Principles of Signals and Systems
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Lecture – 17
Laplace Transform – Definition, Region of Convergence (ROC), LT of Unit Impulse and Step Functions

Hello, welcome to another module in this massive open online course. So, in this module we are going to start on a new concept, a very important concept. In fact, in analysis of signals and systems known as the Laplace transform.

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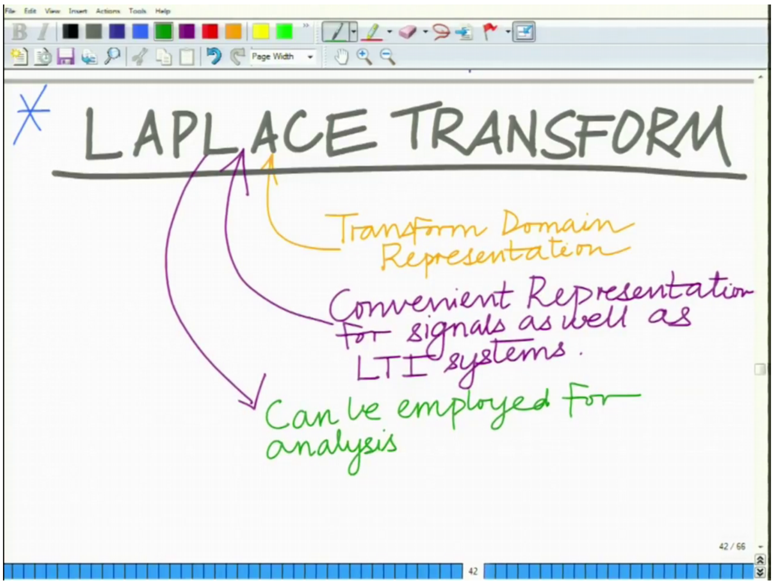

So, we are going to start, looking at various aspects of the Laplace, the various aspects of the Laplace transform and in fact, this is going to be on the first transforms amongst the many transforms that we are going to be looking at. And basically this gives us what is known as a transform domain representation. This gives us our transform domain representation of the system and this is convenient, a convenient representation of for signals as well as LTI systems.

It gives us a convenient representation for signals as well as for LTI systems, and this can be employed for the analysis, and to obtain also it can be employed for analysis, and also to obtain valuable insights regarding the performance.

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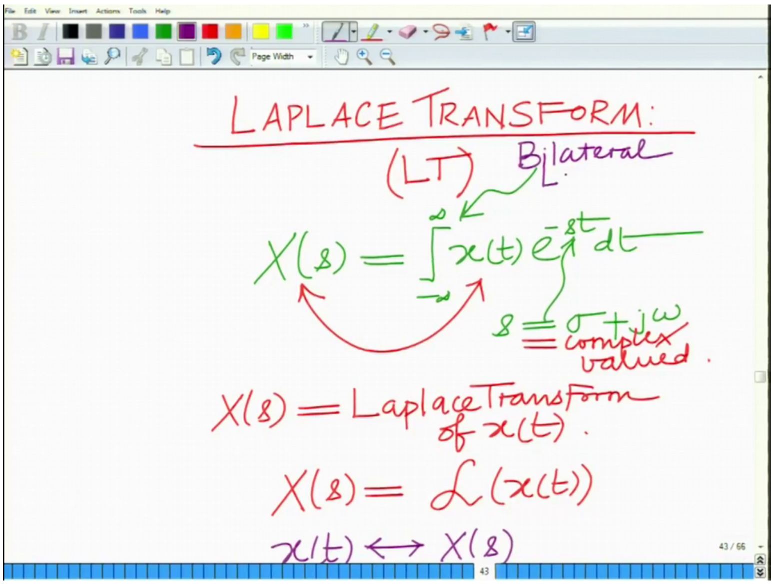

And behavior of LTI signals LTI systems for various signals. So, this can be employed for analysis for a performance analysis, for analysis as well as and to obtain valuable insights into behavior of signals and system. So, it can be used to for analysis and to obtain insights into the behavior of signals and systems.

So, let us look at the definition of the Laplace transform, which for convenience we can abbreviate as L T. So, the l t is defined as excess for a signal, x t is defined as minus infinity to infinity x t e raise to power minus x t. Naturally this is a function of s, where this s equals sigma plus j omega, this is a complex value. So, s is complex value. So, x s is the Laplace transform of x t x s an x t from a signal in Laplace transform pair. So, x s equals x s equals a Laplace transform of x t, and this is also represented by x s

The Laplace transform of x t; that is the calligraphic l denotes the Laplace transform or that x t and x s form a signal and Laplace transform pair. This indicates that they form a this double sided arrow indicates that they form a signal slash Laplace transform pair ok.

So, the Laplace transform. So, this is the definition of the Laplace transform; that is x s equals integral minus infinity to infinity x t e raise to minus stdt. And the signal xt x s is the Laplace transform of the signal xt and the signal, and these form a signal and Laplace transform pair, and this is also known as the bilateral Laplace transform, because it uses an integral from minus infinity to infinity. So, this is also known as a to be more precise, and without ambiguity to avoid any ambiguity. When we say Laplace transform, we will simply mean the bilateral Laplace transform; that is integral from minus infinity to infinity.

Now, an important concept associated with the Laplace transform, is what is termed as the ROC or region of the, region of convergence.

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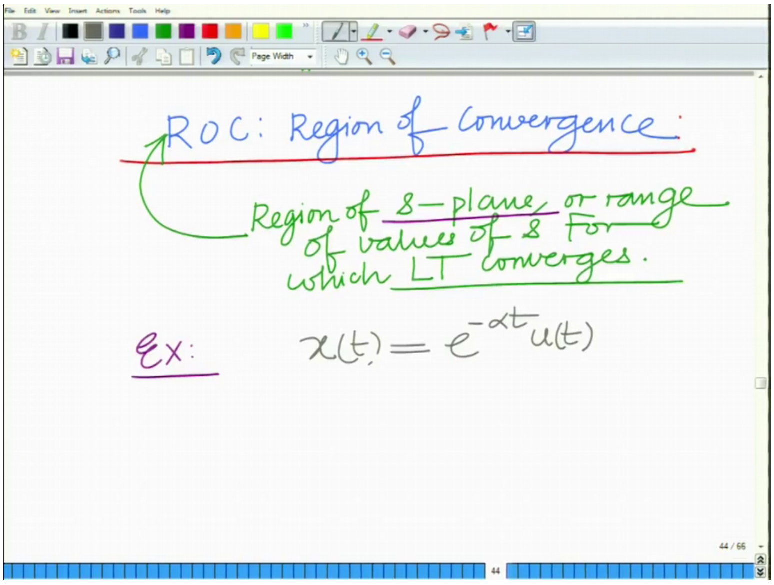

Now, what is this concept of ROC. This simply describes this simply describes region of the s plane or basically range of values of s s, for which the Laplace transform exists or the integral for which the Laplace transform exists or Laplace transform integral converges. So, this has to converge correct. For instance let us understand this. So, this basically describes a range in the, or a region in the s plane which I am going to show you, which I am going to define and demonstrate shortly that is it is a region in the s plane, or basically this characterizes the range of values of h, for which the Laplace transform for a given signal xt converges ok.

Let us examine this using an example. For instance let us say my signal is xt equals e power minus alpha t u t and x s, then becomes e power minus alpha. Let us further assume that alpha is real, alpha is a real quantity. So, this is e power minus alpha t u t e power minus stdt, because u t is non 0 only for t greater than or equal to 0.

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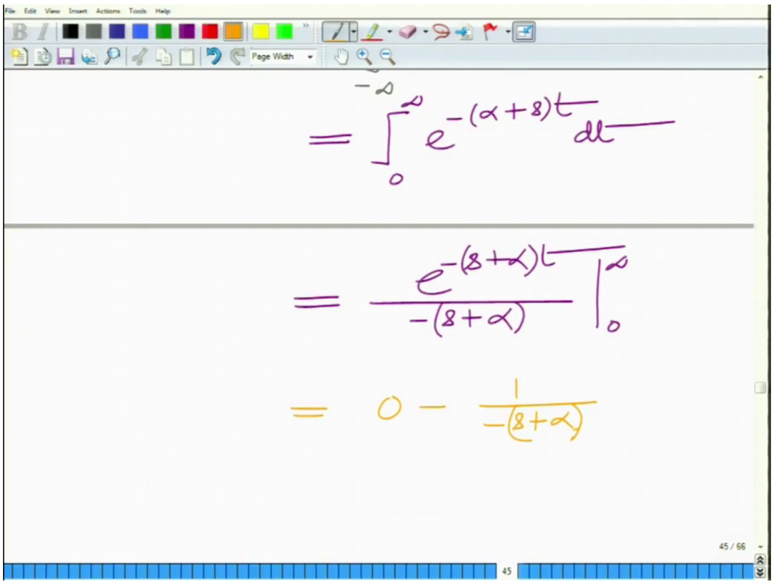

So, this integral can be written equivalently as e raised from 0 to infinity e raised to minus alpha s plus t d t which becomes e raised to minus s plus alpha t over minus s plus alpha between 0 to infinity.

Now, look at this. We have to evaluate e raise to minus s plus alpha t, as t tends to infinity. Now this becomes 0, but only under the condition s plus alpha is greater than 0. So, e raised to minus s plus alpha t at t as t tends to infinity is 0 decays to 0, only if s plus alpha is greater than 0. So, this is equal to. We can write the 0 minus. Of course, minus, this minus 1 over s plus alpha 1 over at equal to 0. This is 1 minus s plus alpha.

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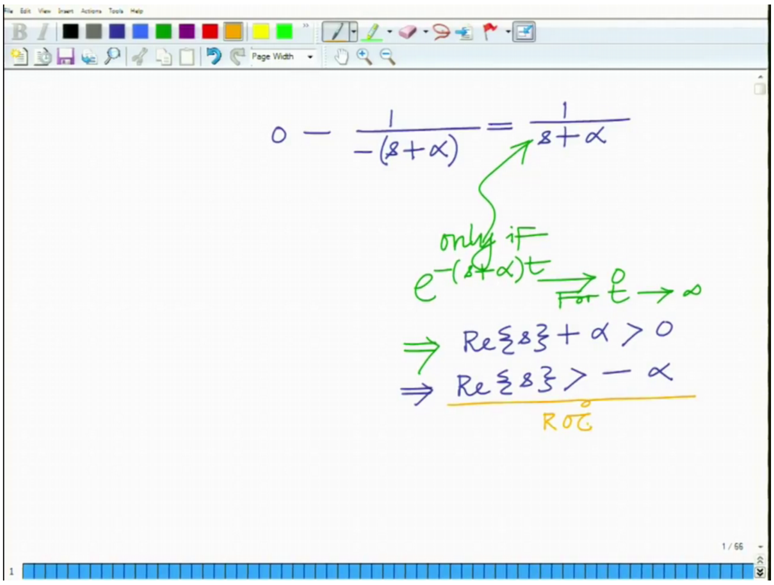

So, this converges to 1 over s plus alpha only if e raised 2 minus s plus alpha times t tends to 0. For t tends to infinity this implies, which implies that we need the real part of s plus alpha must be greater than 0, which implies, which in turn implies that the real part of s must be greater than minus alpha. And therefore, this is the region of convergence. This is the ROC ok.

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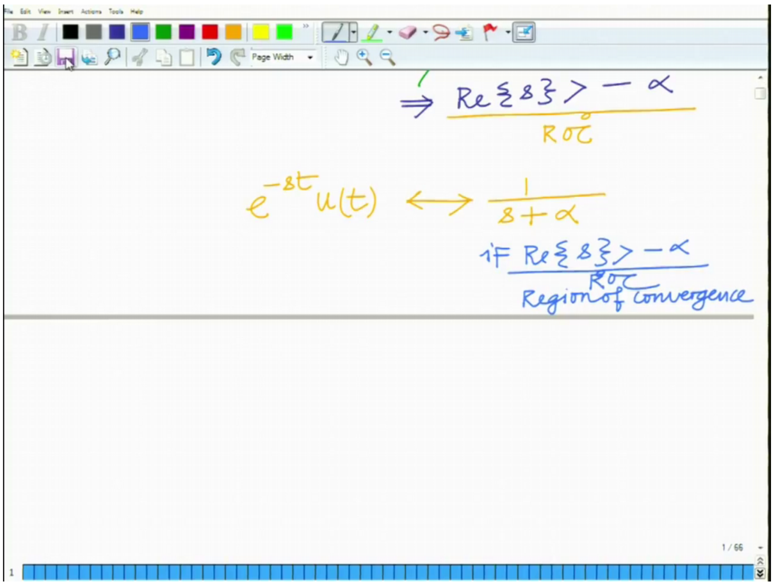

And therefore, the Laplace transform of e raised to minus s t u t is given as 1 over s plus alpha, but this exists only if real part of s greater than minus alpha, and this is therefore, the ROC or the region of convergence. This is your ROC or the region of convergence ok.

And this can be represented. This ROC can be represented in the s plane as follows. So, in the s plane this can be represented s plane representation of the roc. So, the ROC can be represented in the s plane as the follows.

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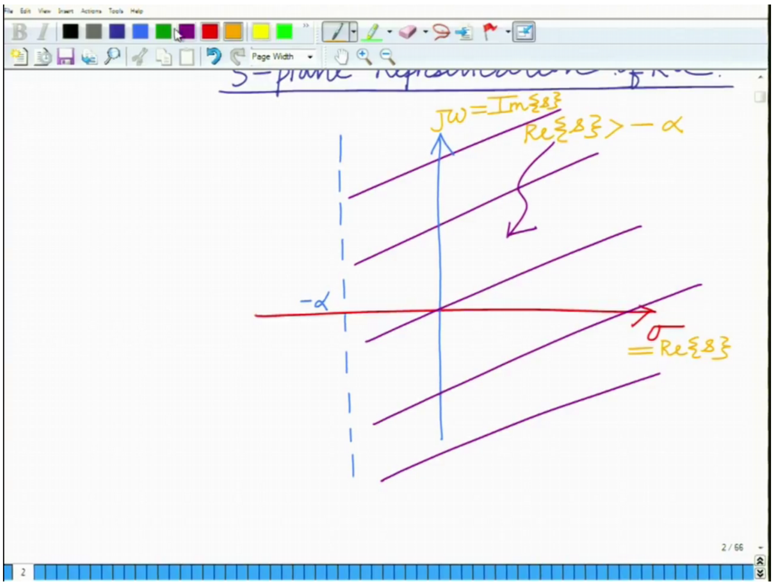

So, the s plane simply contains the real part; that is sigma on the x axis, and the imaginary part that is j omega on the y axis and we have, let us say this is minus alpha, considering alpha to be greater than 0. This is basically your minus alpha and the ROC is basically this. Entire half of the s plane which is basically real part of s without and it does not remember, it does not include the line corresponding to minus alpha, because real part of s is strictly greater than minus alpha.

So, this is real part of s greater than minus alpha. So, this is your j omega axis j omega axis, and real part of s greater than minus. So, this sigma, this is the real part of s, this is the imaginary part of s, and this is basically real part of s greater than my minus alpha, this is the roc. So, this is the ROC representation for. So, this is your ROC s plane representation for e raise to minus alpha t u t. This is the roc, and this is basically the ROC or the region of convergence for the signal above.

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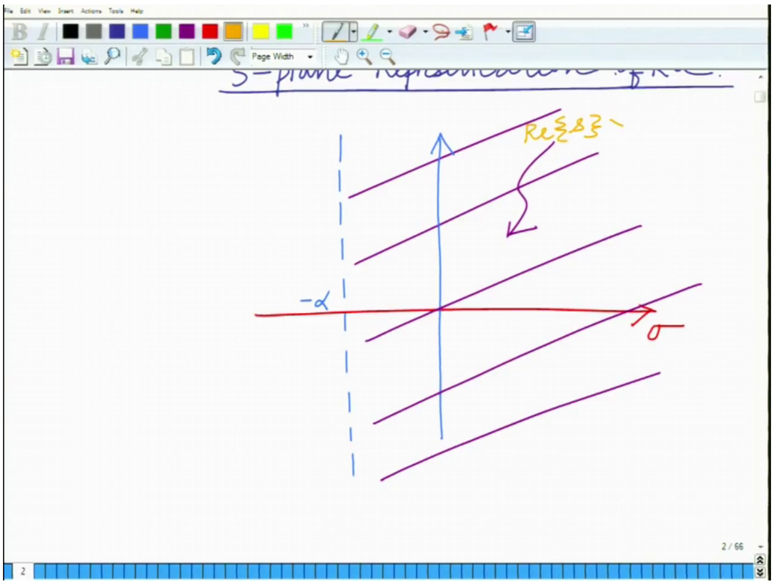

Now, let us look at something slightly different. Let us consider a slightly different signal which is consider now x t equals minus e power minus alpha t u minus t. This is different because.

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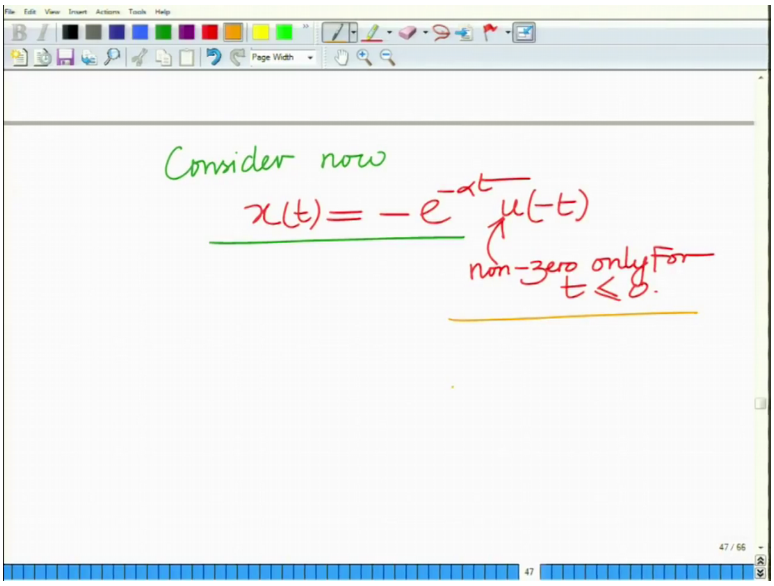

So, non 0 if t less than equal to 0 ok because u minus t is non0, only for t less than or equal to 0, x t equals e raise to minus e raise to minus alpha t u of minus t. This is non0 only for t less than or equal to 0 now. So, this signal. So, while the previous signal was e power minus alpha t u t. this is minus e power e raised to minus alpha t u minus t which exists on the left side alright. So, this is left handed signal that exists only for t less than or equal to 0 ok.

Now, if you look at the Laplace transform of this signal.

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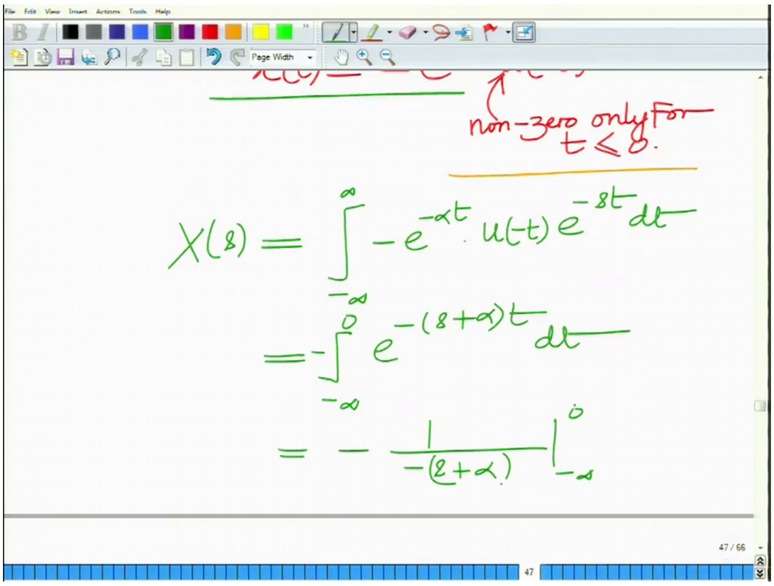

Once again we perform minus infinity to infinity minus e power minus alpha t u minus t e power minus s t d t. Now note that this can be equivalently written as. Now since this is non 0 only for t less than equal to 0. So, this can be equivalent written as minus infinity to 0 minus e power minus, once again s plus alpha t d t and the integral is minus 1 over minus s plus alpha evaluated between the limits. This time it is minus infinity to 0 which is basically. Now I am sorry I have to also write e power minus s plus alpha t e plus e raise to minus s plus alpha t.

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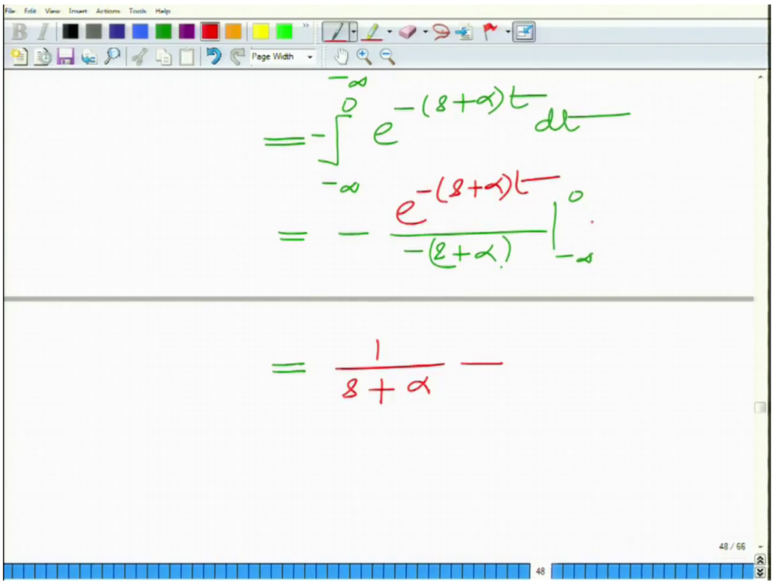

Now, e raise to minus s plus alpha t evaluated at 0 is 1. So, this becomes 1 over s plus alpha minus. Now e raised to minus s plus alpha t as t tends to minus infinity is 0.

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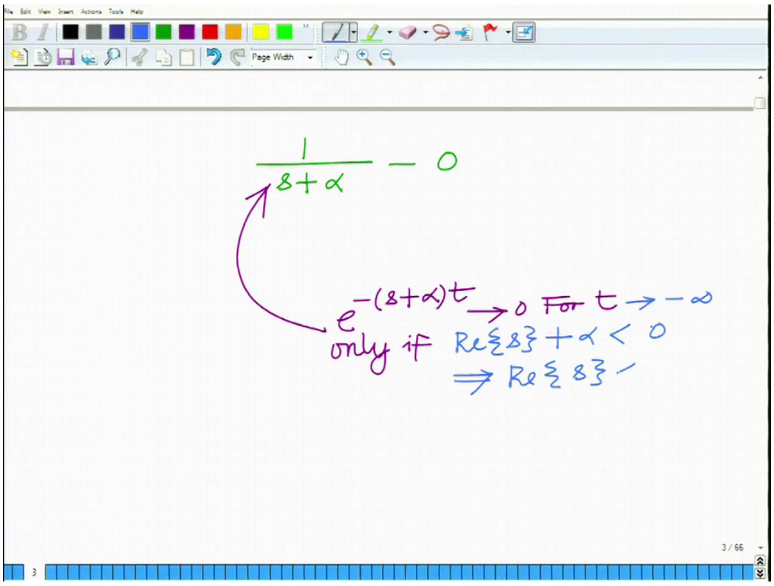

Now, this converges that is e raise to minus s plus alpha t tends to 0 for t less than 0. Only if only if the real part of s plus alpha is for t tends to minus infinity. This tends to 0 for t tends to minus infinity for t tends to minus infinity only if real part of s plus alpha is less than 0, implies real part of s is less than minus real part of s is less than minus infinity alright. I am sorry real part of s is less than minus alpha and therefore, this is your roc. And therefore, the Laplace transform in this case of e raised to or minus e raise to minus alpha t u of minus t.

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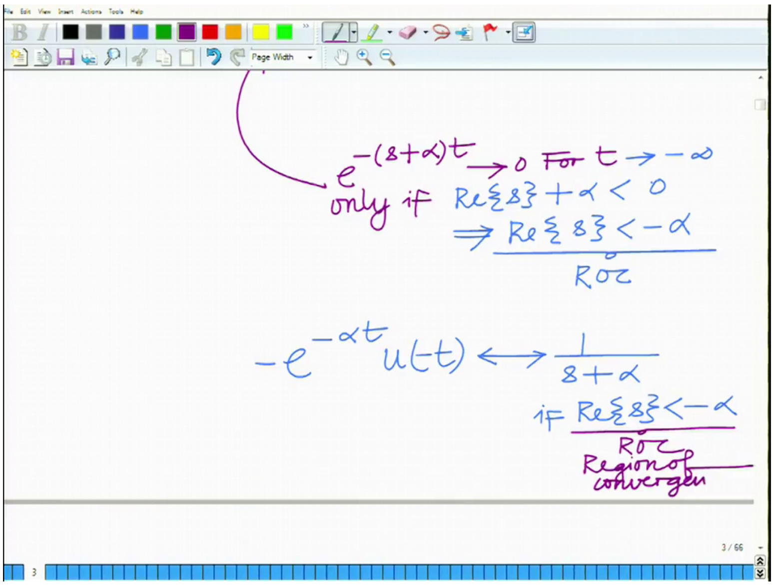

This has the Laplace transform 1 over s plus alpha if real part of f s less than minus alpha, and this is therefore, the ROC or the region of convergence.

And this can once again be represented in the s plane as follows. You have the real axis.

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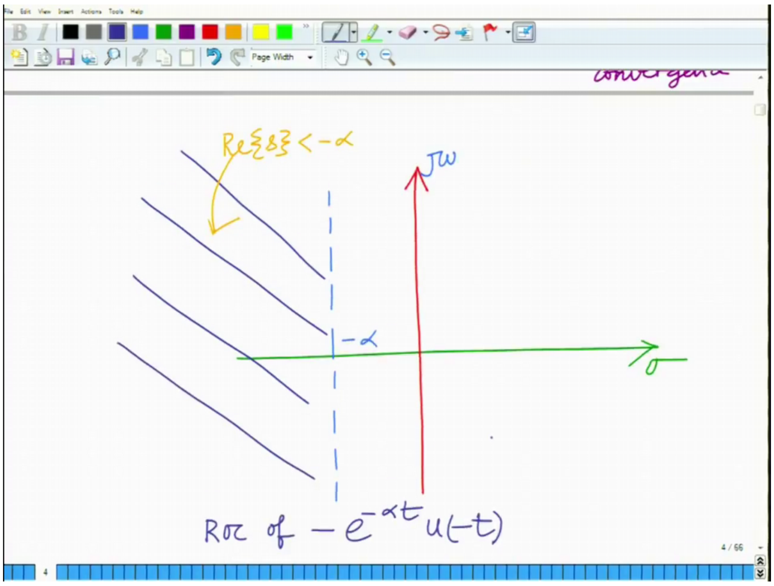

You have the imaginary axis. And now the ROC is the real part of s is less than minus alpha. So, which means you will have 2 ROC in the entire s plane that is less than minus alpha, without including the line that is minus alpha. So, this will be the ROC that is real part of s less that minus alpha and therefore, what you have is, sigma. This is the g omega axis and this is the s plane presentation of the roc.

So, this is the ROC of minus e raise to minus alpha t times u of minus t and. Now, therefore, what you observe is both the signals both e raise to minus alpha t u t and minus e raise to minus alpha t u minus t, both have the same Laplace transform.

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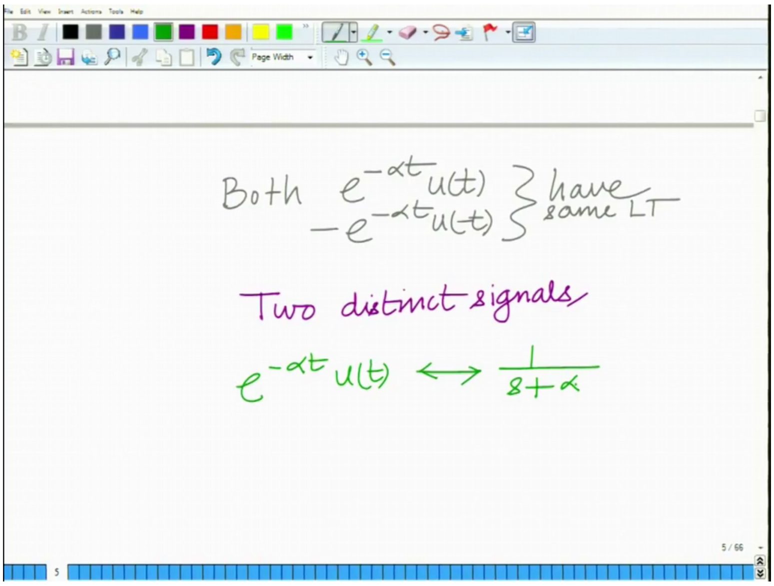

In fact, if you look at this. These are two distinct signals correct. The two distinct signals you have e raise to minus alpha t u t has the Laplace transform 1 over s plus alpha, and e raise to minus e raise to minus alpha t u minus t also has the Laplace transform 1 over s plus alpha, but notice that the ROC is of both are different.

So, the ROC of the first is real part of s greater than minus alpha and the ROC of the second is real part of s less than minus alpha.

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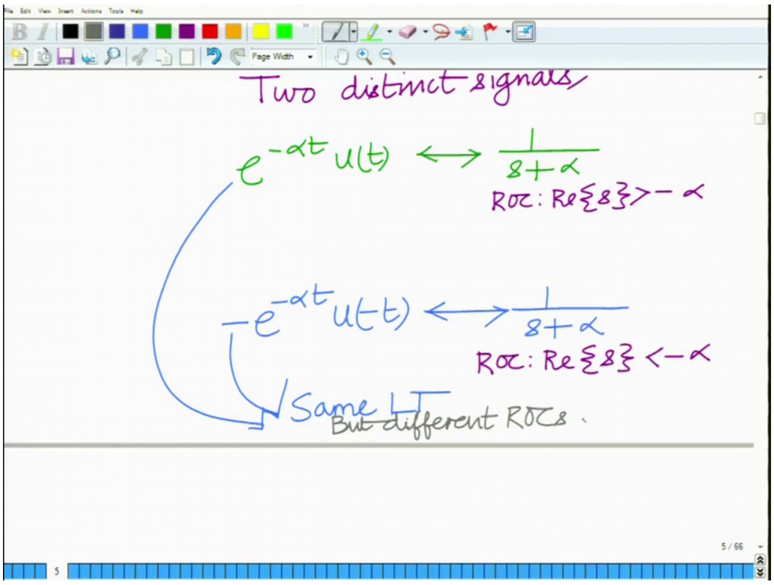

And therefore,. So, they have the same Laplace transform, but different roc. So, same Laplace transform, but different. So, these are the same Laplace transform, but different rocs alright, and therefore, for the Laplace transform to be unique, the ROC must also be specified as part of the Laplace transform.

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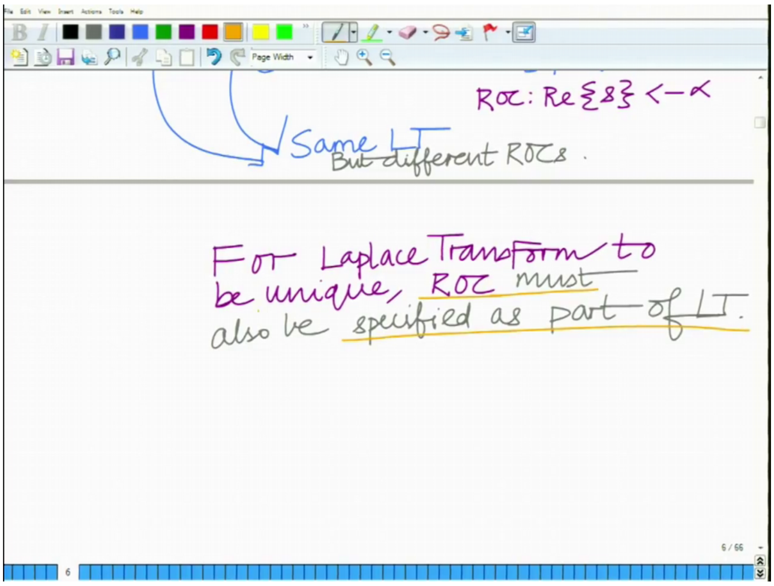

And this is important to distinguish not enough, to just mention the expression for the LT Laplace transform to be unique or the inverse Laplace transform ok.

So, for the Laplace transform or to determine the signal from the Laplace transform uniquely, correct for the Laplace transform to be unique ROC must also be specified as part of the must also be . So, for the Laplace transform to be unique, the ROC must also be specified, must be specified as part of roc, must be specified as part of the Laplace transform. Let us now proceed to compute the Laplace transform of some popular signals, and when you look at. Come to the popular signals one of the most important signals, the two of the most important signals for. Of course, for the analysis or to understand the behavior of LTI systems are of course, the impulse unit impulse and the unit step function ok.

So, let us look at some of the Laplace transforms. The Laplace transforms.

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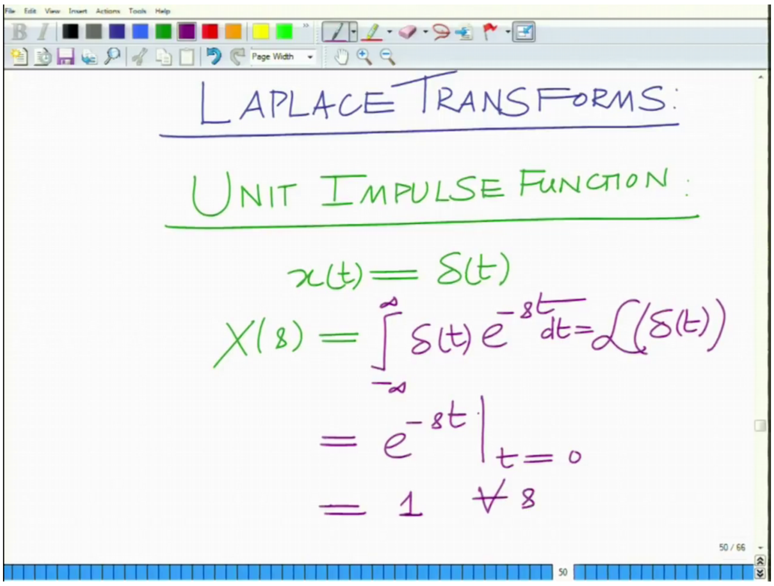

Now, the unit impulse function. So, let us look at the first one the unit impulse function. The Laplace transform of this; of course, we have x t equals delta t and x s equals well integral minus infinity to infinity delta t e raise to minus s t. This is integral dt equals Laplace transform of del t, and this is simply e raise to minus s t evaluated. Remember integral delta t integral minus infinity to infinity delta t e raise to minus s t d t is for any function. The function evaluated to equal to 0. So, this is e raised to minus s t evaluated at t equal to 0.

So, this is equal to 1 for all s. So, there is the region of convergence is all s.

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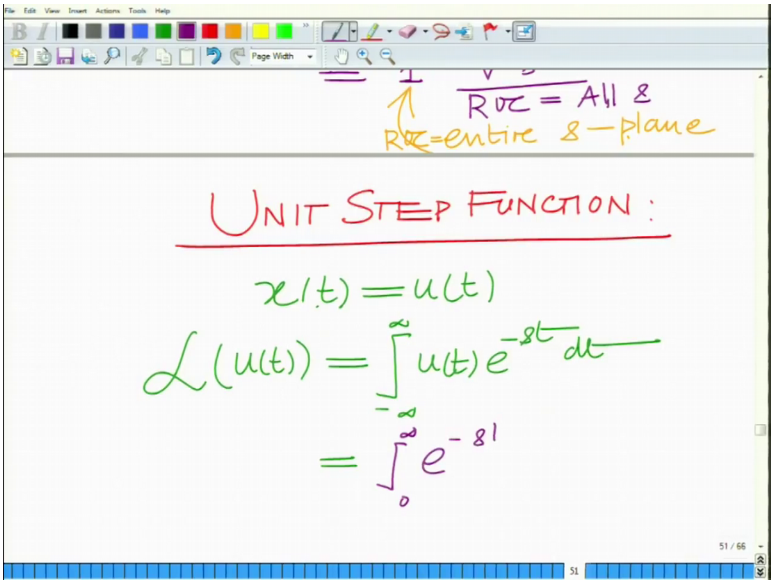

In this case ROC equal to minus infinity; that is equal to all s that is all values of s in the imaginary plane, that is real part sigma minus infinity less than sigma less than infinity and minus infinity less than omega less than. It covers entire s plane in the region of convergence for this impulse function. So, this is entire ROS or entire roc.

So, this is the entire or sorry and the ROC is the entire s plane that is what I mean to say ROC equals the entire s plane for the image. Now let us look at the unit step function now for the unit step function x of t equal to u t and the Laplace transform of u t equals minus infinity to infinity u of t e power minus s t d t, which is of course, since u t is only non0 for t greater than equal to 0. So, this is integral 0 to infinity e power minus stdt and u t equal to 1 for t greater than equal to 0 which is, well e power minus s t divided by minus s evaluated between the limits 0 and infinity.

Now, e power minus s t as t tends to infinity tends to 0, only if s is greater than 0. So, or real part of s is greater than 0.

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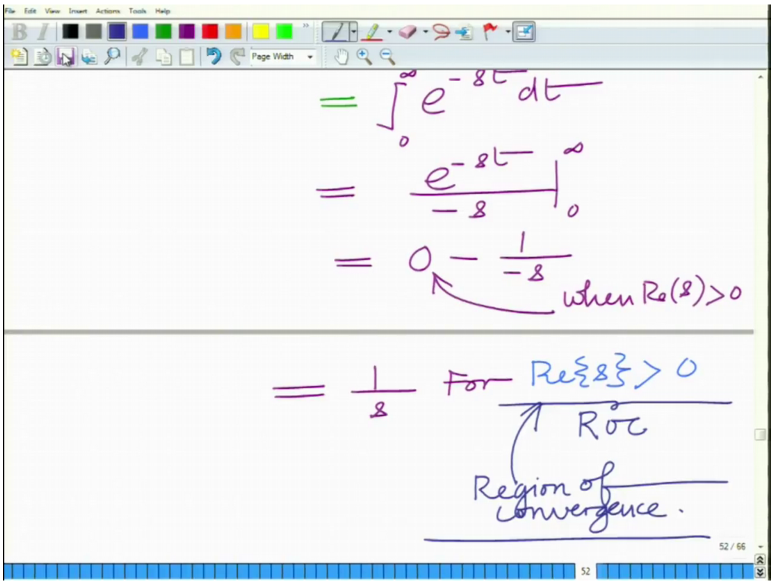

So, this is 0 minus, and at 0 this is e power minus s t is 1. So, this is 1 divided by minus s, but this is 0 only if when real part of s greater than 0, and therefore, this is equal to 1 over s for real part of s greater than 0, and this is basically the roc. This is basically the ROC of this. So, this is basically the region of convergence for ROC for unit stuff for.

So, this is the LT. In fact, we cannot say this is LT, because LT is the complete thing, its the expression plus the roc. So, this is basically you are ROC or also just to write it once more. this is the region of convergence. So, in this module we have basically looked at an important concept, there is a definition of the Laplace transform, which gives us the transform domain representation of several signals, which is very useful to understand the behavior and properties of signals as well as systems.

We have looked at the definition and we looked at examples for the Laplace transform, and realized that the region, its also in addition to the transform for a given signal. It is also important to specify the region of convergence to uniquely determine the signal given the Laplace transform. So, we will stop here and continue in the subsequent modules.

Thank you very much.