Principles of Signals and Systems
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Lecture – 12
Properties and Analysis of LTI Systems – Differential Equation Description, Linearity and Time Invariance

Hello, welcome to another module in this massive open online course. So, we are discussing looking at the properties and analysis of LTI systems. So, today later in this module let us look at another aspect that is the differential equation representation of LTI systems.

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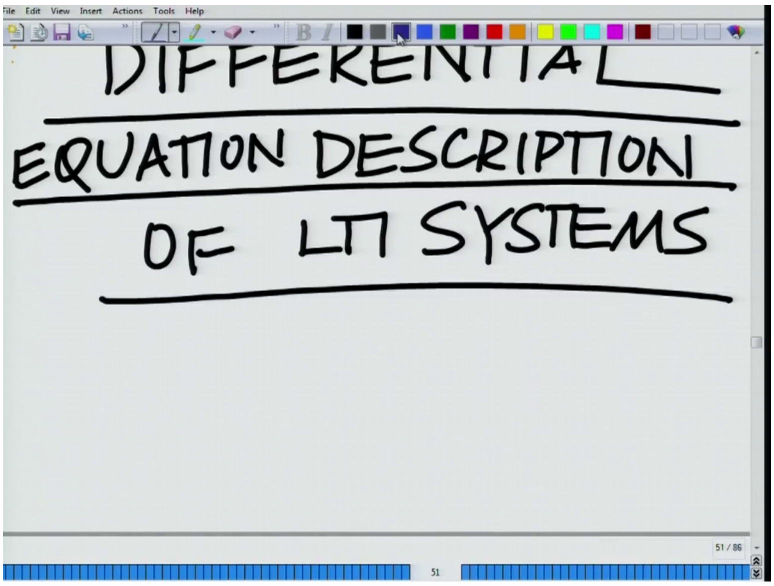

So, let us start by looking at differential equation, differential equation description strike it, let us call differential equation, description differential equation description of LTI systems ok.

And we have seen previously in an example that LTI systems the input output relationship of an LTI system can be represented by a differential equation for instance correct.

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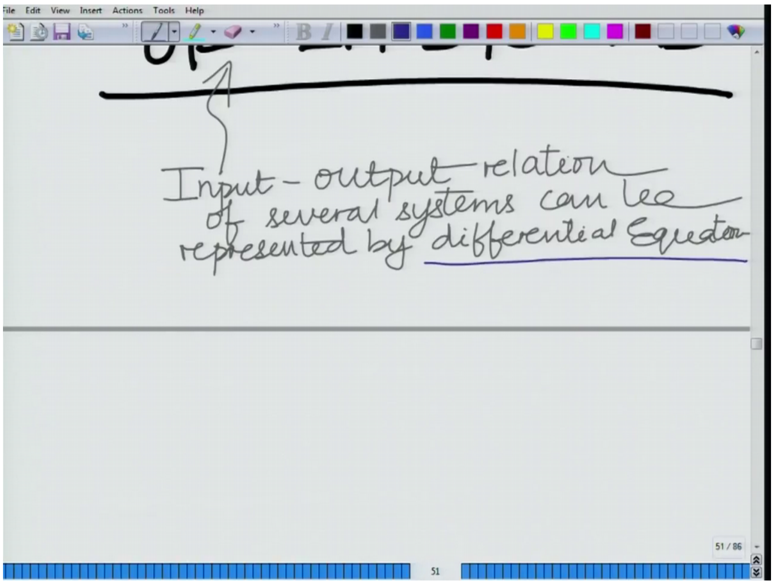

So, these systems, several systems or the input output relationship of several systems can be represented by differential equations. Let us note down that point can be represented by a differential by differential equation.

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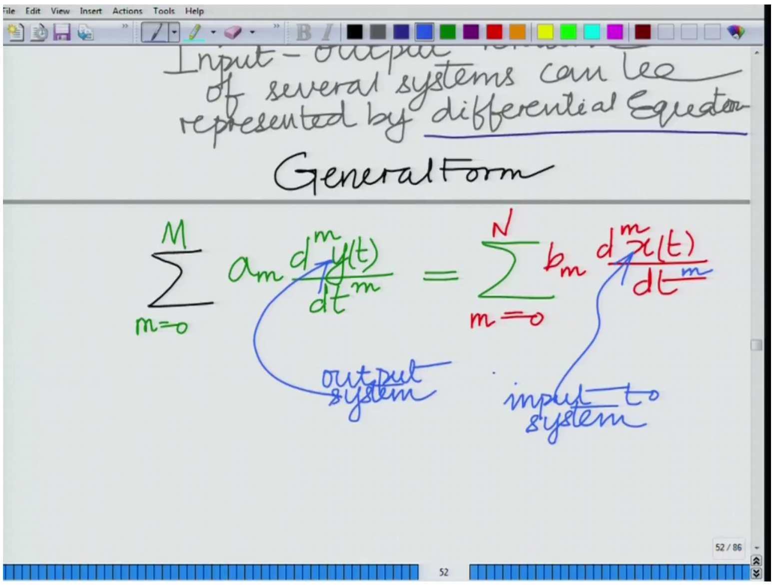

So, we can employ the differential equation, we can employ differential equations to represent the input output relations and this the general structure or the canonical form the general form of such a representation is, that the general form of such a representation is summation M equal to 0 to m a m which is the coefficient the mth derivative d m y t, d t m equals is an mth order differential equation m equal to 0 to N b m, d m x t over d m x t d t. And it is very clear that x t is the input two system need not necessarily be an LTI system and y t is the output we will see under what conditions this becomes this represents in LTI system and y t is the output of system, output of the system.

For instance an example, a simple example of such a system would be we have seen this before I have a current source, I have an inductor and a resistors.

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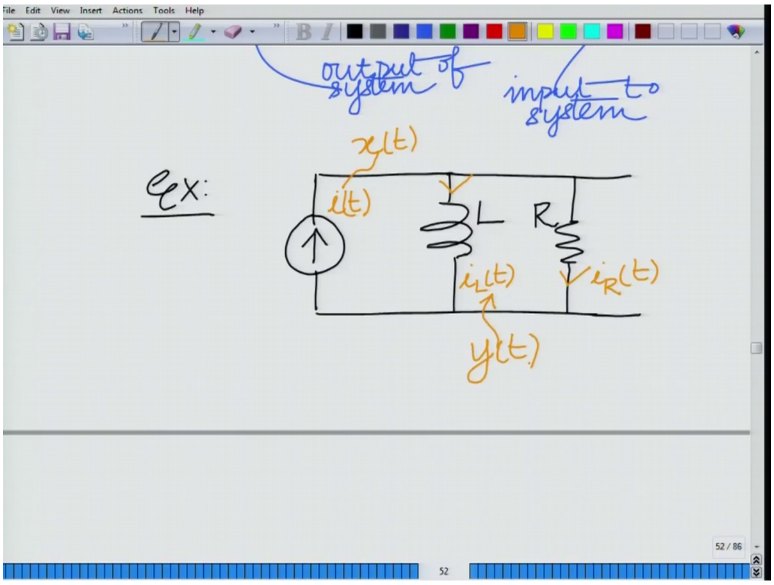

So, I have an inductor inductance L, resistance L, current source described by i of t, the current function is i of t we have the current through the inductor which is denoted by i L of t and current through resistance which is i R of t. And we have shown if you denote i t as the input x t to the system i L of t as the output that is current through the inductor as the output of the system. We have shown that this can be described by the differential equation L over R d i L t over d i L t d t plus i L t current through the inductor equals the net current of the source. So, this is the differential equation.

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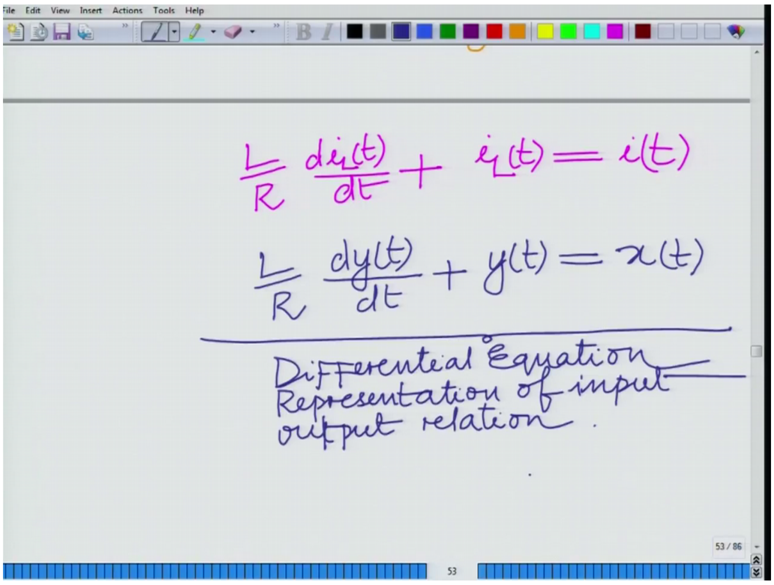

So, this a simple circuit with comprises of a current source in parallel with an inductor and resistance correct we have shown that this can be represented by a differential equation.

Now, since I L t is the output y t and x t i t equals x t which is the input I can equivalently write this as L R d y t over d t plus y of t equals x t. So, this is the differential equation representation of the input output relationship, differential equation representation of the input output relation.

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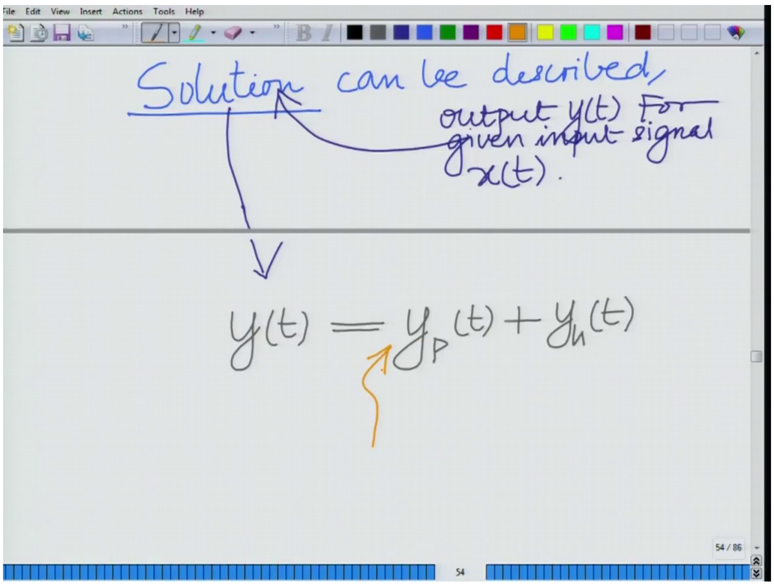

Now, the solution for this differential equation representation is given by, the solution can be obtained as or can be that is; what do we mean by solution. That is the solution here simply means the output signal y of t for the given input signal x of t. So, given an input signal of x of t that a signal x of t input to the system we desire to find the output signal y of t.

So, the solution can be described as y of t or the output for given input output signal y of t for given input signal x of t and the solution can be described as this is the general structure of the solution this differential equation that is y of t equals y of p of t plus y of h of t. So, this is basically the output signal can be described as the sum of two signals y of p of t plus y of h of t this is termed as y of p of t this is termed as a particular solution, this is termed as a and this is y of h of t is termed as a homogenous solution.

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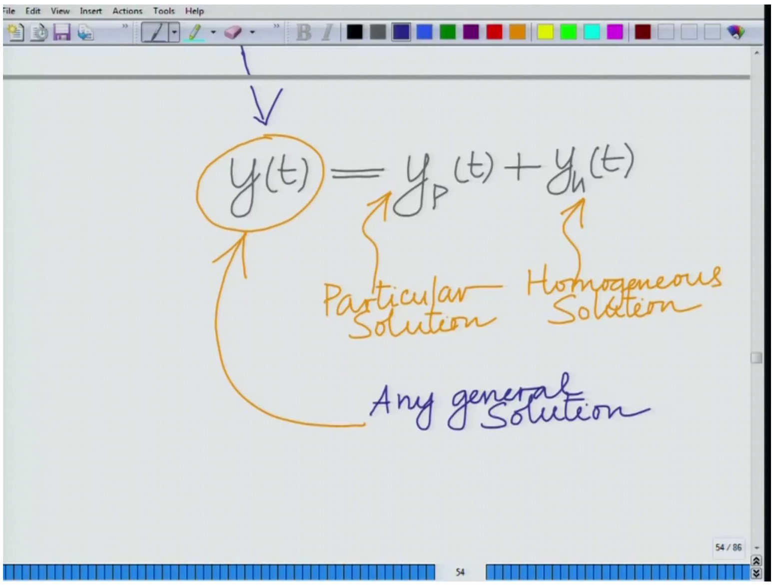

So, we have two things we have the particular solution and we have some homogenous solution. So, any solution y of t any general solution can be expressed as the solution as the sum of a particular solution and a homogenous solution. So, y of t any general solution to this differential equation which describes the input output relationship of this LTI system how can be represented as the sum of particular solution and a homogeneous solution. So, that is what we have over here.

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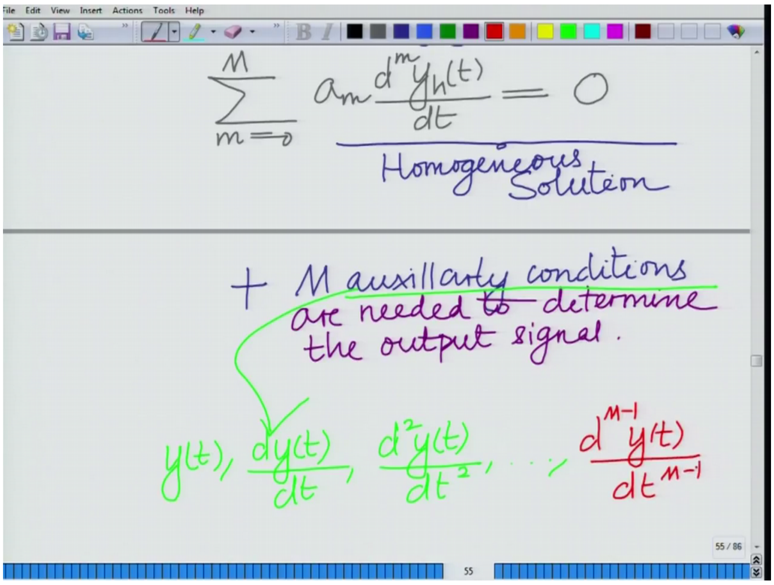

And in the homogeneous solution in particular is given as the homogeneous solution is summation M equal to 0 to m, a m d y h t divided by d t. So, you take the left hand side, you take the left hand side of this differential equation. So, this is the left hand side. So, you are taking the left hand side, you are taking the left hand side of this differential equation and you are basically setting the left hand side to 0. So, that describes the homogeneous solution to this differential equation which gives the input output relationship of the system.

So, set this is equal to 0, set it equal to 0 and this describes the this gives the homogeneous solution, this gives the homogeneous solution and plus we need M auxiliary conditions plus to solve this we need M auxiliary conditions also termed as the initial conditions or boundary conditions or so on. M auxiliary conditions are needed to determine the solution. M auxiliary conditions are needed to determine the output signal, M auxiliary conditions are needed to determine the output signal and these auxiliary conditions are given as well these are the auxiliary conditions. These auxiliary conditions are given as basically the values of the output signal or the derivatives, derivatives of various orders d y of t divided by d d o d square y of t d t square so on, d m the m minus 1 th third order d m minus 1 y t d t. m minus 1 at some point. So, these have to be given at some point, at some time t naught let us say these are the auxiliary conditions. So, these are basically your.

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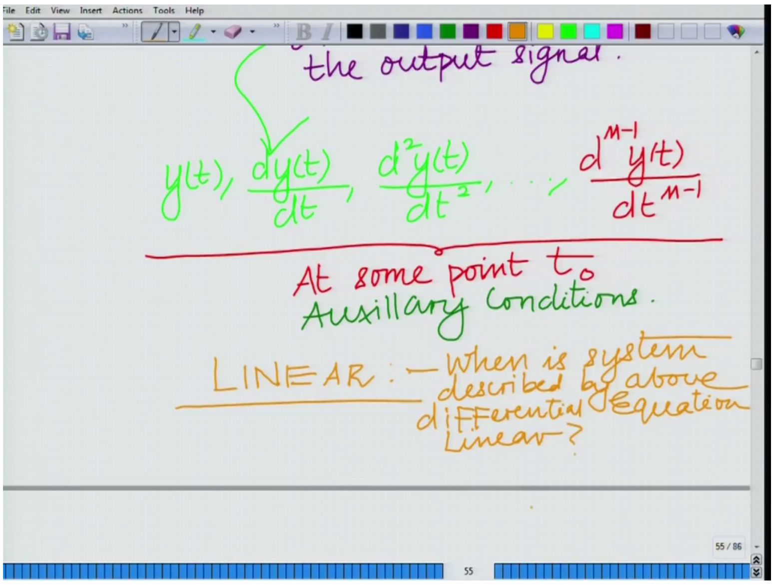

So, we have the various auxiliary conditions which are necessary to determine the solution these are the values of the signal output signal y of t or its various derivatives, derivatives of various orders that is d y t d t d square y t d t square or d m minus 1 y t d t m minus 1 and these have to be specified at some point d naught. So, as to uniquely determine, as to determine the solution for this output signal y t; for this system described by the differential equation.

Now, linearity now when can we say the system is linear. Now, how do we; now for this system described by the differential equation when is the system linear. So, we want to answer to the question when is system described by the above differential equation. When is this linear? This is linear if auxiliary conditions are 0, if all the auxiliary linear, if all auxiliary conditions equal to 0; that means, the values of d y t d t, y t, d y t d t, d square t, d square y d t squares. So, on d m minus 1 y t d t m minus 1 all these auxiliary conditions are 0 at some point only then the system will be linear. If auxiliary condition, that is it means to say that the auxiliary conditions have to be 0 auxiliary conditions have to be 0.

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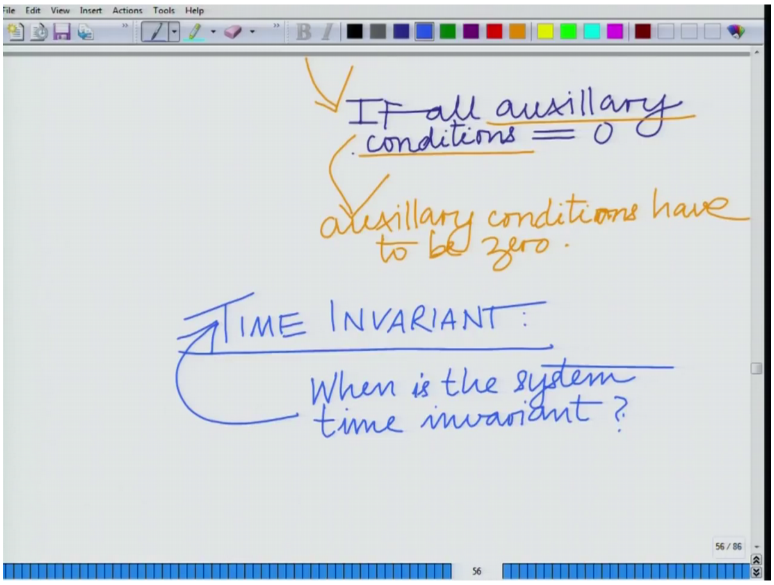

Now, further when can we say when is the system time invariant, when is the system described by the differential equation above. So, we would like to answer the question when is the system, when is the system time invariant.

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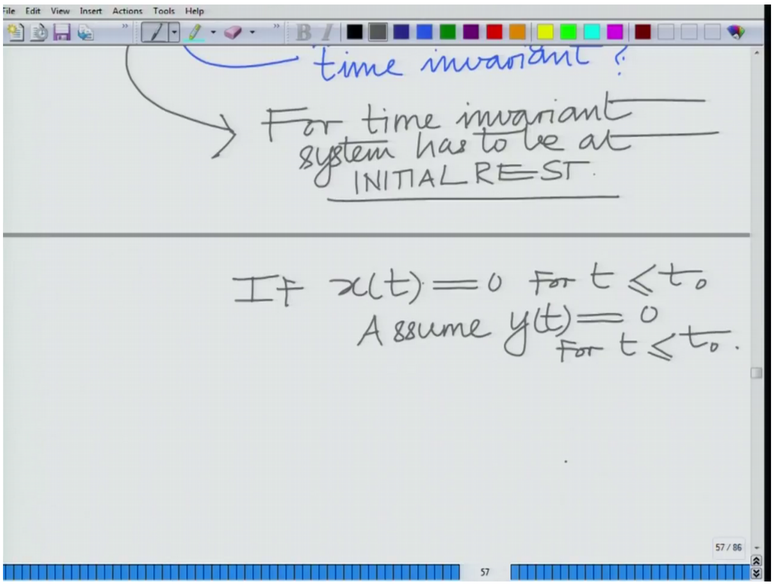

Now, for the system to be time invariant this system has to be at initial rest, for time invariance that initial, this is known as the initial rest condition which basically implies that if x t equal to for t less than or equal to t naught then assume y t equal to 0 for t less than or equal to t less than or equal to t naught.

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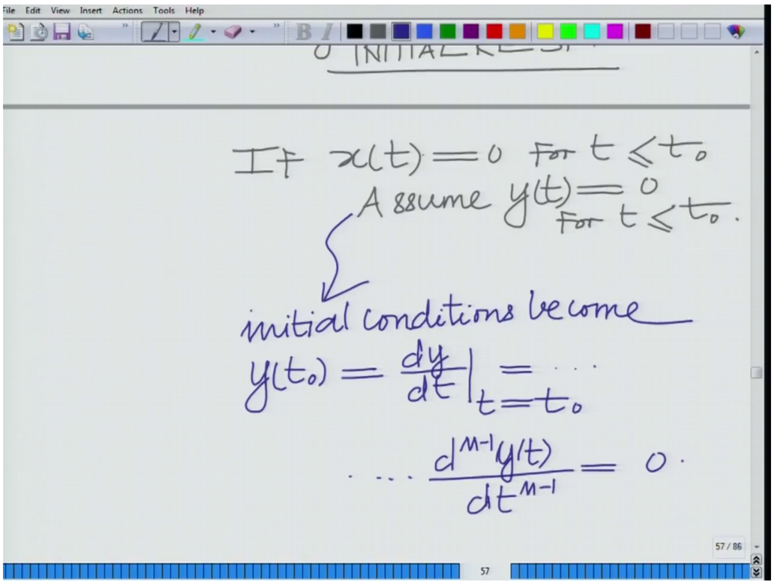

And therefore, initial conditions become the initial conditions become that is y of t naught, it was d y over d t evaluated at t naught and so on that is the value of the that is the output signal at t naught and its various derivatives of various orders until the mth m minus 1 equal to 0. So, these are the, these become your initial condition.

So, the system has to for the system to be time invariant for the system to be time invariant correct; that means, it has to satisfy the initial rest condition which we have specified over here. So, the auxiliary conditions are 0 it becomes linear system, if it is at initial rest if its satisfies the initial rest condition then it becomes a time in that then the system described by this differential equation becomes a time invariant system. And if it satisfies both naturally it will be a linear time invariant system, that is auxiliary conditions are 0 as well as it is at initial rest it becomes a linear time invariant system.

So, I think that completes our discussion on the differential equation representation of a linear system. So, we will stop here and look at other aspects in the subsequent modules.

Thank you very much.