Principles of Signals and Systems
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Lecture – 16
Example Problems Discrete time LTI Systems – Output of System, Causality, Stability

Hello, welcome to another module in this Massive Open Online Course. So, we are looking at example problems for the analysis of LTI systems, let us continue looking at these problems.

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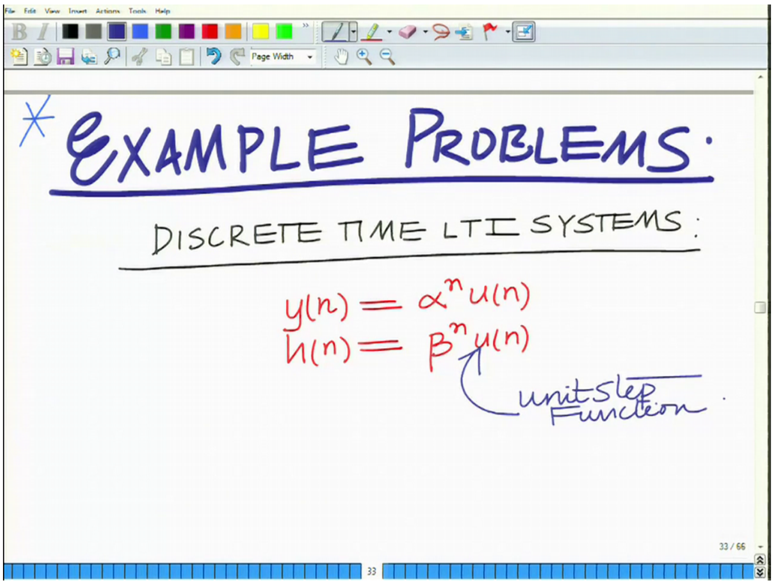

Let us look at a few more problems for LTI systems, so the analysis of LTI systems. So now, let us consider discrete time LTI systems. So, so far we have looked at problem solving for continuous time LTI systems; let us now look at Discrete Time. So, in discrete time systems let us say we have an input y n equals alpha n u n, and we have an impulse response h n that is given as beta n times u n; where u n is the unit step function or the unit step signal.

Now we would like to find the discrete time convolution.

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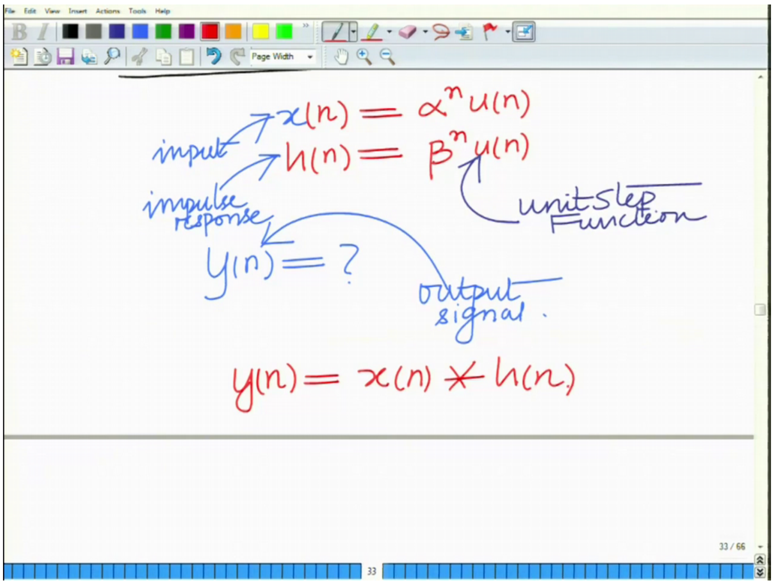

That is we want to find the output. So, we are given I am sorry, this is x n. So, what is the output y n? So, this h n is the impulse response of the discrete time system, x n is the input signal. So, what is the output signal y n for the system?

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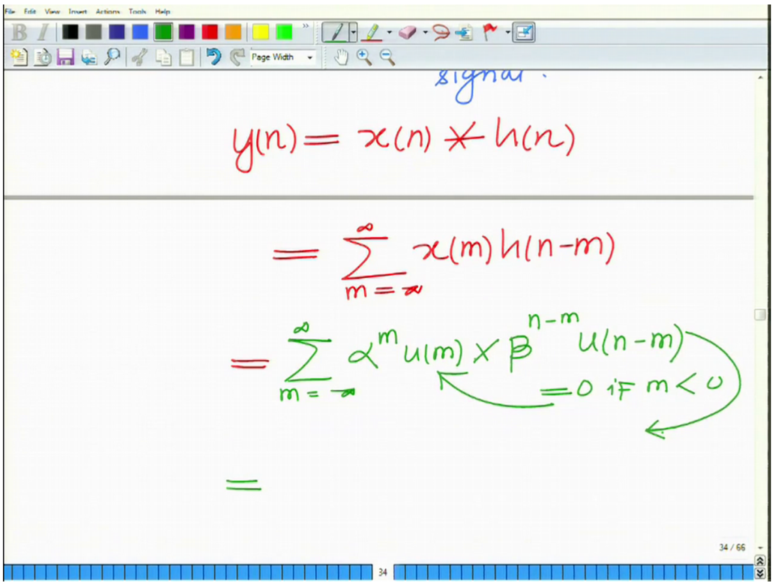

And we know that y n is given by x n convolved with h n which can be expressed as the discrete sum m equal to minus infinity to infinity x m times h n minus m, which can be represented as summation m equal to minus infinity to infinity substituting the expression for x m we have alpha to the power of m x m is alpha to the power of m u m times h n minus m is beta to the power of n minus m u n minus m u n minus m; which now u m recall that u m is greater than equal to 0 if m greater than equal to 0 or u m a is equal to 1 this is equal to 0 if m less than 0.

And u n minus m similarly, this is u of n minus m is equal to 0 if n minus m is less than 0 or basically m is greater than m.

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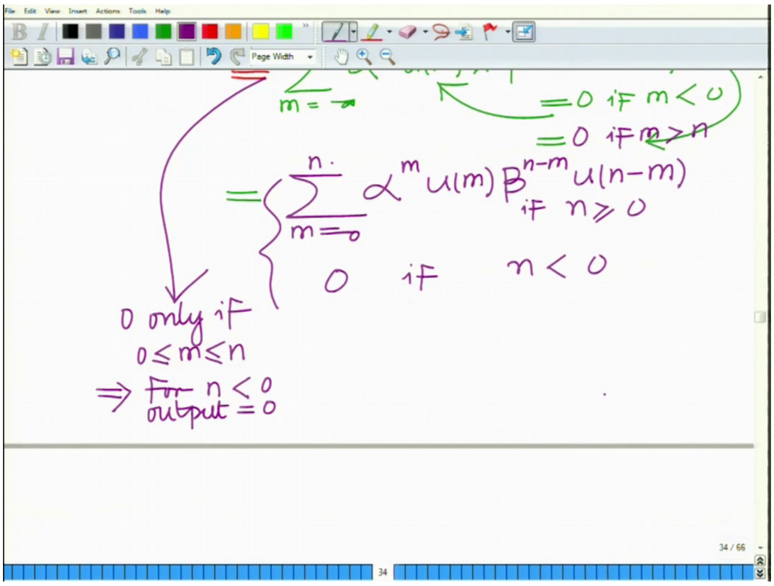

So this is equal to 0, again u n minus m equal to 0 if m greater than n. Hence this integral is nonzero only for m lying between 0 to n. So m equal to 0 to n, this is alpha to the power of m u m beta to the power of n minus m u n minus m. So, this is 0 only if 0 less than or equal to m less than equal to n. Which automatically implies for n less than 0; for n less than 0 output equal to 0. So, if this is the summation 0 to n, if n is greater than or equal to 0 and equal to 0 if n less than 0. So, this is if n greater than or equal to 0; if n is less than 0 then this is naturally 0.

And you can also see this from the nature of the input signal and the impulse response, because the input signal is causal; that it is 0 for n less than 0, the impulse response is causal, right impulse response is 0 for n less than 0 and therefore the output is naturally it is a causal. And the output is naturally going to be causal, that it is nonzero only for n greater than equal to 0 and 0 otherwise.

Therefore, this is equal to alpha m. Now we simplify this further.

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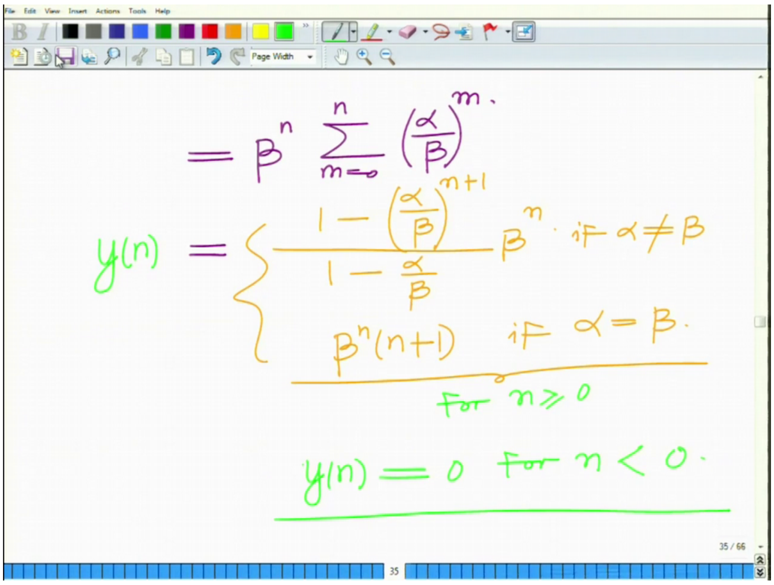

Further now, if you bring m outside; so this is m or if you bring n outside I can write this as summation m equal to 0 to n alpha over beta to the power m which is equal to. Well, again here we have two cases which is equal to 1 minus alpha over beta to the power n plus 1 minus over 1 minus; alpha over beta into beta over n if alpha is not equal to beta. If alpha is equal to beta then you can see; if alpha equal to beta each term is alpha over beta is 1. So, summation m equal to 0 to n alpha over beta to the power of raised to m there is simply be summation m equal to 0 to n summation of 1. So, this will simply be n plus 1. So, this will be beta n into n plus 1, if alpha equals beta.

And both these are for n; for n greater than equal to 0 and y n, so this is y n equal to 0 for n less than c. That is the complete expression for the output signal y n. We have derived the expression for the signal y n, we have seen that y n is 0 if n is less than 0 and for n greater than or equal to 0 the expression the resulting output depends on whether alpha equals b or whether alpha does not equal b. If alpha equals b it is b raised to the power of n into n plus 1, if alpha does not equal b it is 1 minus alpha over b raised to the power of n plus 1 over 1 minus alpha over b times b raised to the power of n, ok.

Let us consider; so this is one of the examples.

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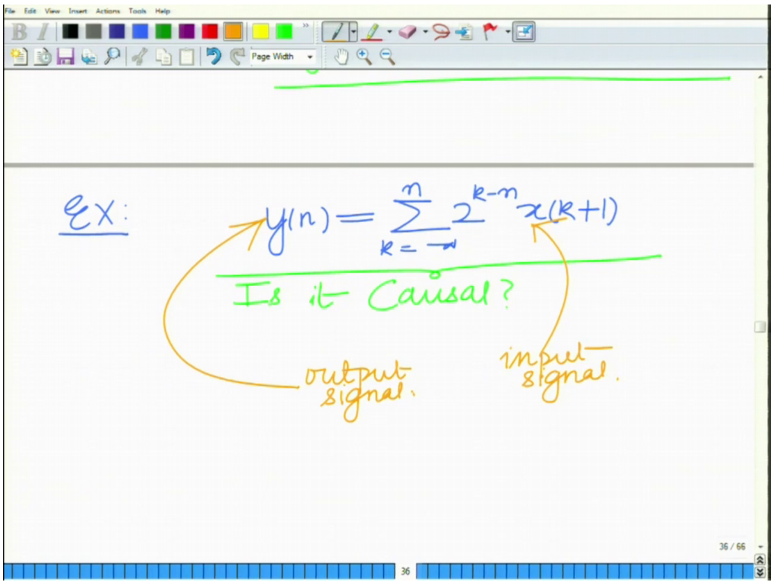

So, let us now consider another example: that is y n output of y n is given in terms of input x n as 2 to the power of 2 raised to k minus n x k plus 1. So, we want to ask the question is this causal. Is this a causal system? X is the input signal, y n is the output signal, is this a causal system.

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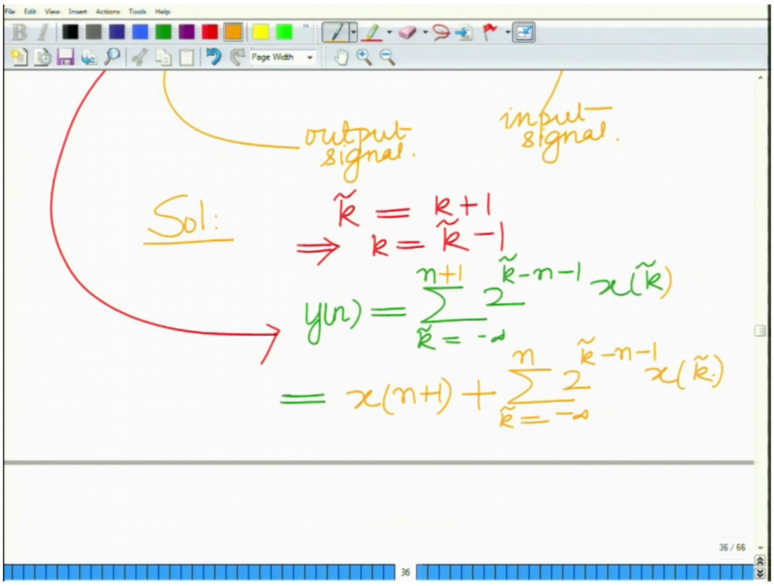

Now you can see I can rewrite this as; so the solution will be to rewrite this as y n equals. Now set k tilde equals k plus 1 implies k equals k tilde minus 1, which implies this can be quality written as y n equals summation k tilde equals minus infinity; that is lower limit remains minus infinity, upper limit becomes. Since k goes from 0 to n, so k tilde equals k plus 1 so this becomes n plus 1 2 raised to k minus or k tilde minus 1 k tilde minus n minus 1; k minus n becomes k tilde minus n minus 1, x k plus 1. Which now if you separate the term corresponding to k tilde x; x of k tilde plus 1 now separate term corresponding to k tilde; I am sorry this k tilde goes from 0 to a correct, this is correct this is n plus 1, this is simply k tilde; x of k tilde.

Now if you consider the term k tilde equals n plus 1, so this becomes 2 raise to k tilde equals n plus 1 2 raise to k tilde minus n minus 1 that is 2 raise to 0, so this is 1. So, this becomes x of n plus 1 plus summation; since we have already written the term corresponding to k tilde equals n plus 1 what remains is the summation k tilde equals minus infinity to n 2 raise to k tilde minus n minus 1 into x of k tilde.

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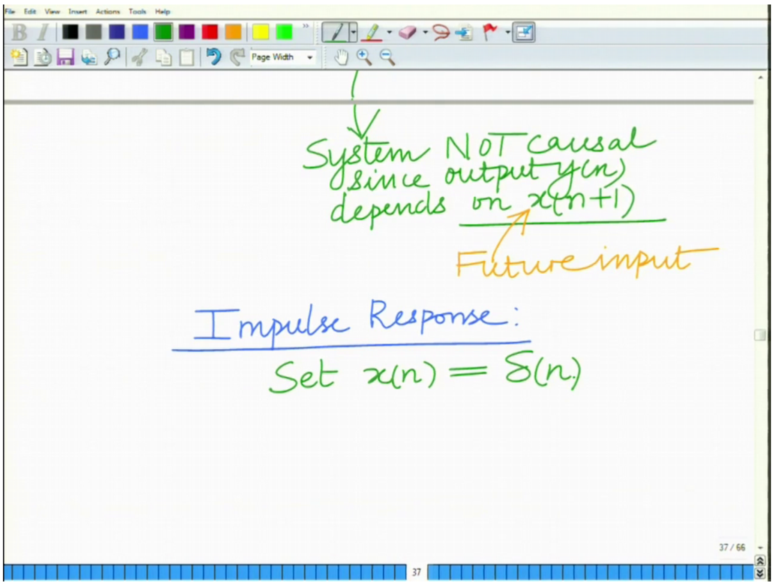

And now if you look at this expression you can see that the output the y n depends on x n plus 1. Remember the output while the system is causal only if y n the output at any time instant and depend only on x n and the previous inputs. Since this depends on the future input that is x n plus 1, it is not causal. So, system is not causal also depends on x n plus 1. And this, remember is a future input; input at a future time instant n plus 1.

So, this is not causal. And, also to find the impulse response of this system; one can also find impulse response. If you look at the impulse response approach: to find the impulse response at the input to the impulse, set the input to the impulse what we will have is y of n.

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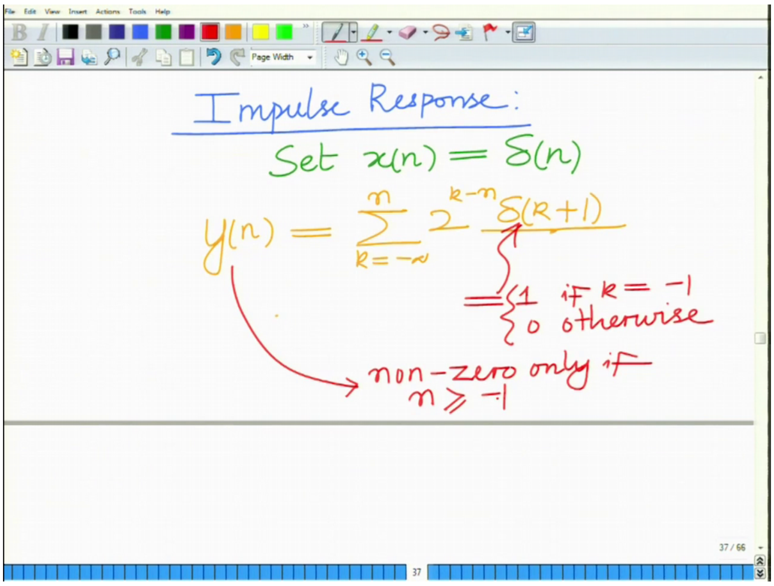

Equals summation k equals minus infinity to n 2 power k minus n delta k plus 1. And remember delta k plus 1 equals z equals, I mean this is equal to 1 if k plus 1 equals 0 implies k equals minus 1 and equals 0 otherwise. So, this implies y n; now this implies y n equals nonzero only if n is greater than or equal to minus 1. Because we have to have k plus 1 equal to 0 which means k must take the value of minus 1, so this is nonzero as long as only k takes the value minus 1, which means n must be greater than equal to minus 1.

So, you can see this summation for this summation to be nonzero, the index k must go from minus infinity to at least minus 1, alright. Because if n is less than minus 1 then this will be 0, because delta k plus 1 will be 0 for all k plus 1 less than 0 or k less than minus 1. So, this is nonzero only for n greater than minus 1.

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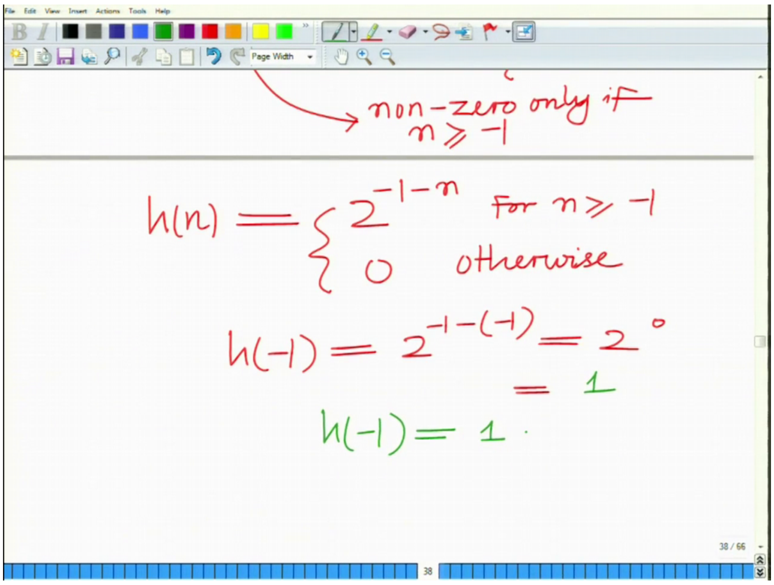

And in fact, for n greater than minus 1 the y n or the impulse response h n equals; you can see it is nonzero only for k plus 1 equals 0 or k equal to minus 1. So, this will be 2 power minus 1 minus n 2 for n greater than or equal to minus 1 and equal to 0 otherwise.

Now, which means if you look at h of minus 1 equals 2 raised to minus 1 minus minus 1 equals 2 raised to 0 which is equal to 1; so h of minus 1 we have equal to 1, which means h of n is not 0.

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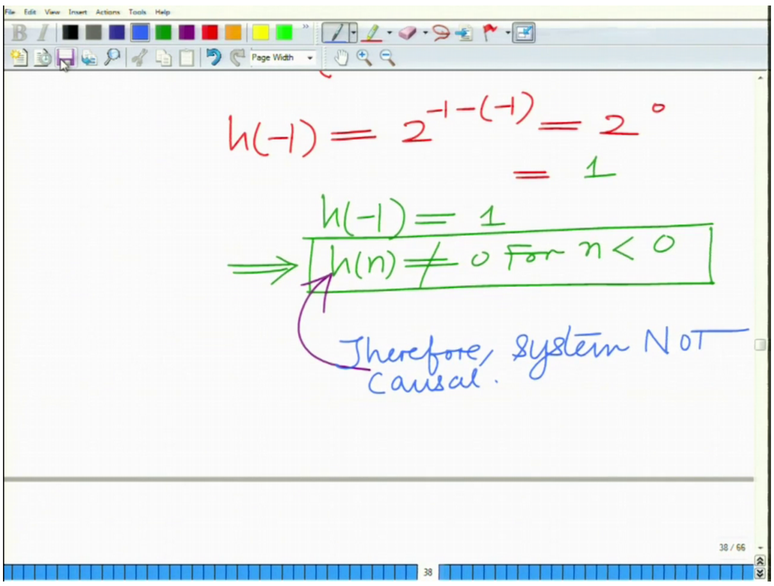

So, this implies since we have h of minus 1 the impulse response at time instant n equal to minus 1 is 1, this implies that h of n is not 0 for n less than 0. Since the impulse response is not 0 for time less than 0 this implies that the system resulting LTI system is non causal. This follows from the property of LTI systems, therefore system is not causal.

So, this depends on future input, here also we could have set the same thing. Therefore, and we can say the same thing here also. We can note the same thing here also. Therefore, the system is not a causal system. So, this is not a causal system, so that completes that example.

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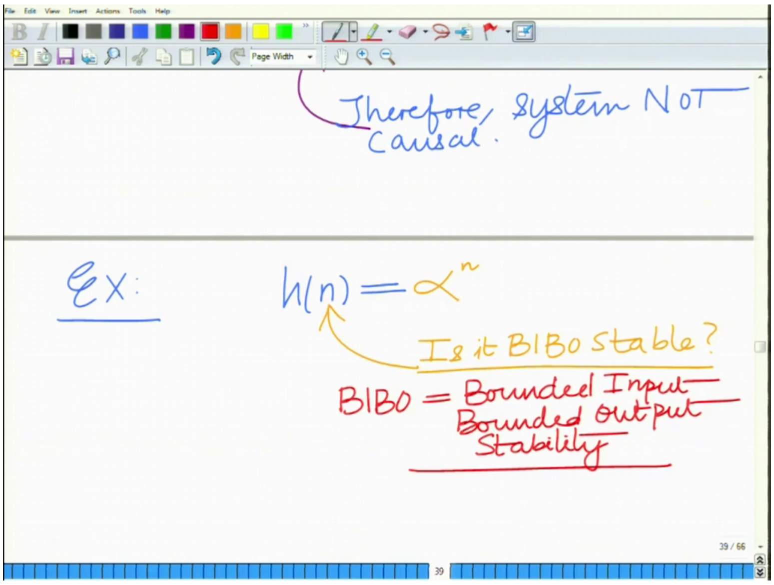

Let us do another example. And in this example we will examine the concept of BIBO stability for a discrete time LTI system. Remember BIBO stability refers to bounded input bounded output stability. So, we are going to check if the given LTI system is BIBO stable. So, we are going to check if h of n with impulse response given by alpha n is this, is it BIBO stable remember BIBO implies Bounded Input Bounded Output stability.

So, we want to ask is this system BIBO stable and the solution is as follows.

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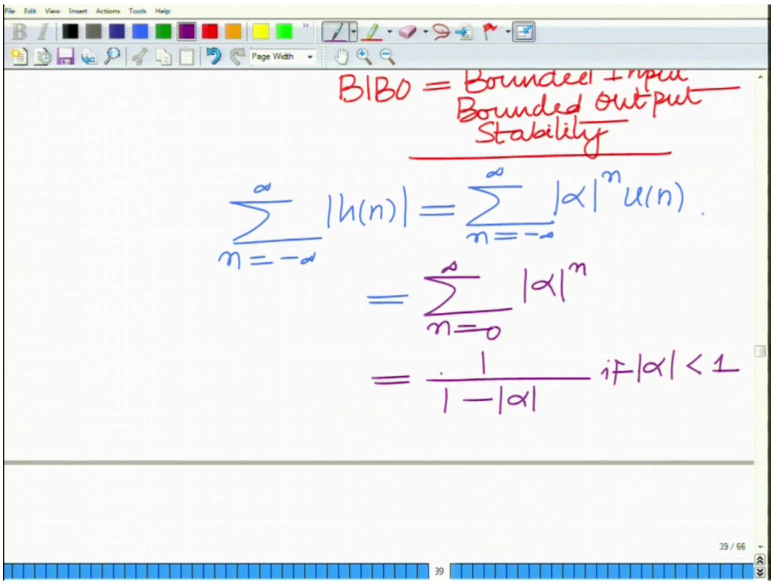

We have to check the absolute sum n equal to minus infinity to infinity, mod of magnitude of h of n which is summation n equal to minus infinity to infinity magnitude alpha raised to the power of n which is summation; alpha raised to the power, alpha raise to n u n. I believe forgot to add u n, which means this is going to be summation n equal to 0 to infinity magnitude alpha raised to the power of n which is 1 over 1 minus magnitude alpha, but only if magnitude alpha less than 1, equal to infinity. So, this sum will diverge to infinity. This is equal to infinity otherwise.

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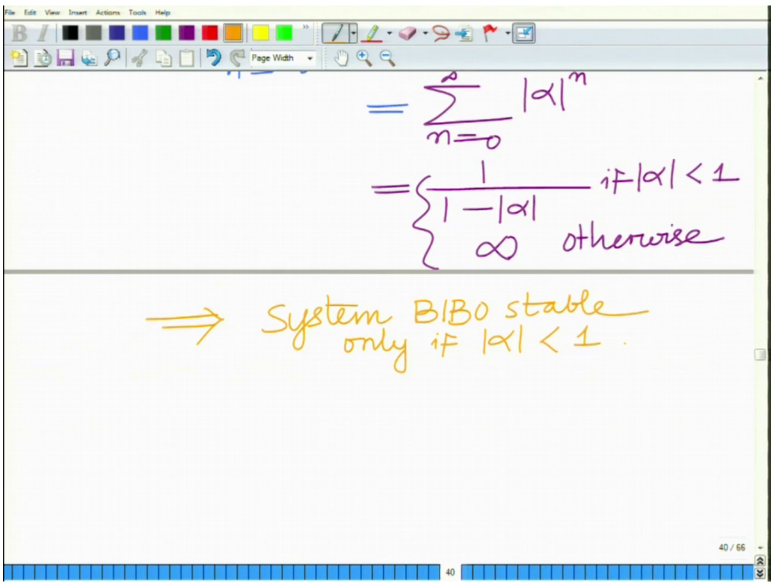

So, the absolute sum of the impulse response is finite only if magnitude alpha is less than 1; which implies that the system is BIBO stable only if magnitude alpha is less than 1. Implies, only if magnitude alpha is less than 1 otherwise it is an unstable system; otherwise it is an unstable system, BIBO stable only when magnitude alpha is less than 1.

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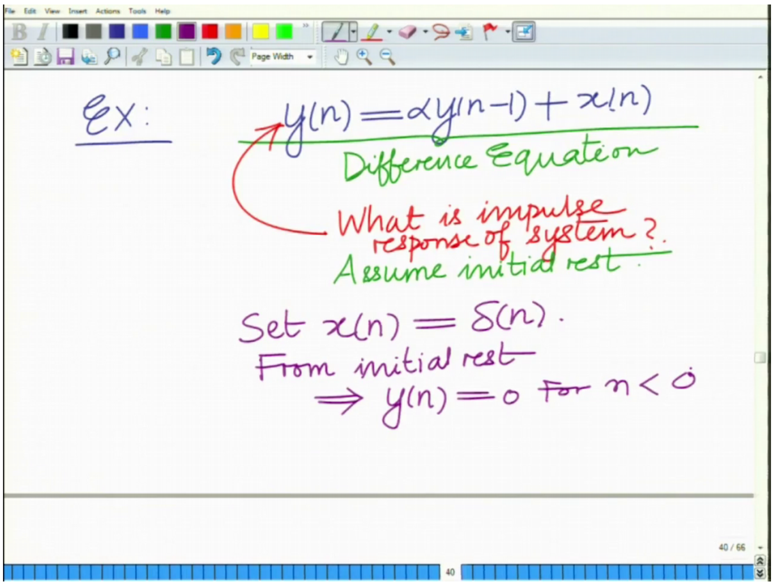

And let us do one more example which is related to. So, this is the system, we are giving a difference equation. Since this is a discrete time system we have a difference equation. So, this is a difference equation description of the system we want to find; what is the impulse response of the system. And the solution can be approached as follows assume initial rest and we can also assume initial rest. Initial rest, if you remember that implies that the output is 0 the output that is if the input is less than t equal to t naught correct then the output for t less than t naught is 0, ok

Now since we are looking at the impulse response if the input is 0 for t less than 0 which means the output is also 0 for t less than 0. So, let us set x n; since we want to find the impulse response set x n equal to delta n from initial rest this implies y n equal to 0 for n less than 0. Since input is 0 for n less than 0.

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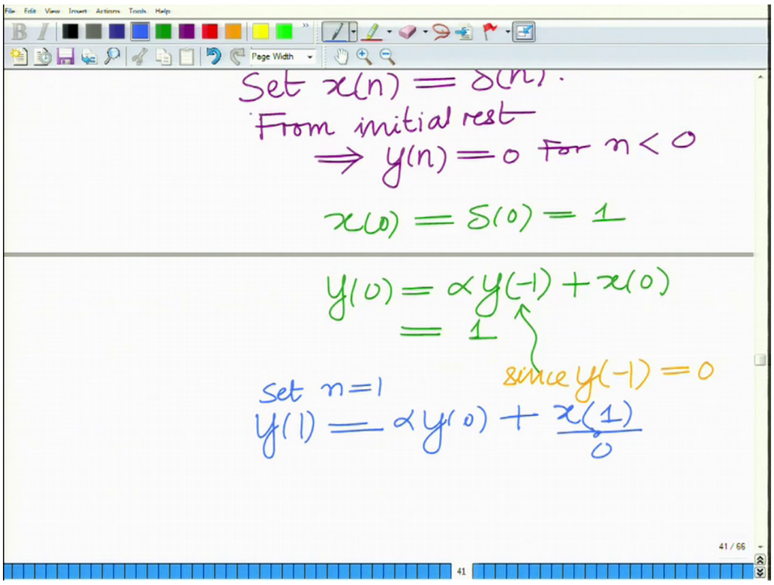

And therefore, now set x n equal to delta n which means x 0 equals delta 0 equals. We have y of 0 equals alpha times y of minus 1 plus from the equation setting n equal to 0 we have y of 1 equals alpha times y of minus 1 y of 1 equals; well alpha times y of minus 1 x of n equals y of n minus 1 plus x of n. So, y of 0 equals alpha times minus 1 plus x of 0 which is equal to simply 1; since y of minus 1 equals 0.

This implies now, if you want to find y of 1 set n equal to. Now set n equal to 1 y of 1 equals alpha times y of 0 plus x of 1.

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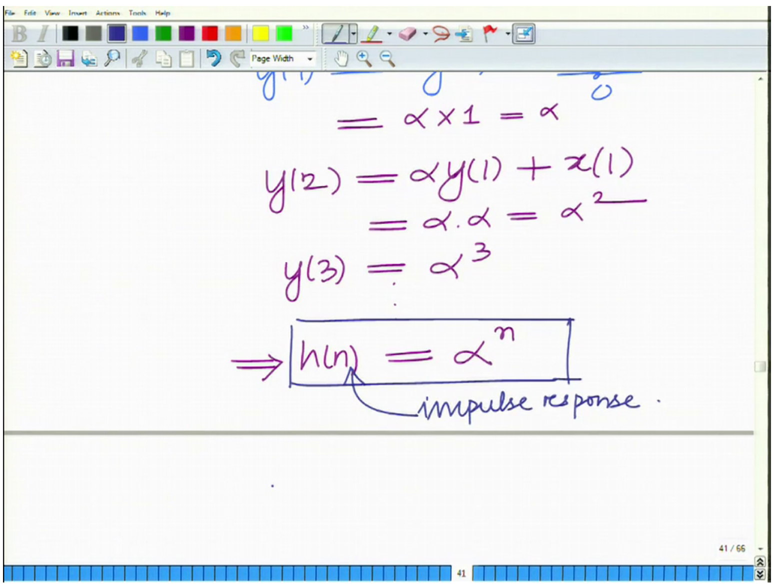

Now x of 1 is 0, so this becomes alpha times y of 0 which is alpha times 1, which is equal to alpha similarly y of 1 equals alpha times y of 1 plus x of 1; x of 1 equals 0. So, this becomes alpha into alpha equals alpha square y of three equals alpha cube and so on, ok.

So, this implies the output y of n with this system with initial rest is alpha power or the input. In fact y of n, because is a response to the unit impulse this is alpha raised to the power of n, this is the impulse response of the system described by the difference equation.

So, I think that completes the set of examples that we want to examine to describe the properties of LTI systems. So, we will start stop this module here and we will look at other aspects in the subsequent modules.

Thank you very much.