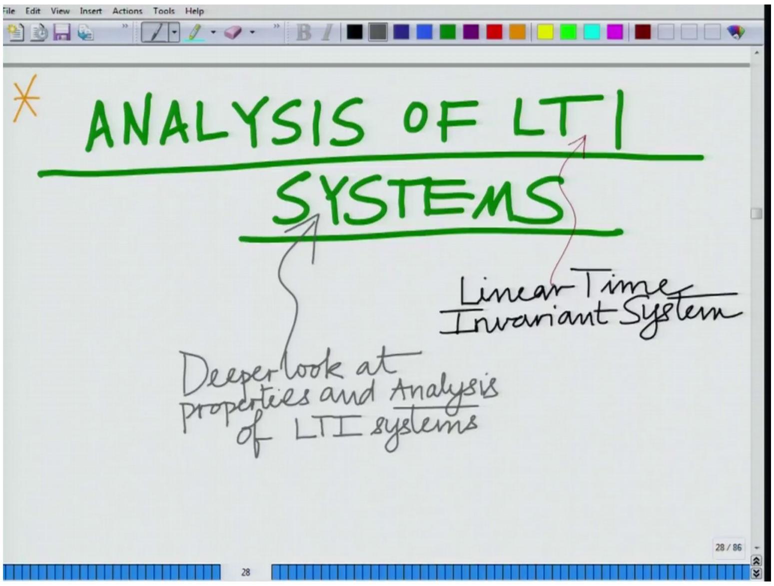
Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 10
Properties and Analysis of LTI Systems – Impulse Response, Response to Arbitrary Input, Convolution and Properties

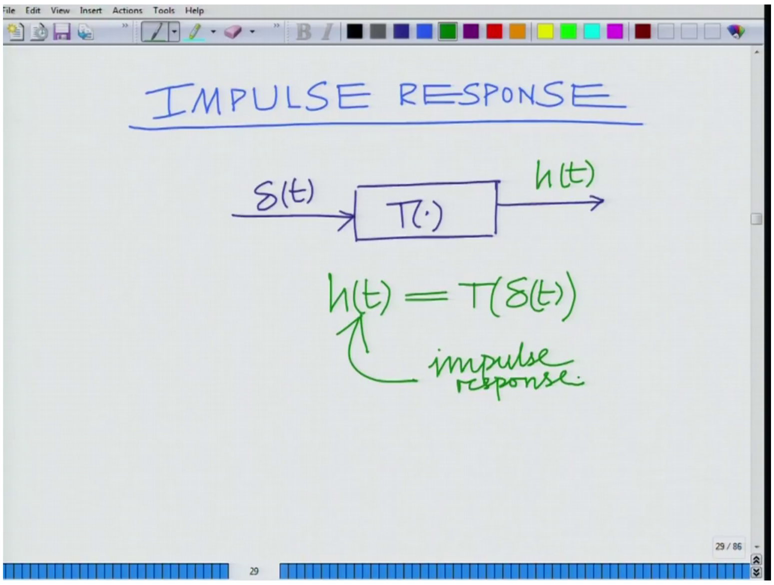
(Refer Slide Time: 00:27)



Hello, welcome to another module in this massive open online course. So, in this module, we are going to start looking at a new topic that is the analysis of LTI systems or the analysis of linear time invariant systems. So, in this module, we want to start looking at the analysis of LTI systems, where we are well aware at this point that LTI is stands for the term LTI is stands for a linear time invariant system.

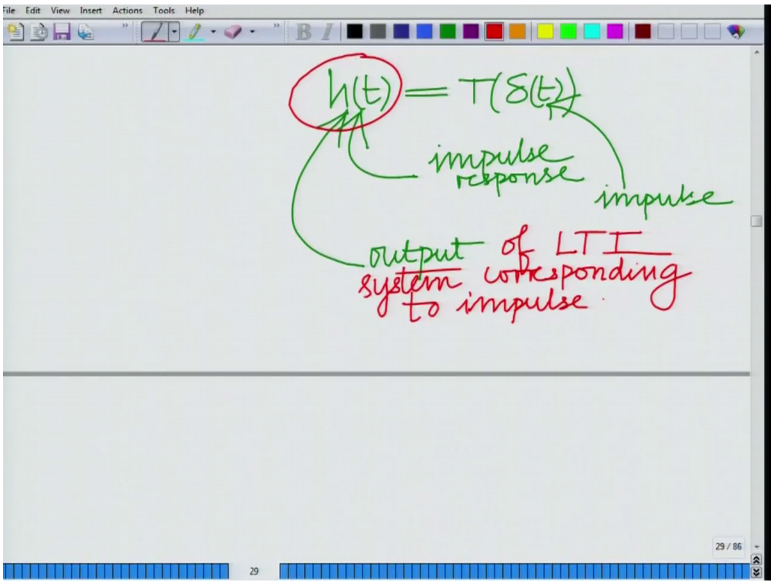
So, what we want to do is we want to take a in depth look, we want to take a deeper look at the analysis and properties of these LTI systems. Since, these are the kind of systems which we most frequently encounter in practice and which we are also going to keep encountering throughout the rest of this courses, these form a key component a central component of this course, it would be wise to take a deeper look at the properties and analysis of this system. So, we would like to get in take a deeper look or in depth look at the properties and analysis deeper look at the properties of an analysis of LTI systems.

(Refer Slide Time: 02:28)



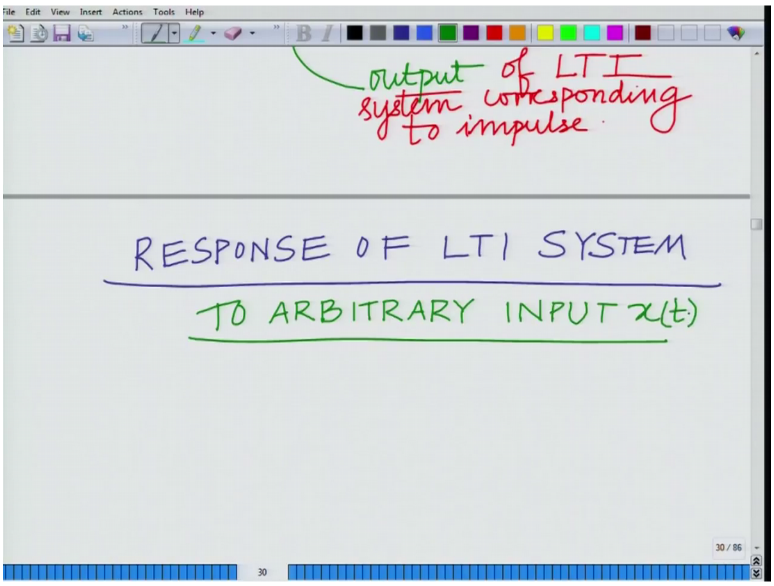
Let us start with the concept of an impulse response. So, I would like to start. So, the first thing fundamental concept in an LTI system is what is known as the impulse response that is an impulse response. The impulse response of an LTI system; and as the name implies the impulse response of an LTI system is nothing but the response or the output signal of the LTI system corresponding to an impulse that is delta t that this output signal is denoted as the impulse response. So, the impulse response of an LTI system is simply if you look at it, this is simply h of t that is if you have an LTI system T then represented by the transformed T. And to which we give the input delta t then the output h of t that is h of t it was T of delta t. This is the impulse, this is the impulse response of the system.

(Refer Slide Time: 04:08)



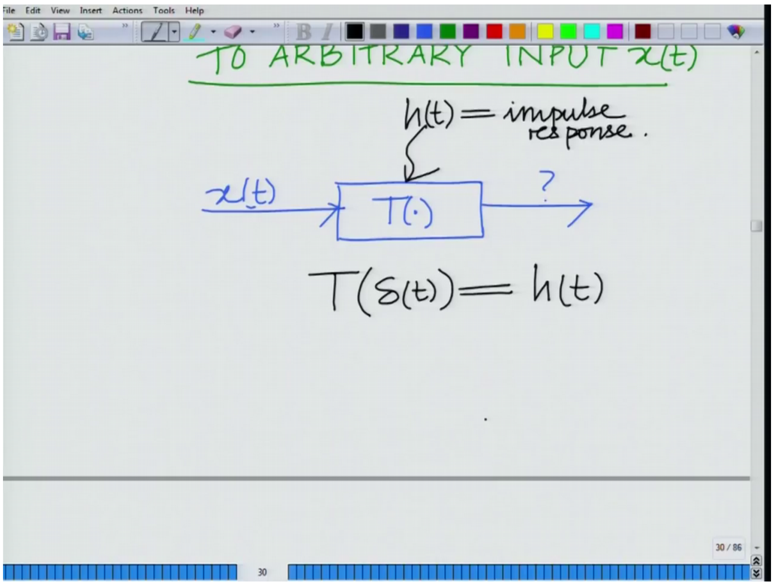
So, the response of the system correct. So, h of t, so this is the impulse. So, delta t this is the impulse, this is the impulse response. This is the output corresponding to a impulse output that is clear output of LTI system corresponding to an impulse response to an arbitrary. Now, we will see later that this h of t has a very important role to play because h of t is fundamental role to play in determining the output of this LTI system corresponding to any arbitrary input. And how is that that is something that we have we will look at in the future subsequent.

(Refer Slide Time: 05:14)



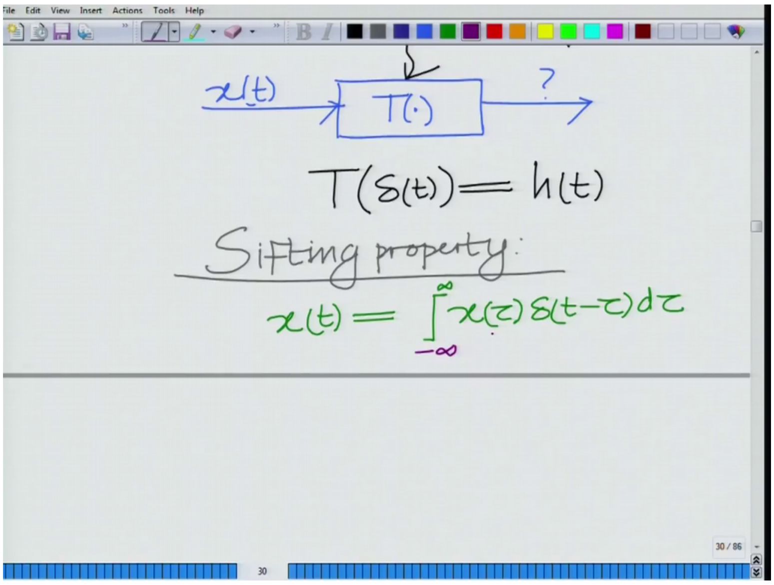
Now, let us look at the response. Now, given any arbitrary signal. So, what we want to look at now as I have alluded to is the response of LTI system to an arbitrary input signal response of LTI system to an arbitrary input signal. Let us say x of x of t that is I have an LTI system what we want to do is basically I have an LTI system, which is characterized by T.

(Refer Slide Time: 06:06)



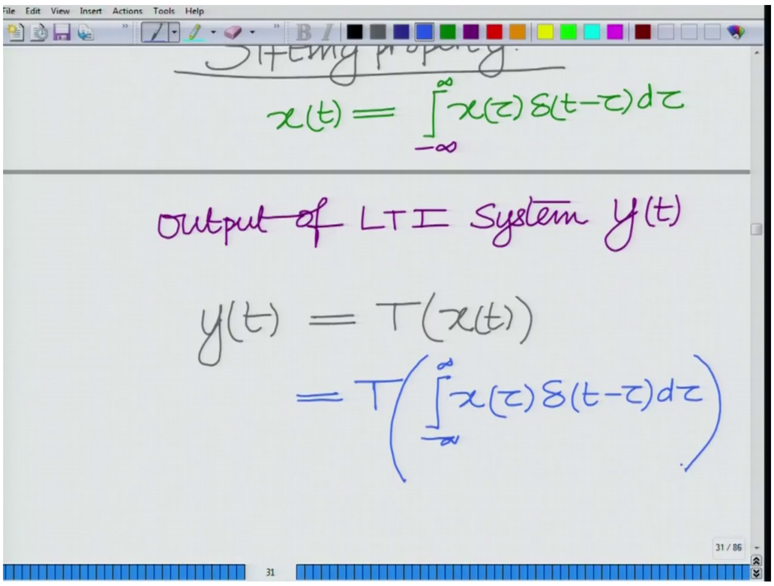
I have an input x of t what is the corresponding that is what is the corresponding output given an arbitrary input signal, and given that T of delta t let us also assume that we know the impulse response T of delta t equals h t for this system. So, let us say impulse response of this system equals impulse response of this system is h t. So, now we would like to characterize the output of the system for any arbitrary input signal x t.

(Refer Slide Time: 07:08)



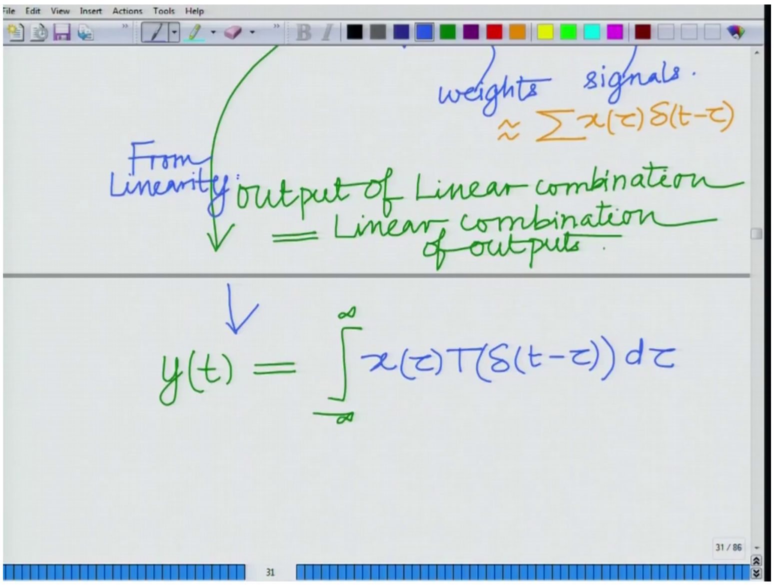
Now, to do that first let us start by using the sifting property, the sifting property for a continuous time signal, which is something that we have seen previously. Using the sifting property, we can write x of t as well integral minus infinity to infinity x of tau delta t minus tau d tau. I can write the sifting using the sifting property correct, I can write this is something that we have seen before x of t equals integral minus infinity to infinity x of tau delta t minus tau d tau.

(Refer Slide Time: 07:59)



Therefore, the output of the LTI system y t is given as y t equals T of x t. Now, I am going to use the sifting property to substitute the above expression for x t that is minus infinity to infinity T of x tau delta t minus tau d tau. Now, remember we are looking at an LTI system. Now, if you look at this integral x of tau delta t minus tau this is a weighted combination of impulses this is a linear combination of impulses. This is of the form alpha 1, x 1, alpha 1, x 1, t plus alpha 2 x 2 t. So, the alphas these are the weights.

(Refer Slide Time: 09:09)

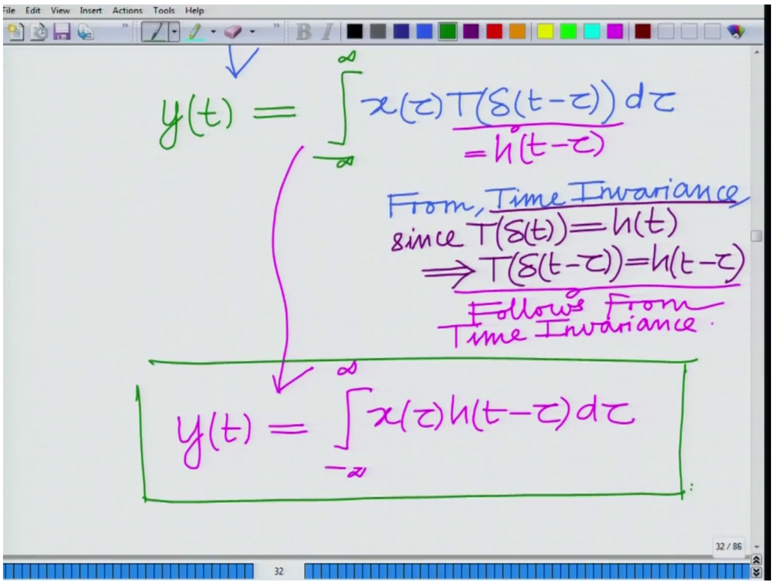


So, if you look at this, these are the weights, and these are the signals in the combination. So, you can think of this approximately is an integral it is nothing but a continuous sum. So, you can think of this approximately as summation x of tau that is this is a linear combination of several signals the delta t minus tau it is a linear combination of several signals. And since this is a linear time invariant system, the output of a linear combination is basically a linear combination of the outputs of the individual signals in the linear combination. So, we have from the basic property of linearity what we have is combined additivity and homogeneity additivity and homogeneity we have output of a linear combination of signals that is equals linear combination of outputs.

Therefore, which means this quantity here can now be simplified as y of t equals minus infinity to infinity x of tau T of that is output corresponding to each delta t minus tau because it is a linear combination of the delta t minus tau signals. Therefore, the output of this linear combination is basically the linear combination of the outputs corresponding to the delta t minus tau signals.

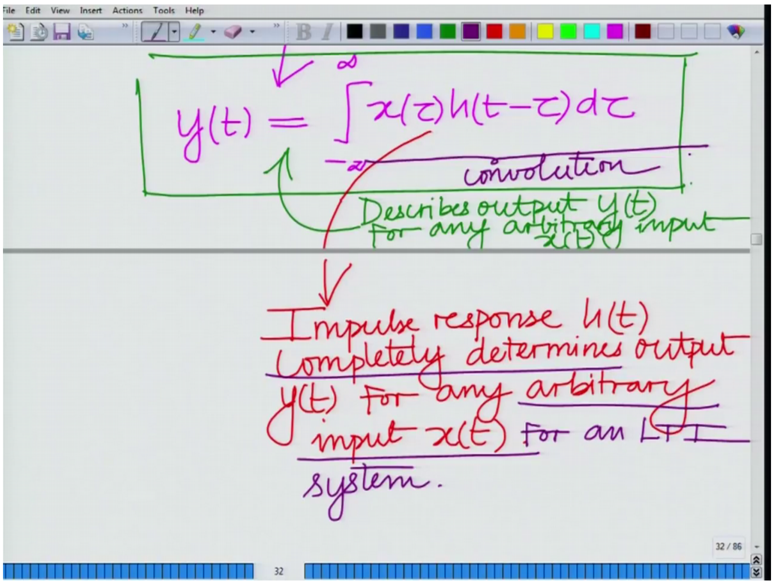
And further now you can see the output for each delta t minus tau also follows from time invariance, because we know the output for each delta t is h of t. Therefore, from time invariance, the output for delta T of an output to delta t minus tau is h of t minus tau, this is a consequence of time invariance. So, this property this follows from, so this first step follows from linearity, this is an important aspect.

(Refer Slide Time: 12:04)



Now, further from time invariance, since T of delta t equals h tau or t this implies T of that is an output to delta t minus tau equals h of t minus tau since this follows from. So, since the output to T of the output to delta t, this T of delta t is h of t output to t delta t minus tau that is T of delta t minus tau is h of t minus tau, this follows from time invariance. So, we have used linearity we have used time invariance. Now, therefore, this can be replaced by this is equal to h of t minus tau. And therefore, now we can further simplify this interestingly as y of t equals minus infinity to infinity well x of tau h of t minus tau this describes the output y t to any arbitrary input x of t.

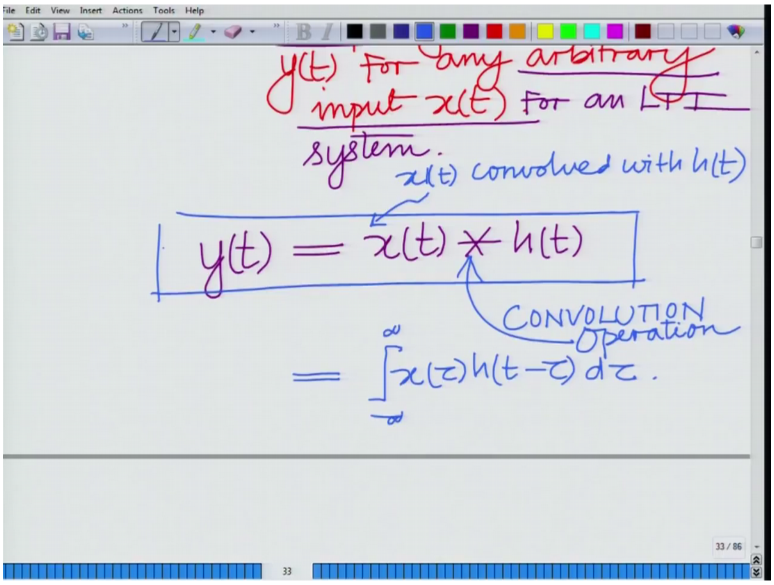
(Refer Slide Time: 13:49)



So, this equation describes given the impulse response for any arbitrary input x of t. And therefore, knowing the impulse response the impulse response h of t remember look at this, this is given the output y t is given in terms of the impulse response and the arbitrary input signal x. So, knowing the impulse response, one can completely characterize the output signal y of t corresponding to any arbitrary input signal x of t. So, impulse response, so this is important impulse response, impulse response h of t completely determines output y of t for any arbitrary input x of this is the property of the impulse response completely determines output for any arbitrary input x of t that is the interesting thing.

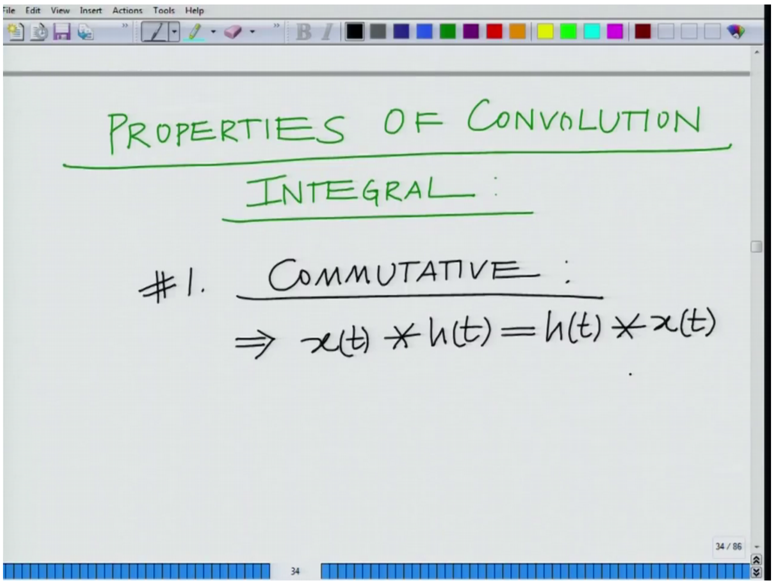
So, the impulse response fundamentally characterizes the output yeah. And keep in mind this only for an LTI system; for any arbitrary input x t for an LTI system. Remember that is fundamentally important because that is the premise on which this whole that is the edifice on which this whole thing is built that is it is an LTI system. We have used the property of linearity, and we have used the property of time invariance. In fact, this form of this relation integral minus infinity to infinity x of tau h of t minus tau d tau this has a very fundamental role to play in the analysis of LTI system. This is known as the convolution integral or simply termed also as a convolution, this integral is known as the convolution integral, this is also simply termed as the convolution.

(Refer Slide Time: 16:32)



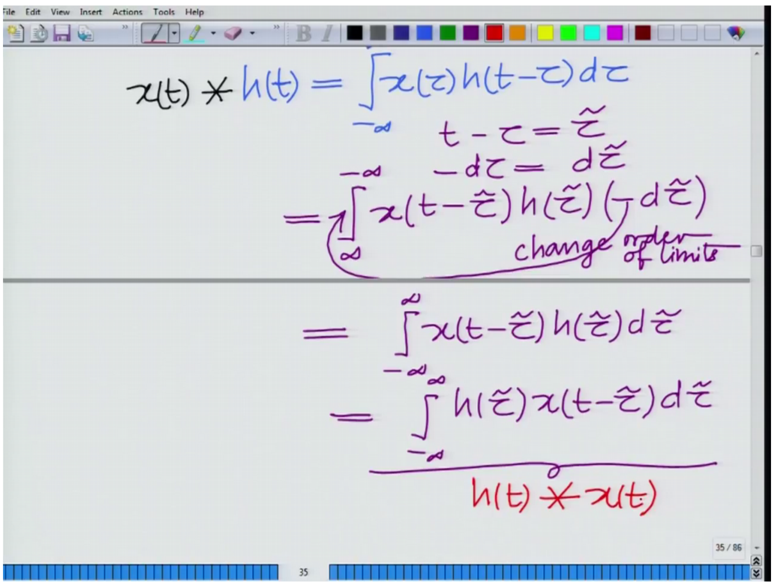
So, y t is simply written as x t convolved with x h d. So, this is termed as the convolution operation, h t convolve with x t, this is termed as the convolution. Let me just write it correctly, since this is a very important topic. This is a convolution operation which is of course, equal to x of tau h of t minus tau d tau, this x t convolved with the input convolved with output, x t convolved with the way to read it is x t convolved with h d. The input signal x t convolved with the impulse response h which determines the output corresponding to the arbitrary input signal x t. So, this convolution integral or this convolution operation has an important role to play in the analysis and properties of LTI systems. Let us look at some of the properties salient properties of this convolution integral because this is going to keep occurring very frequently in fact to determine the output of the LTI system.

(Refer Slide Time: 18:22)



So, let us look at some of the properties, properties of convolution, properties of convolution, for that matter properties of the convolution, properties of the convolution integral. So, we have the first property, we would like to show is that the convolution is commutative, it is a very important property. This means that x t convolve with h t equals h t convolved with x t that is for any two signals x t and h d t or any two signals x t and y t let us say x t convolved with y t is y t convolved with x. And this can be simply shown as follows.

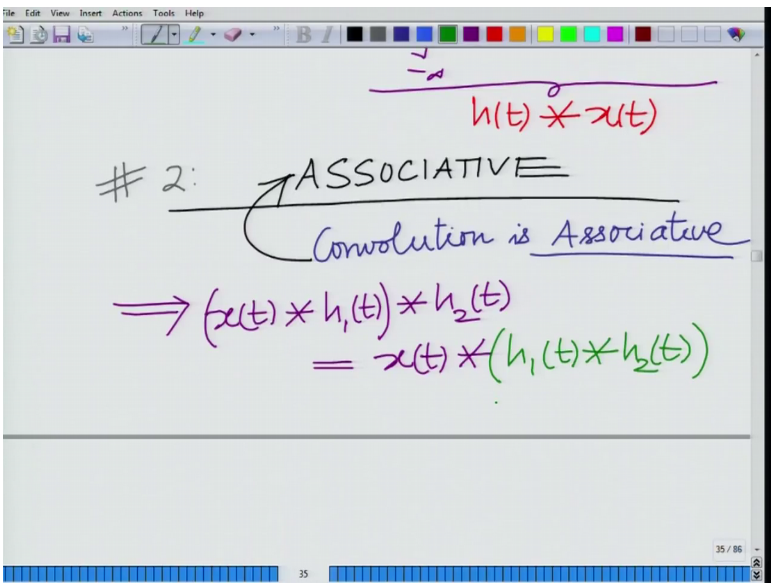
(Refer Slide Time: 19:38)



We have x t convolved with h t from our definition above x t convolved with h t this is equal to minus infinity to infinity x of tau h of t minus tau d tau, now set t minus tau equals tau tilde. So, this will become, so this implies minus d tau equals d tau tilde. So, this will become now the limits will be tau tilde is this t minus tau when tau equals infinity t minus tau for a given value of t any given finite value of t, this will be minus infinity. When tau is equal to minus infinity t minus tau will be plus infinity x of tau is x of well t minus tau tilde h of t minus tau is h of t minus tau is h of tau tilde and d of tau will be minus d of tau tilde. Now, here we have the integral going from infinity to minus infinity. And we have a negative sign.

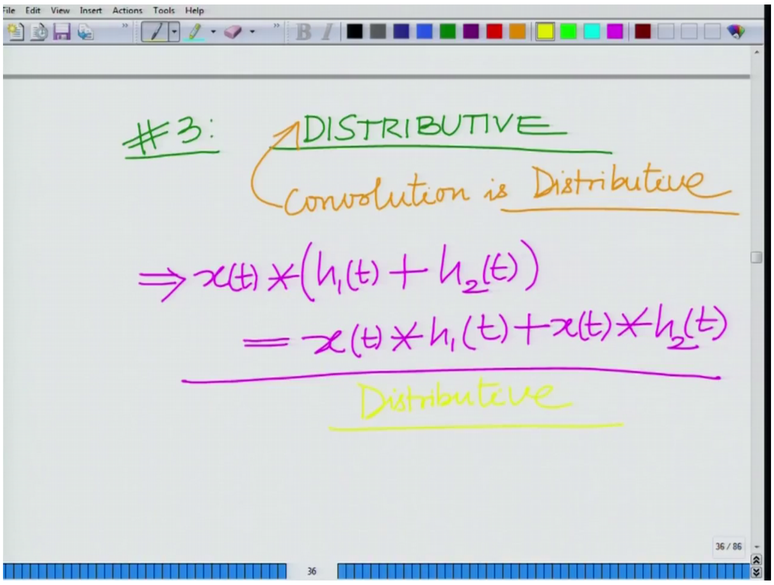
Therefore, this negative sign can be used to, so this will be negative of the integral from minus infinity to infinity which will cancel with the negative sign. So, this negative sign can be used to change the order, change limit change the order of limits. So, this will be integral infinity to minus infinity is negative of the integral from minus infinity to infinity. So, this is equal to integral minus infinity to infinity x of t minus tau tilde h of tau tilde d of tau tilde, which is basically again just to write belabor this point a little h tau tilde x of t minus tau tilde d tau tilde which is nothing but your h of t convolved with x of t, so that is what we have so the commutative. So, the convolution operation of the convolution integral is commutated that is x of t convolved with h of t is same as h of t convolve with x of t.

(Refer Slide Time: 22:23)



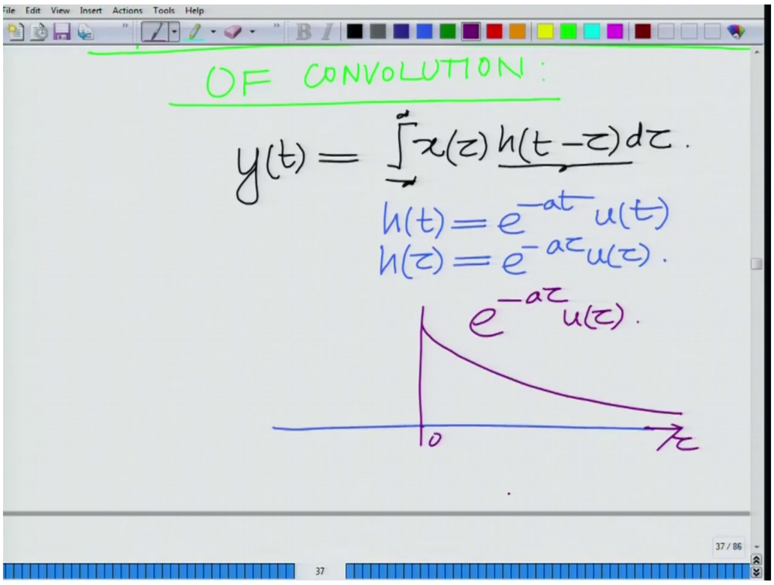
In addition, you can show a few other simple properties of the convolution such as the other property that naturally follows is known as associativity. So, the convolution operator is associative. So, the convolution is associative. This implies if you have two signals x of t convolved with h 1 t convolved with h 2 of t x of t first convolved with h 1 then convolved with h 2, this is equal to x of t convolved with that is first convolve h 1 t and h to t and then convolve it x 1 t. So, x 1 t convolved with h 1 and subsequently convolved with h 2 is equivalent to x 1 t convolved, so h 1 convolved with h 2 h 1 t convolved with h 2 t and subsequently convolve with x of t. So, this is basically the associate property of associate.

(Refer Slide Time: 23:54)



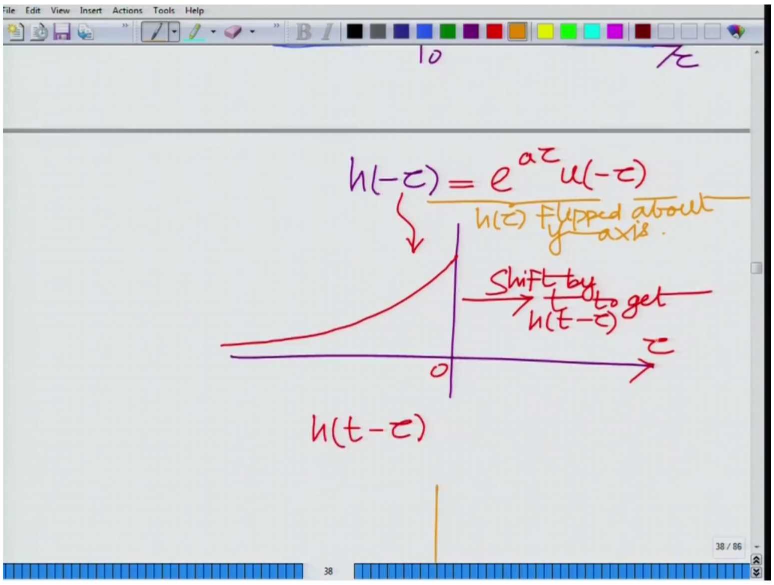
Third property is the distributed nature of conversions, so the convolution is distributive. So, the third property convolution is distributive. This implies that again for two signals x t convolved with h 1 t plus h 2 t equals x t convolved with the some h 1 to h 2 t is equals x t first converted with h 1 t plus x t convolved with h 2 t. So, this is basically from the distributive property. And this is basically distributive property. So, these are the three fundamental properties of the convolution.

(Refer Slide Time: 25:30)



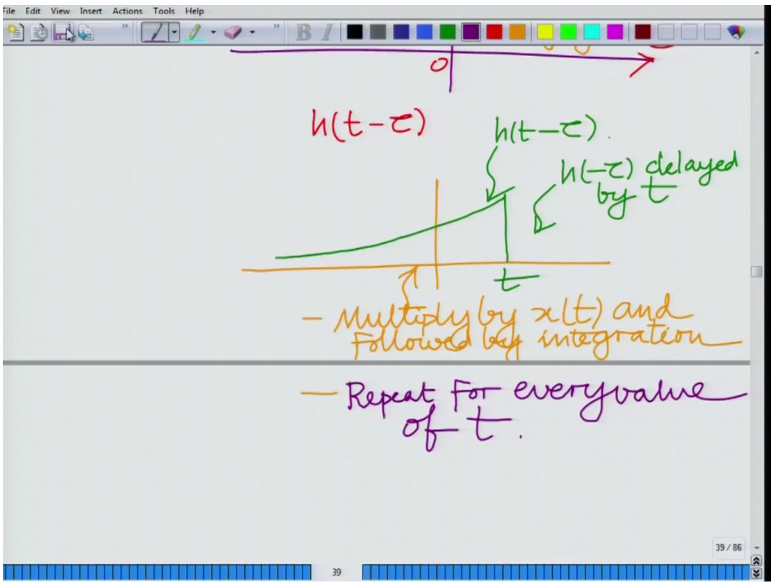
Now, let us look at something interesting. Let us look at a graphical representation of this convolution operation. So, let us look at graphical representation of this convolution operation. We have y t equals integral minus infinity to infinity x of tau h of t minus tau, let us look at this quantity h of t minus tau, for instance, let us take h of tau or h of tau or h of t equals e power a t that is an exponential decreasing exponential e power minus a t into u t, which means h of tau equals e power minus a tau into u tau that is nonzero for tau greater than equal to 0. And it is exponentially decreasing for e greater than 0, it is exponentially decreasing. So, this is e power minus a tau u tau. So, this is exponentially decreasing. This is 0, this is the tau axis. Now, let us look at h of minus tau. So, we are trying to graphically interpret this convolution operation. Let us now look at h of minus tau. So, h of minus tau corresponds to flipping this h of tau about the y axis.

(Refer Slide Time: 27:38)



So, if I flip it, so basically h of minus tau will look like something like this. So, this is h of minus tau which is equal to e to the power of a tau replace tau by minus tau e to the power of a tau u of minus tau. So, this is tau, this is 0, this is h of now h of t minus tau will be taking this h of minus tau and shift, shifting it, shift by t to get h of t minus tau. So, h of t minus tau, so this is basically you can see this is flipped h of tau flipped about the y axis.

(Refer Slide Time: 29:01)



Now, if you shift it to the right or delayed, shifted to the right or delay, if you delay this by t, so this is somewhere you have t this is your h of t minus tau. So, this is basically h of minus tau delayed by h of minus tau delayed by t. So, what we are doing is basically first flipping about the y axis, and shifting to the right by t that gives you h of t minus tau. Now, multiply this by x of tau and integrate from tau, integrate tau from minus infinity to infinity, and do this for every delay or every point.

So, next step is multiply this with x of t and integrate, multiplied by x of t followed by integration, and repeat for every value of t repeat this for every value. So, what you are doing is your flipping, your delaying it by t, and you are slowly moving it ahead for every value of t, and in each point t you are computing the value of this convolution that gives you the net output of this convolution. So, flip, keep shifting to the right computing it for every value of T, of course, if t is negative because remember t can go anywhere from minus infinity to infinity, if t is negative automatically a delay becomes an advance. So, start from t equal to minus infinity to infinity keep shifting towards the right move towards t equal to infinity, and keep calculating the value of this integral at every point that is that basically is a graphical representation of this convolution operation.

So, basically that so what we have seen in this module is we have started looking at the properties and started to analyze write analyze LTI systems, start to take an in depth look at the properties of an analysis of LTI systems. We have introduced the concept of impulse response and describe how the impulse response can be used to characterize the response of an LTI system to any arbitrary input signal x t, and following that we have looked at several properties of this convolution and the graphical representation. So, we will stop here and as already said convolution is a very important concept in the analysis of LTI system. So, it is important to understand this. So, please take a look at these various concepts and make sure that you understand them thoroughly.

Thank you very much.