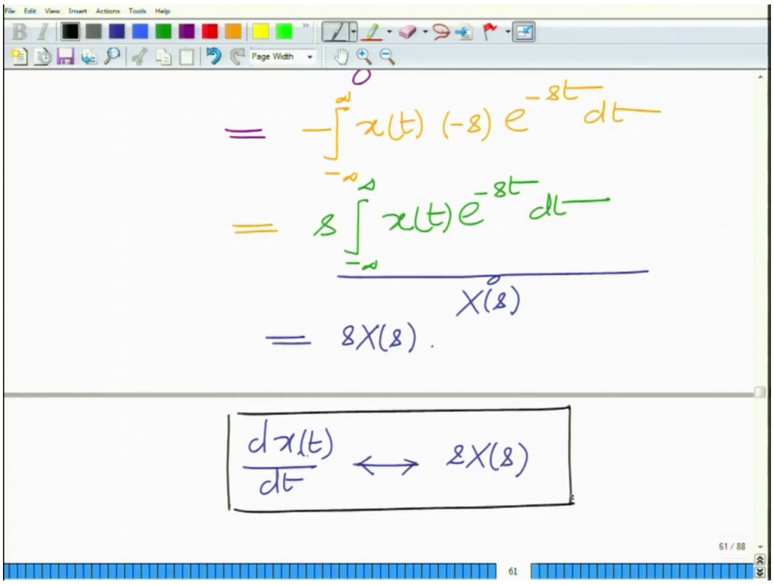

Which are basically s times minus infinity to infinity x t e raised to minus st dt and you can see this is the Laplace transform simply of x t. This is s times x of s and the region of convergence will be the same because X s exists when or the Laplace transform of x takes this or region of convergence is the same as that of the region of convergence of x t.

Therefore, the derivative has the Laplace transform, if x t is Laplace transform capital X s the derivative as Laplace transform X s X s.

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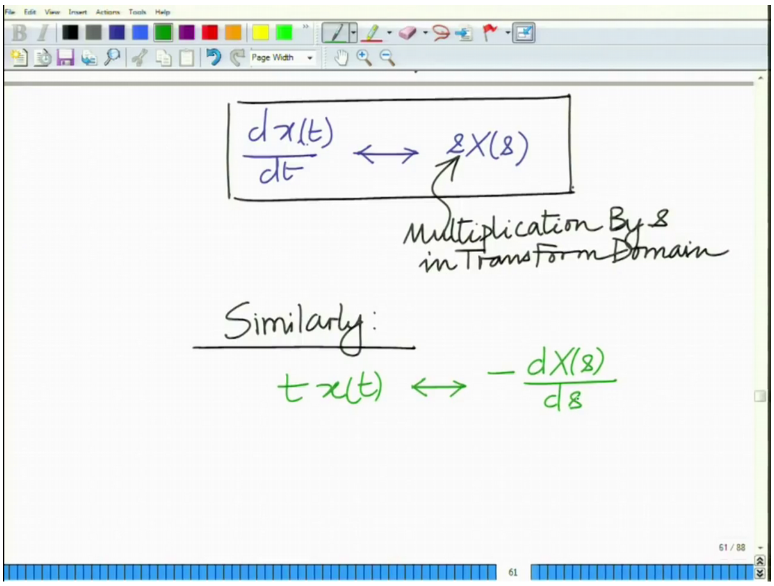

The differentiation in time multiplication by s in the transform domain this is equivalent to multiplication by s in the transform domain ROC equal to R which is equal to ROC of the same as the ROC of x t. Similarly, you can show that similarly you can explore other properties there are several properties of the Laplace transform. You can exploit leave some of these for you to explore you can easily see that t x of t has Laplace transform minus d X s over ds, that is, if you multiply by t in the time domain take a signal x t multiplied by t the corresponding Laplace transform is minus d X s over ds, where X s is the Laplace transform of x t.

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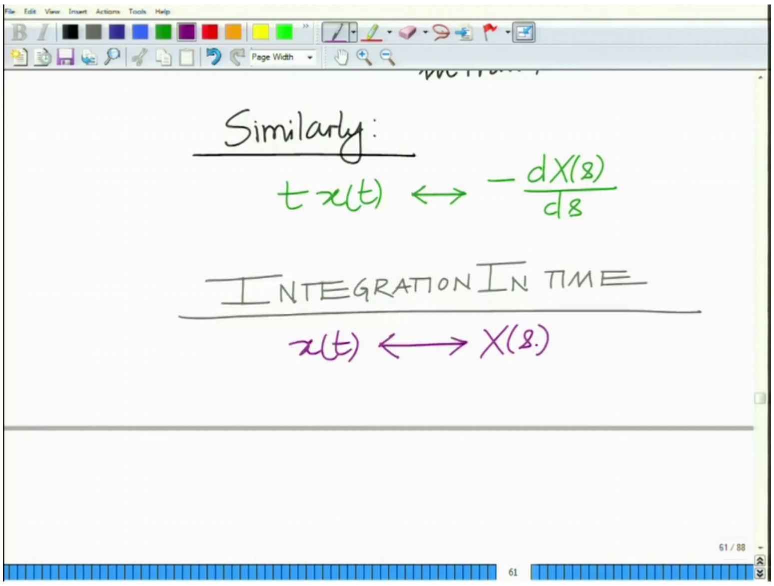

Let us look at one last property which is that of the integration, that is, integration in time. Let us consider an integrator; for instance, an integrator circuit which performs integration in time and integration you can show that it is a low pass filter. Basically, integration in time or basically output of an integrator circuit, that is, let us consider x t which has Laplace transform X s.

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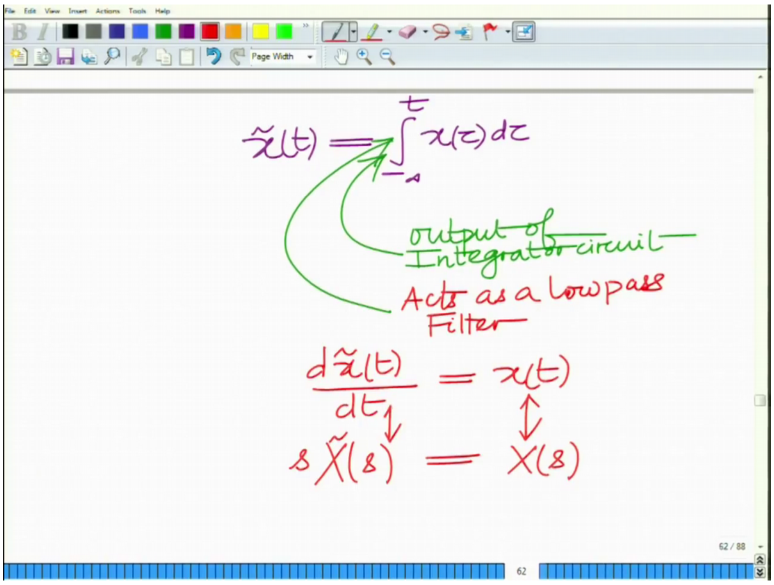

Then, if you pass it through an integrator the output is x tilde t equals minus infinity to t x of tau d. tau, this is basically the output of an integrator circuit and you can also show that acts as a low pass filter this acts as a low pass filter.

This acts as a low pass filter and you can see that well dx tilde t dt is equal to x t, that is, if I differentiate x tilde t, I get x t, naturally integration of x t is x tilde t. If I differentiate x tilde t, I get x t, that is, differentiate the output of an integrator circuit I get back the original signal and if x tilde t has a Laplace transform X tilde s. Now, from the previous property the derivative of x tilde t has Laplace transform s X tilde s, which basically must be equal to the Laplace transform of X of s, that is, taking Laplace transform on both sides because x t as Laplace transform X s d x tilde dt must have Laplace transform s X tilde s. Where, X tilde s is a Laplace transform of x tilde t which means X tilde s has Laplace transform X s over s and the ROC will be the same.

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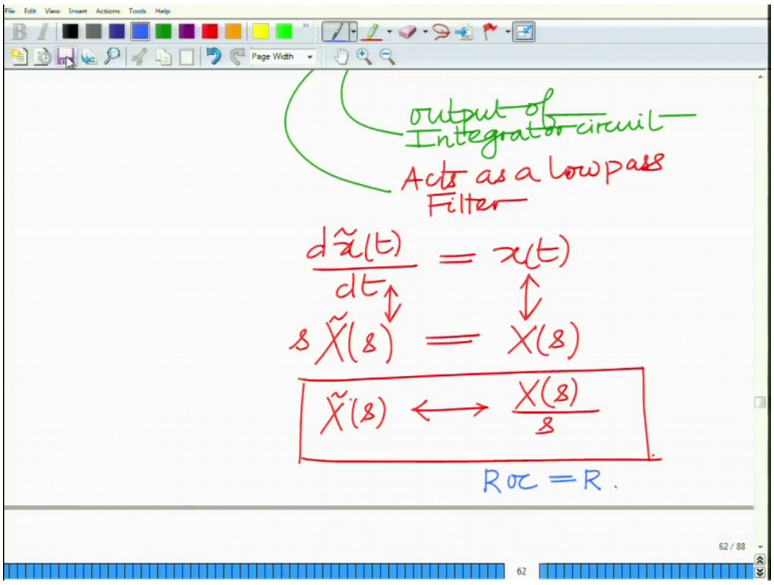

Using the differentiation property we have derived the Laplace transform of this and the ROC will be the same. Basically, that wraps up some of these interesting properties we have looked at the Laplace transform of some signals, such as the cosine signal and some interesting properties. Important properties, such as what happens when you delay a signal, what happens to Laplace transform, what happens to when you take a linear combination of signals and finally, when you differentiate a signal and when you integrate a signal, what happens to the Laplace transform. There is one other important property of the Laplace transform which is the convolution what happens to the Laplace transform of the output of a convolution of 2 signals we look at that property in the subsequent modules, stop here.

Thank you very much.