Principles of Signals and Systems
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Lecture – 13
Properties of Discrete Time LTI Systems – Impulse Response, Stability, Eigenfunction, Systems Described by Difference Equation

Hello, welcome to another module in this massive open online course. So, we are looking at the properties of LTI systems. So, far we have looked at the properties of continuous LTI systems. Let us now look at the properties of discrete LTI system. Several properties that we have looked at in the context of continuous LTI systems can be extended in a straightforward manner to the scenario of discrete LTI systems.

But nevertheless let us take a look at those properties ok.

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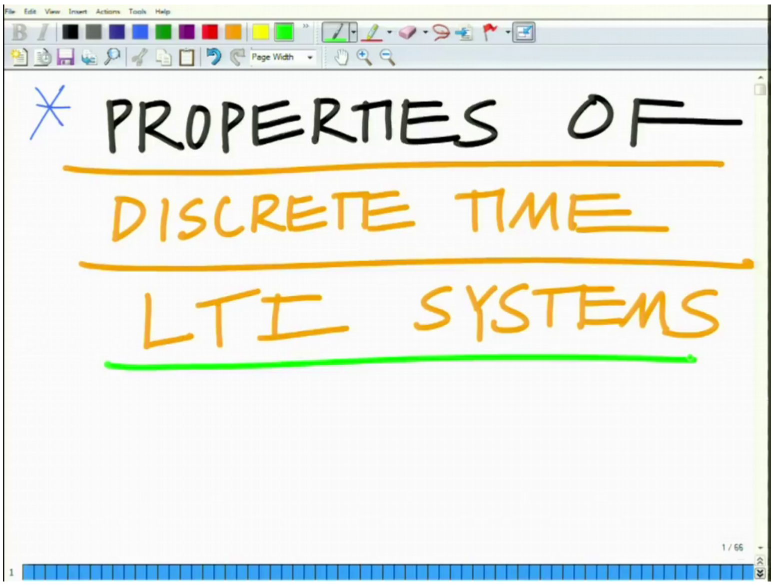

So, we want to look at the properties, properties of discrete time, LTI systems and similar to what we have done, similar to what we have seen in the context of continuous time LTI system.

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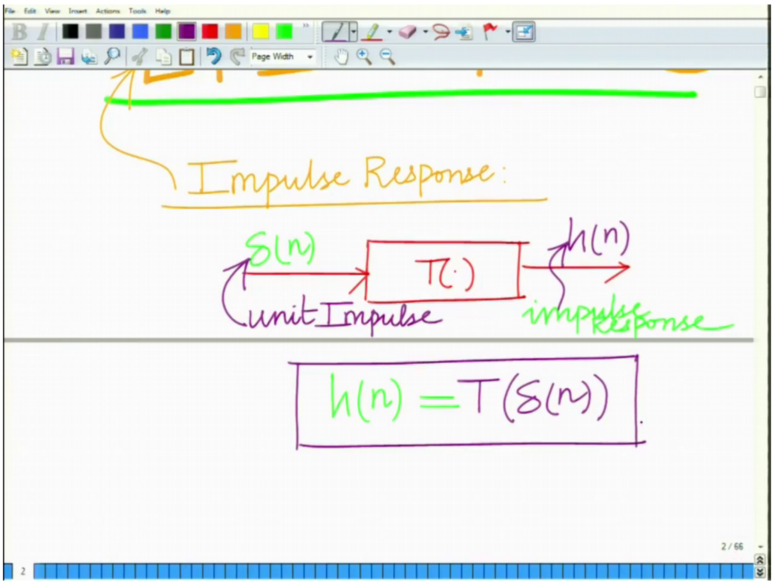

Let us start with the fundamental quantity that is the impulse response. The impulse response of a discrete LTI system is basically if you look at the discrete to consider the discrete system then the response to the impulse, the impulse delta n is basically your impulse response. So, this is discrete time impulse or the unit impulse. So, this is your so called unit impulse and this is your impulse response.

So, we have h of n which is the impulse response of the discrete time system is discrete time LTI system is T of delta n. So, this is your impulse response, impulse response or the response to the unit impulse.

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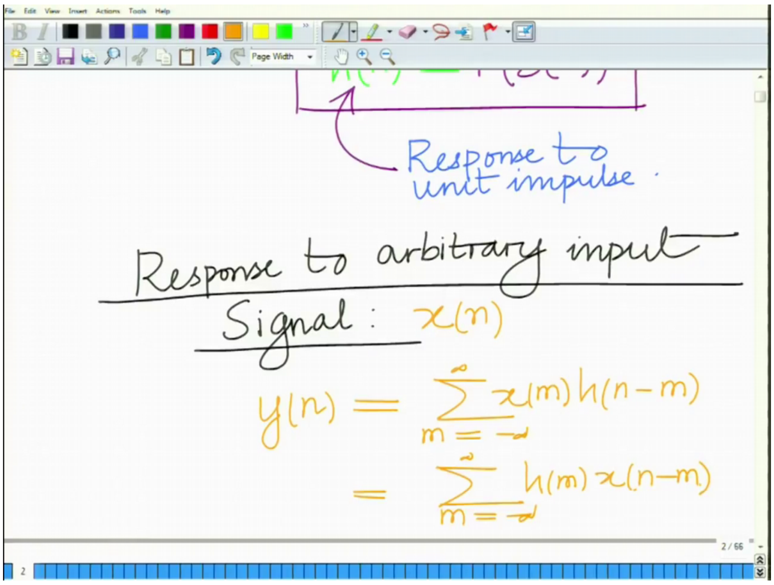

Response, response to unit impulse and similar to what we have seen in the context of continuous time LTS s this is of fundamental importance because the impulse response helps characterize the response or help helps characterize the output for any arbitrary input signal x of n and this is given by the convolution, in this case it will be the discrete convolution or basically that convolution sum. So, in the integral is replaced by a summation ok.

So, the response to an arbitrary input signal, response to an arbitrary input signal x of n is y of n equals summation m equal to minus infinity to infinity x of m into h of n minus m, which is equal to summation m equals minus infinity to infinity, h of m times x of n minus m very good excellent and this is of course, is known as the convolution sum.

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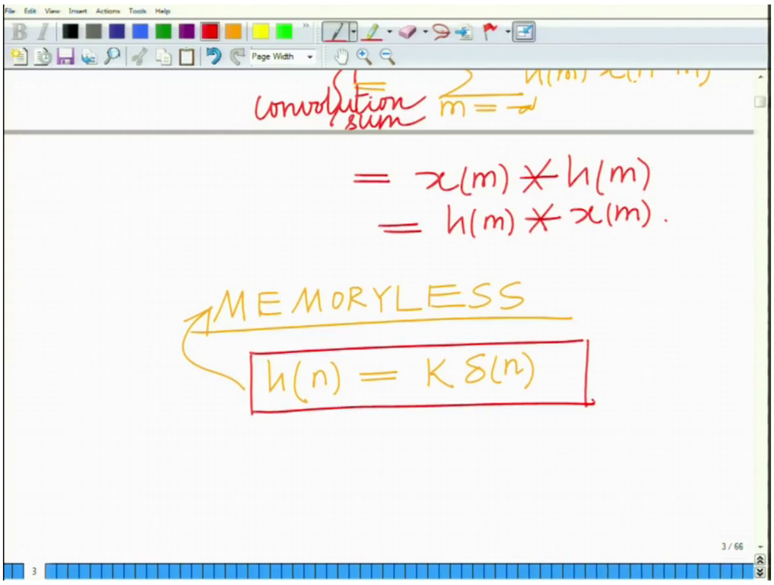

This is a convolution operation and this is known as the convolution sum and it is represented as, either as you can see as the convolution is commutative it is x m convolved with h m or the impulse response h m convolved with the input signal x m. So, this is the response to any arbitrary input signal x m, which can be obtained by its convolution with the impulse response h m of the discrete time LTI system.

Similar to that of the continuous time LTI system and similarly there are several other properties which can be extended. I just give a briefly describe a few of them, a few a few of them which are salient a few of the salient properties of discrete time LTI systems, for instance a memoryless discrete LTI system in which the input depends only on the. In is the output depends only on the current input and does not depend on the past or the future outputs that is characterized by an impulse response h of n which is key of delta.

So, this is a memoryless impulse response of a memoryless discrete LTI system.

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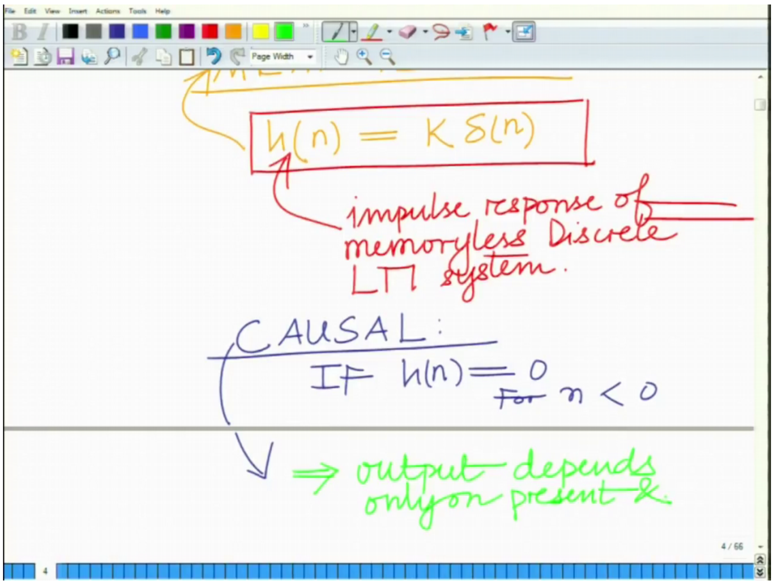

Impulse response of a memoryless discrete LTI system, now the system discrete LTI system is causal which means output depends only on the present and past values of the input, if h n equal to 0 for n less than 0. So, as you remember causal implies that output depends only on present and past.

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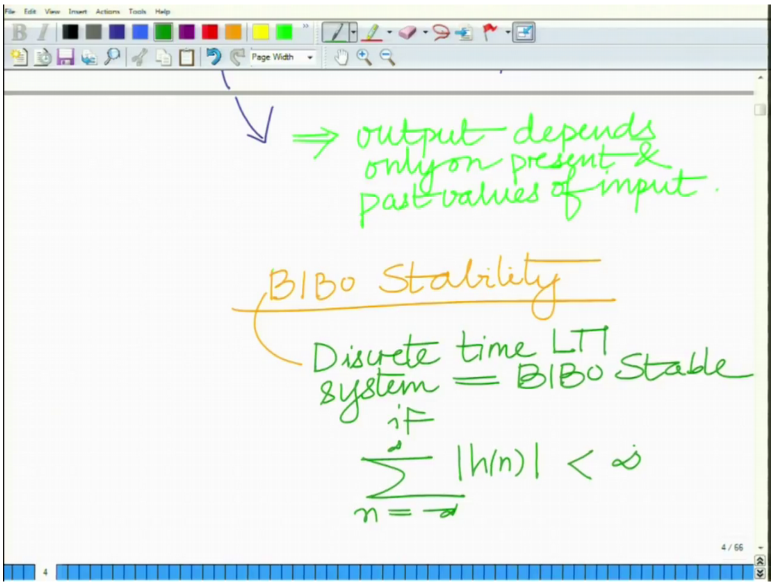

Output depends only on present and past values of the input signal.

Bibo stability stable system in particular one of the most important criterion for stability is BIBO stability. System is BIBO stable if or your discrete time LTI system is BIBO stable, if summation n equal to minus infinity to infinity mod h magnitude h of n is less than infinity implies that this quantity is a finite this quantity is absolutely summable.

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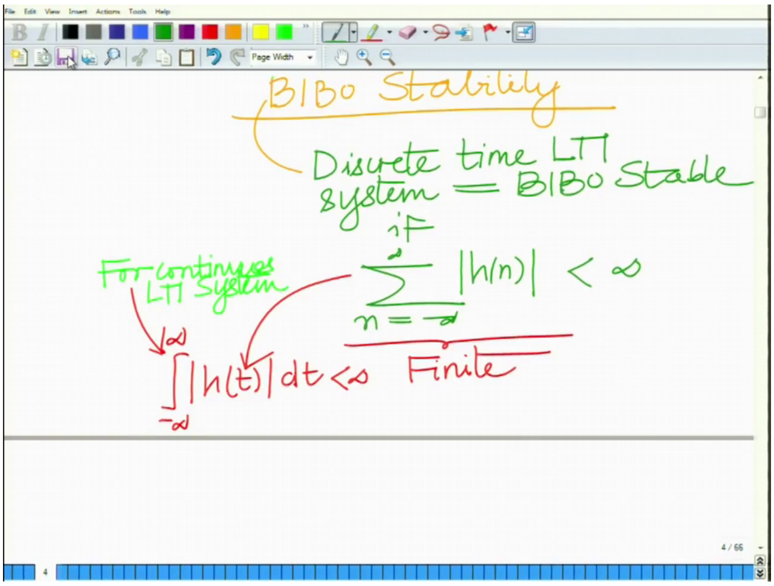

Implies this quantities is a, this quantity is a finite quantity remember for a continuous time system LTI system this required condition is integral minus infinity to infinity magnitude h of t d t integral magnitude h of t d t is finite, this is for a continuous LTI system this is for a continuous LTI system.

Similarly, the equivalent for a discrete LTI system is that summation n equal to minus infinity to infinity magnitude of h of n should be less than infinity that is the equivalent condition for your continuous time LTI system. For a discrete time LTI system the equivalent condition is summation n equal to minus infinity to infinity, magnitude of h of n should be for finite quantity that is strictly less than infinity. That is the required condition for a discrete time LTI system to be BIBO stable, BIBO stand for bounded input bounded output stability.

Now, let us look at the Eigen function of discrete time LTI systems.

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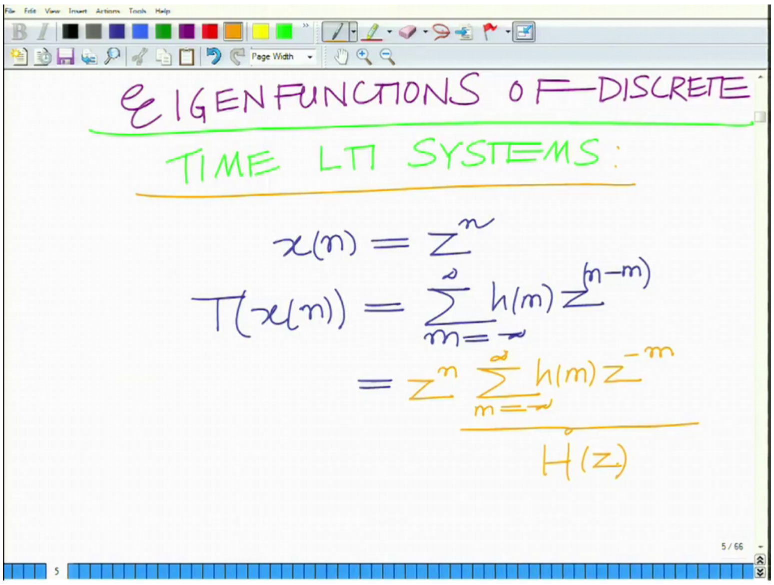

Let us look at Eigen functions of discrete time LTI system, consider the function x of n remember e power lambda t was an Eigen function of discrete time LTI systems of a discrete time LTI system. Let us consider x of n equal to Z power n then T of x of n equals summation m equal to minus infinity to infinity, h of m where h of m is the impulse response time Z of n minus m. Now, I can bring the Z of n outside, so this will simply be Z of n summation m equal to minus infinity to infinity, h of m Z of minus m and which is a function that depends only on the impulse response is known as H of Z ok.

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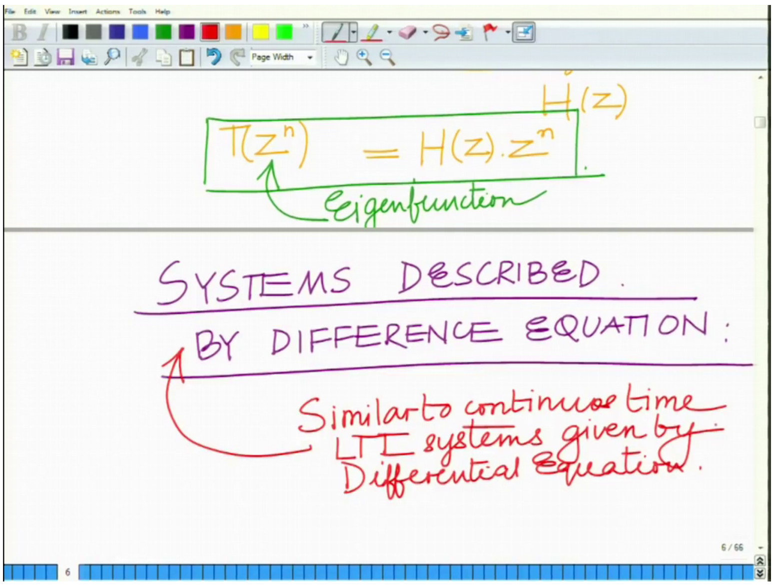

So, this you can see this is H of Z times Z to the power of m, if the input x of n is z to the power of an output is simply H of Z times Z to the power of n and therefore, we have said that this is an output, this is an Eigen function because the input is simply a scaled version of the output. If the input is z of n output is simply t of z, output is simply h of z times the times Z of n that is a output is simply h of z times the input signal which is Z of n this is scaled version. So, this is known as the Z of n is an Eigen function, this is known as an Eigen function of the transfer ok.

Now, systems described by a difference equation or similar to a continuous time LTI system which can be described by a differential equation, a discrete time LTI system can be described by a difference equation, systems described by a, systems described by a difference equation. Now, similar to continuous time LTI system, given by a differential equation discrete time LTI systems can be described by a discrete time LTI systems can be described by a difference equation similar to continuous time LTI systems, which can be described by a differential equation.

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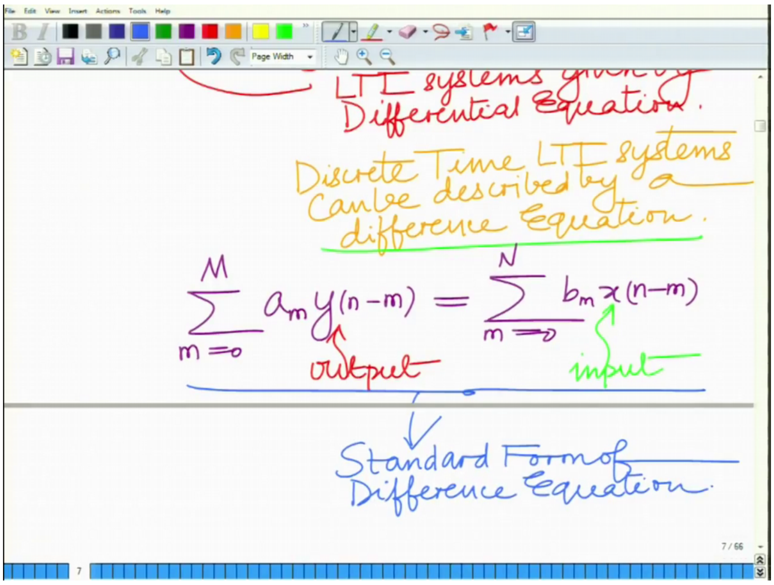

And the difference equation the general structure of a difference equation is if y of n is the input of a and x of or y of m is the output and x of m is the input, then I have m equal to 0 to capital m a of m y of n minus m equals m equal to 0 to N b of m, x of n minus m. So, this is your input this is your output and this is the canonical form or the standard form of a differential or a difference, this is the standard form of difference equation for a; this is the standard form of a difference equation that is summation ei m by n minus m equals summation b m X n minus M, where a m and BM are basically the coefficients of the difference equation. And this difference equation this general form of a difference equation can be used to describe discrete time LTI systems, linear time invariant system.

So, this basically summarizes several of the key property several of the salient properties of discrete time LTI systems you have not looked at them elaborately because these are very similar in nature to the properties of analogous properties for continuous time LTI systems and I trust that by simple extension you will be able to understand this and. In fact, you will be able to derive these properties yourself all right.

So, now, let us start looking at some examples to better understand the properties of both continuous time as well as discrete time LTI systems that we have looked so far in the various modules. So, let us start looking now at the various examples for continuous time as well as discrete time LTI system.

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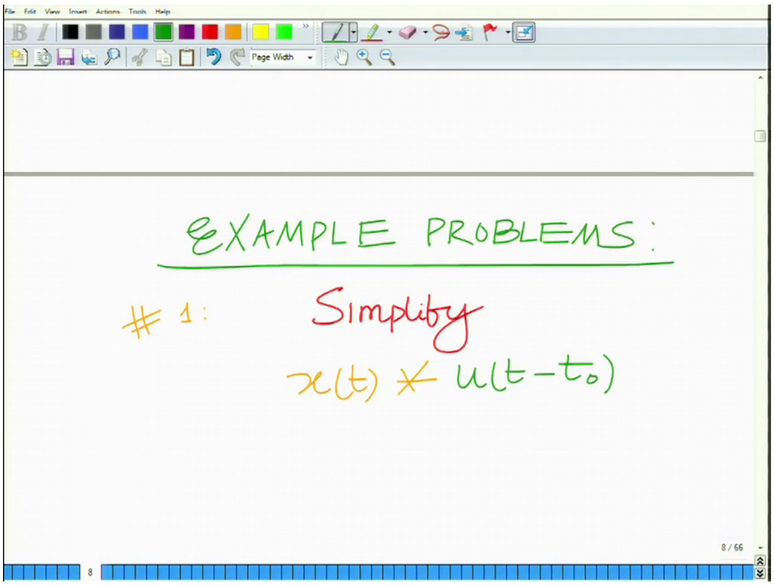

So, let us start looking at example problems, let us start looking at example problems. Now, the first problem will be let us look at a very simple problem for a continuous time LTI system simplify, simplify x t convolved with shifted or delayed version of the unit, unit step function u t delayed by t naught ok.

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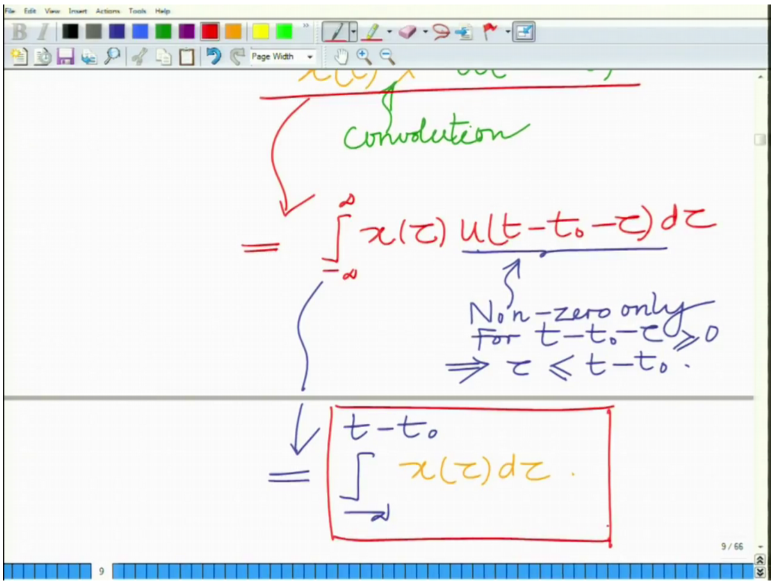

So, this is basically as you might recall this is your convolution operation and this can be simplified as follows, this is basically as you have seen this is simply x of tau integral minus infinity to infinity x of tau h of t minus tau or x t y t x t convolution with y t is x of tau mu of t minus tau d tau.

Now, remember mu of tau this is non 0 only for only for t minus t naught minus tau greater than or equal to 0 implies tau less than or equal to t minus t naught and for tau less than or equal to t minus t naught, u of t minus tau minus t naught is equal to. Therefore, this can be simply simplified as this is equal to, this is equal to integral minus infinity to infinity the upper limit can now be replaced by t minus t naught x of tau d tau. So, this is the final simplified answer to the, final simplified answer to the problem integral minus infinity to infinity t minus t naught x of tau d t.

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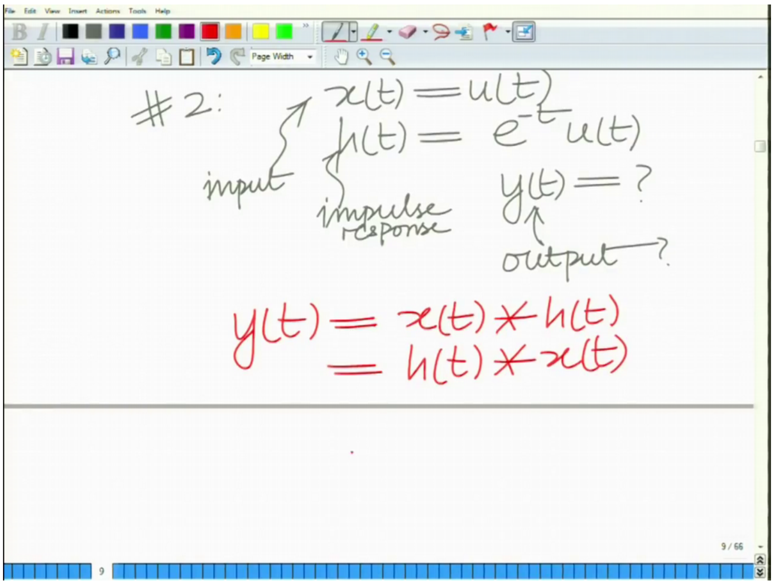

Now, let us look at another simple problem, let us say given or input signal x t equals u t and a system with impulse response h of t equals e to the power of e raise to minus t, u t now what is the. So, this is the input to the system this is your impulse response, what is the, question is what is the, question is what is the corresponding output signal y t ok?

So, we are given 2 signals that is x t equals e to the power x t equals u t the unit step function the impulse response h t is erase to minus t u t what is the corresponding output signal y t. Now, as you know, the output for an arbitrary input is given by the convolution of the input with the impulse response h t which I can also write as h t convolved with x t.

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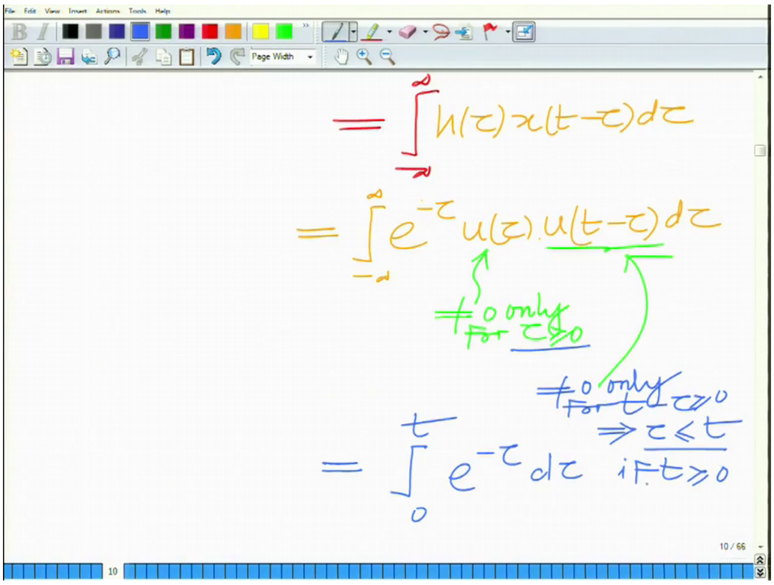

Which if you recall is basically integral minus infinity to infinity h of tau x of t minus tau d tau. Now, x of tau a a h of tau is minus infinity to infinity e power minus tau, u tau x of t minus tau is u of t minus tau d tau. Now, this is greater than or equal to 0, non 0 only for tau greater than or equal to 0, now this is non 0 u of t minus tau u of t minus tau is non 0 only for t minus tau greater than or equal to 0 implies tau less than or equal to t. So, this integral will only survive if. So, this is only non 0 for tau greater than or equal to 0 and tau less than equal to z.

So, this integral will be equal to minus infinity. So, it can be replaced by the equivalent limits 0 to t because it is non 0 only if tau is greater than or equal to 0 and tau is less than or equal to t. So, this integral can be replaced from 0 to t, minus infinity to infinity the limits can be replaced as equivalent limits can be replaced by 0 to t, but only if t is greater than or equal to 0 because if t is less than 0 then it is not possible that tau is less than or equal to t, but tau is greater than or equal to 0.

So, for t greater than equal to 0 this is integral 0 to t e power minus tau d tau if t greater or equal to 0 and equal to 0 if t is less than 0.

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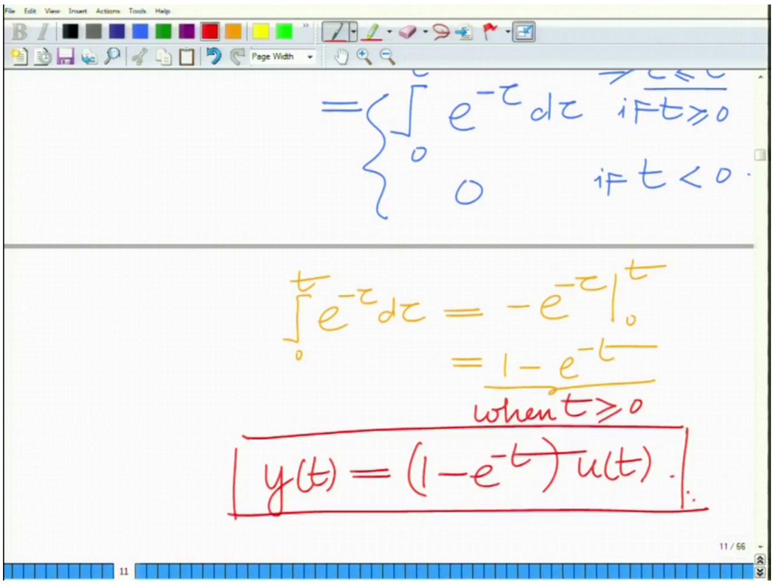

Now, let us simplify this integral, integral 0 to t e to the power of minus tau d tau that is minus e to the power of minus tau 0 to t which is 1 minus e power minus t, if t is greater than or equal to 0. This is only the case when t is greater than or equal to implies y of t and 0 otherwise implies y of t equals one minus t power minus t into u, this is the final output signal expression for the, expression for the final output signal all right.

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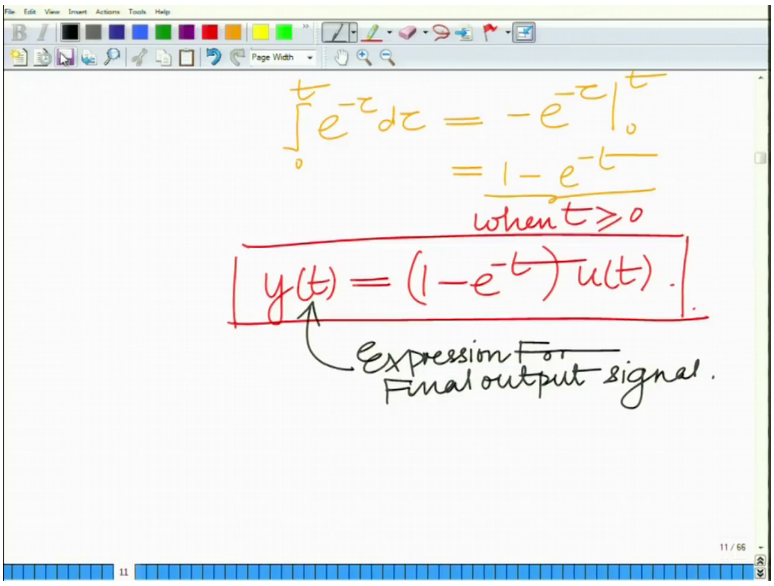

So, in this module we have looked at the properties of discrete time LTI systems and we have also started exploring some example problems to illustrate the application of the various concepts we have learnt regarding the properties of continuous as well as discrete type LTI systems. And we will be continuing this for at least a few of the upcoming subsequent modules in which we are going to look at further problems regarding the analysis of both continuous as well as discrete time LTI systems.

Thank you very much.