Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 08
Standing waves on T-lines

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Hello and welcome to the mooke on applied electromagnetics for engineers. In this module we will discuss some important effect called standing wave on a transmission line and how to quantify that one using what we call as voltage standing wave ratio or simply standing wave ratio. We will see the importance of standing wave ratio and its relation to input impedance of a transmission line and refraction coefficient of a transmission line.

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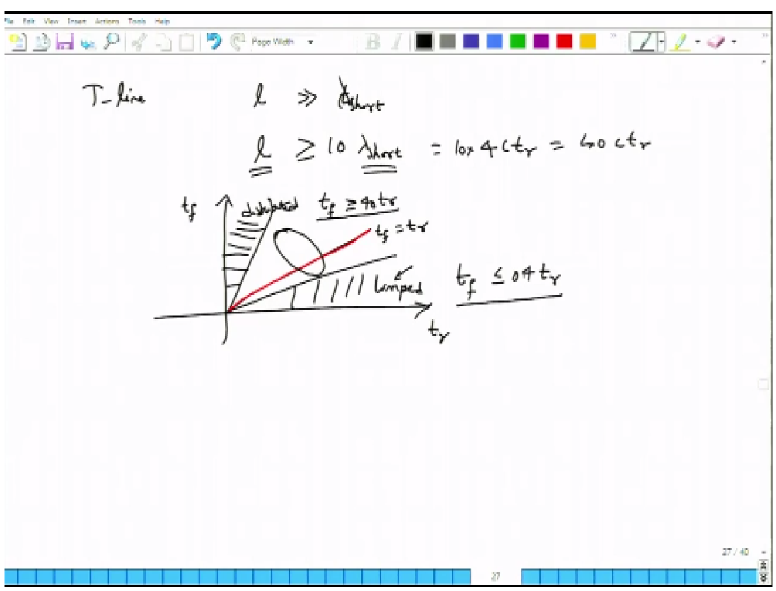

Before doing that let me just take couple of minutes to go back to the discussion that we had in the last module lumped that were distributed, to refresh your memory we talked of lumped regime when the time of flight was less than or equal to 0.4tr and we talked of distributed when tf was greater than or equal to 40tr. So if I plot tf versus tr you see that the slope of this train and any lines that is there in this region here in the lumped regime will be less than or equal to 0.4 tr and this is the lumped regime.

So the lines that are here will have a slope which is greater than or equal to 40 times tr and that to correspond to the distributed or the transmission line regime. This line not very well drawn, but this line has a slope of equal to 1 tf=tr this is the grey area, here you can apply both lumped regime and the transmission line theory both will give you reasonable accurate answers okay. with this let us move on to study the standing wave ratio which comes up in a very important aspect of power matching or the impedance matching which we will take up later on during graph study okay.

What is this standing wave ratio, let us first setup the problem, I have a transmission line over here which I will terminate with the load okay, this time the load could be any general load that I am considering, this load could also be frequency dependent, because this could have some resistive as well as capacitive or inductive components to it. I have a voltage that is coming in right, so there will be a source kept at some distance okay, I would not specify where the force is kept through that, and one of the thing that I will talk to you about how to setup these problems.

So all I am looking at is from the load site, so this could for example be the situation when you are working with an antenna okay, you know that source is somewhere out there okay, and your antenna is residing the signals or your antenna is connected and now the sending out signal font to the source maybe the TV set which is kept in your home no, far away from you, and you are working at the antenna site.

So what you are interested and what you are doing is to measure something at the antenna site and you want to understand what is happening at the behaviour okay. So that is the problem that we are looking at, so what we are looking at is I have a load here and it is connected to some source which is kept at a distance which we do not really known, but we do not really care also. And this, the characteristic impedance of your transmission line is some Z0 and you have terminated the transmission line with some load impedance ZL.

Clearly if ZL is not equal to Z0 there will be some reflection, you know the voltage that is coming in after same ZL will be reflected partly, similar in the current that will be flowing in would also be reflected when constitutes are reflected current. So there is a reflection coefficient associated with this one, but if I were to, you know move along this transmission line and make my measurements okay of the voltage here, I would not just find what is V0+ I will actually find the forward going voltage as well as the backward travelling voltages.

Similarly, if I make the current measurement of course I do not do that, but if I were to hypothetically make the current measurement at any point on the transmission line I would again see the forward going current I0+ and the backward travelling current I0- okay. So that is what it is and what I am interested is to follow or try to find out the magnitude of this voltage as we move along the transmission line.

This magnitude of the voltage along the transmission line will actually tell me the kind of load that we have connected, supposing the load is unknown which happens in most cases, if I want to find out the nature of the load then I need to find out what track the magnitude of the voltage along the transmission line. How would that magnitude of the voltage change, it would change depending on the different types of load that I would terminate the transmission line with. So I would like to get a slightly general expression and from there develop the concept of voltage standing wave ratio.

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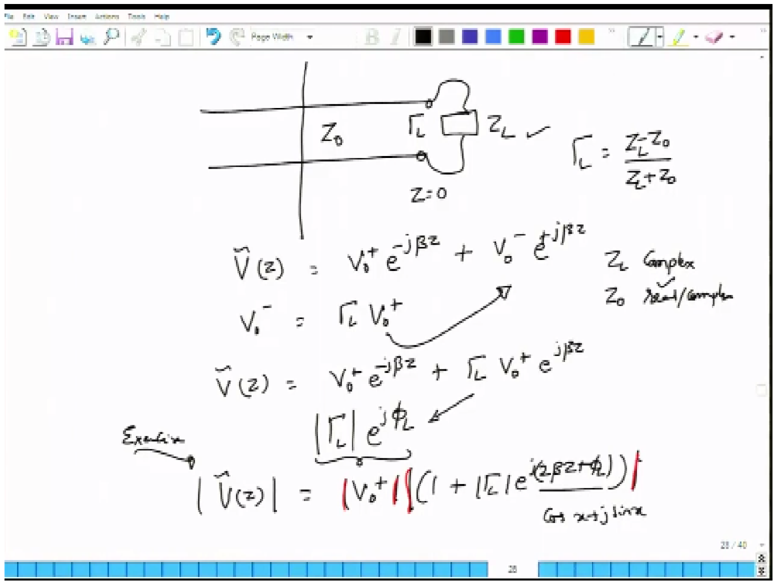

I already know that if I take a uniform transmission line and terminate it with some load impedance ZL here and consider the location of the load to be at Z=0 and then move backwards towards the transmission line of some length, you know I do not really know what the length is, I do not even know where the force is terminated. But at any particular Z on the transmission line that is location on the transmission line I know the total line voltage okay, which will be a phasor which I can denote it as Vbar of Z.

And this voltage will be the sum of the forward going voltage I am considering the transmission line of no loses that is the characteristic impedance Z0 is completely real it is a loseless transmission line. So the total voltage at any location Z will be V0+$e^{-j\beta z}$ we do not individually measure this, we only measure the total voltage which would be given by V0+ $e^{-j\beta z}$+V0- $e^{-j\beta z}$, but V0- is related to V0+ because I know what is, suppose I know what is the load, of course I even consider the cases where I do not know the load.

But let us say because there is a load here and there is a characteristic impedance of the transmission line Z0 there will be a reflection coefficient at the load maybe unknown, but there will be certain reflection coefficient okay which we denote by ΓL. So V0- the amplitude of the reflected voltage is ΓL times V0+ at the load and you can make that substitution back into this one okay. So your total voltage V(Z) will be V0+ $e^{-j\beta z}$+ ΓL V0+ $e^{-j\beta z}$. If ZL is complex that is it will have both resistive and reactive components then if ZL is complex then ZL and Z0 which could be real or it could be also complex, but in this case we are only considering real cases.

The reflection coefficient ΓL will definitely be complex, because ZL is complex and just to refresh your memory ΓL is nothing but ZL-Z0/ZL+Z0 alright so this ΓL will also be complex. Any complex number can be represented or return in the polar form by giving its magnitude and the phase angle which I will call this as ϕL. So I can substitute for ΓL into this expression and rewrite the magnitude of the voltage after taking the magnitude of the voltage I mean on both side as V0+ magnitude 1+| ΓL| $e^{j2\beta z}$+5L this entire thing magnitude okay.

So let me denote the magnitude points by writing this with a red line okay. I will leave this again as an exercise to you it is a very simple exercise please find out and verify that, the magnitude of the voltage that I have written is given by this I can of course simplify, because it is e^{jx} here which I can simplify, because e^{jx} is nothing but cosx+jsinx simplify it 1+|ΓL| is present here it is multiplying cos 2βz+5L and there is a imaginary component +jsin2βz+BL and after taking the magnitudes what you end up with is this expression.

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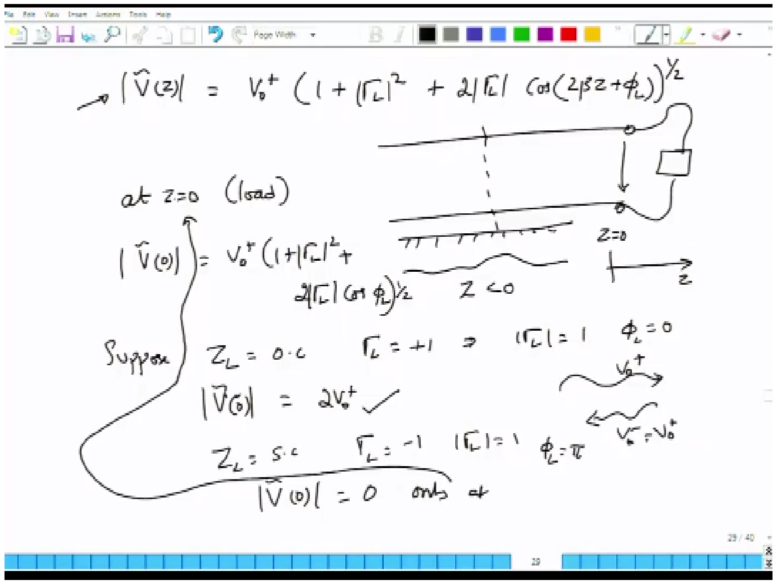

Again this is just one step did I know verification, I did not consider V0+ to be real therefore I will remove the magnitude sign from that one and in the bracket what I get is magnitude ΓL^2+2 magnitude ΓL cos2βz+5L you can verify this and there will be a 1/half on top of it, because this is a root of this entire expression okay. Now let us analyze this equation okay, I have the transmission line at Z=0 I have connected the load right, so if I substitute Z=0 what do I get in this equation. So at Z=0 that is at the location of the load, because my Z axis is going here all these transmission line is actually with Z less than 0 that is to say the numbers here if you were to mark this one by a ruler these numbers will all be negative okay. So that is the meaning of Z less than 0 over here.

So at the load position which happens to be a right at Z=0 what is this expression tell me, the expression will tell me that magnitude of the voltage at the load location is given by V0+ here I have to put Z=0 which means 2βZ is 0 so I get cos ΓL so I get 1+magnitude of ΓL^2+2 magnitude ΓL cos of $5L^{1/2}$. Of course I need to know what kind of load I have terminated. Suppose my load happens to be open circuit, so for an open circuit in case I know that ΓL=+1 which also implies the magnitude of ΓL is 1 and the phase angle ϕL will be equal to 0 or 2ϕ does not matter.

So with 5L=0 cos of 5L will be equal to 1 and what you get is 1+|ΓL^2|+2|ΓL| which is nothing but 1+ because |ΓL| is 1 so this would be 1+1=2 $2+2^{1/2}$ which is √4 which would be 2 and the magnitude will actually be equal to 2V0+ okay. So the magnitude of the voltage at the load will be 2V0+ which is what we know, because the entire voltage that is launched will be reflected. So

V0+ and V0-, V0- will have exactly equal to V0+ and at the load the point they will add together and they will be, you know they will give you magnitude of 2V0+.

Suppose I consider the load to be short circuit for the short circuit I know that ΓL will be equal to -1 you go back to the expression and find this out. The magnitude of ΓL will still be equal to 1, but this time the angle 5L is equal to ϕ. So you go back to this expression and write this. So you get -2 which would be 0 and the magnitude of the voltage here will be equal to 0.

This is true, but this is also true only at the load location, because that is precisely at which point we are evaluating my voltage magnitude using this expression okay. As you move away this expression predicts that the voltage magnitude will change okay. But there is a certain point where the voltage magnitude will become maximum, when will that happen.

If you look at the equation closely as Z changes 2βZ also changes. And the total argument of the cosine function which is 2βZ+5L must go through some integer multiple of 2ϕ at some point when it goes to the integer multiple of 2ϕ I see that the magnitude of the voltage would have become maximum. Similarly, when it goes through a multiple of, you know when will the cosine function go through its minima when it goes through an odd multiple right.

So when it goes through an odd multiple the argument of the cosine function then the sign of cos becomes negative and the voltage would actually be minima.

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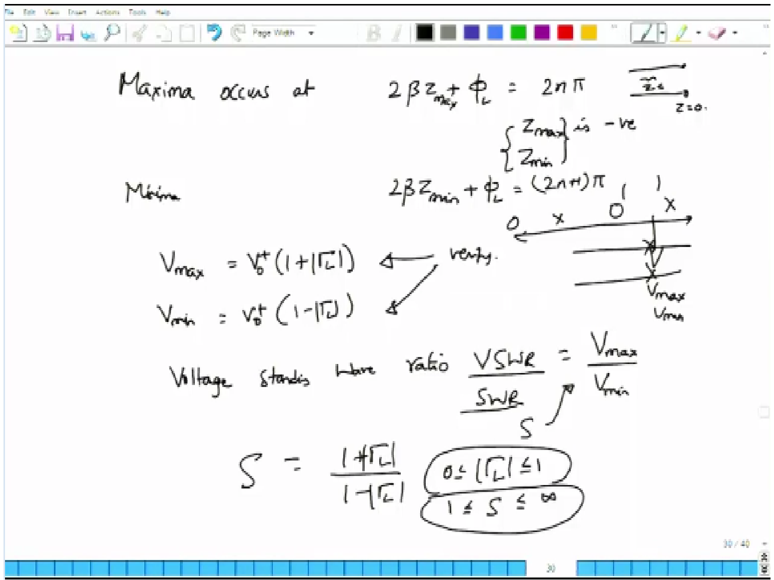

So the maxima are attained or we say that voltage maxima occurs at locations when 2βZ+ let us call this as Z max okay 2βZ max at the location Z=zx+ϕL happens to be some integer multiple of 2ϕ and please note that you have to choose the value of N such that zmax is negative, because of the notational convention that we have used the transmission line is distract by the values of Z which is negative, because the low preceding at Z=0 okay.

When will we have minima, minima occurs when 2βZ I mean I just want to denote the differences between maxim and minim, so this must be equal to 2n+1(ϕ) again you have to choose in such a way that Z min is also negative okay, both Z max and mZ min must be negative. What is the maximum value of this voltage or what is the value of the voltage maxima?

The value of the voltage maxima happens to be 1+ |Γl | x v0+ one short verification please verify this one okay similarly verify that the minimum voltage will be v0+ (1-|Γl) please note that magnitude of Γl depends on load type that you have connected again you can verify this that these two are expressions are okay, now we defined a quantity called as voltage standing wave ratio okay, we call this voltage standing ratio and we either write this as VSWR or simply SWR with the meaning that it is voltage that we are considering okay.

The current standing eave ratio will be del with an exercise and that value will just be – VSWR okay so this voltage standing wave ratio as you might guised is the ratio of the maximum voltage to the minimum voltage that can occur along the transmission line please note that both maxima and minima occur at different positions okay they both do not occur at the same position so this position will not have V max and V min unless we are considering the very special case to which we will come okay.

So usually Vmax occur here and the Vmin occurs again Vmax again Vmin occurs this is a periodic way in which it you know changes for a loss less transmission line okay and we will also calculate what is distance between a voltage maxima and voltage minima shortly, so this standing wave ratio or VSWR some time denoted by S is given by the ratio of the maximum voltage to minimum voltage from the expressions we can easily substitute and then show that this is given by $1+ |\Gamma l| / 1- |\Gamma l|$ okay.

What is the range of S well Γl can go anywhere from 0 to 1 right so 0 corresponds to no reflections 1 corresponds to full reflection therefore S correspondingly must go from 1 right to infinity so S must be = 1 when Γl = 0, so clearly this would be = 1 when Γl = 0 S will be =1 when Γl = 1 for a open circuit or short-circuit or for a reactive termination then S will be = infinity, so this is the range over which S can vary this is a range over which reflection coefficient can vary.

These are for passive loss less transmission lines there are no active amplifiers in the line that we have consider what is the important of standing wave ratio it terms out that you cannot actually measure the reflection voltage or reflection coefficient so easily because we will see later class but what is reasonably okay to measure is the voltage maxima, okay and the voltage minima this is a classic experiment to actually determine the type of load you might perform this experiments in your microwave laboratory okay.

You take transmission line you do not know what is the load we will talk about the proper measurement later on but you have a load here and what you do is you connect a volte meter or rather a power meter, and then you keep moving this you know along the length of the transmission line and record the voltage magnitude that you are going to see not the phase but only the voltage magnitude so you record this voltage magnitude at some point each or max keep going you will reach a minimum okay.

When you take the ratio of maximum to minimum you will be able to find out the magnitude of the reflection coefficient okay also by noting where the maxima and minima occurs on the transmission line you will be able to find out the phase of the reflection coefficient, so you would have found out the magnitude you would have found out the phase knowing the value of z0, and the value of full reflection coefficient Γl you will be able to calculate the value of the unknown load impendence so this is in fact the important of voltage standing wave ratio.

You can make this measurements and determine the unknown standing unknown voltage unknown load that you have terminated the transmission line another important application or another important thing that occurs with standing wave ratio is that this is the quantity that is specified by most RF device manufactures okay, when SWR if the devices are well matched we will talk of matching later on if the devices are well matched then this SWR will be about 1 that is the ideal value.

Any way from 1 to 1.25, 1.5 is also okay but if this standing wave ratio have pushed to larger say by 3 or 4 or 5 may be 10, then we know that the 2 systems are not very well matched and there is considerable reflection occurring between the two components leading to lot of problems first your power your efficiency goes down the because the voltage is circulating and trap the wire will also have to able to with stand this higher voltage.

So all this problems can occur to put it simply standing wave ratio is a quantity that is measurable and we will lot of inside about the type of the load that you have connected as well as what is happening on the transmission line or in terms of the matching between 2 components okay we will go back to few special cases so that this concept is clear to you.

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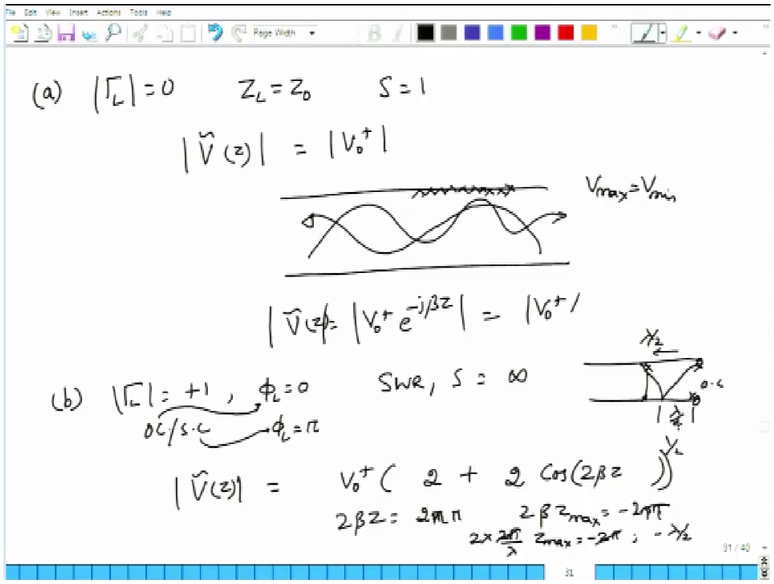

Consider what happens you know in the case when Γl = 0 okay when can I have this satiation when I terminate the transmission line with it is own characteristic impendence Z0 in this case since Γl = 0 the standing wave ratio = 1 what does it mean it simply means that if you go to the expression of the magnitude of the line voltage this turns out be = V0+ you know magnitude of V0 + what it means is that on the transmission line there is only propagating voltage and there are no back reflected voltages.

So when there are no back reflected voltages your voltage phase would simply be = V0+ $e^{-\beta Z}$ okay this is the only wave that is prorogating there are no reflections Γl = 0 and the magnitude of this will simply be = V0+ so there are so in this case the maximum voltage will be = minimum voltage and this location of maximum voltage and minimum voltage occurs at all points on the transmission line so this is only a very special case when S = 1.

Consider the case when Γl magnitude = +1 okay and the phase angle 5l = 0 in many case the standing wave ratio SWR in this case are S as we continued will then be equal to infinity because Γl magnitude = 1 okay we can have the situation either by a open circuit or the short circuit for open circuit the phase angle is 0 for short circuit the phase angle will be = Π but if I find out what is the magnitude of the voltage V(z) we will see that this would be given by V0+ (1+ magnitude Γl^2 + 2 magnitude Γl cos 2 βZ + ϕl) correct to the power ½ in this case Γl magnitude can be equal to 1.

So I can replace them with 1 here and I can replace this one also by 2 so this would be 1+ 1 which would be =2 okay so this would be 2+ 2 cos βZ + ϕl clearly ϕl will be = 0 for the open circuit case and this is how your able to find out is the voltage where will the maxima occur here when 2βZ = 2 Π or rather 2N Π N must be chosen such that Z will be negative, so I cannot choose N = 0 I cannot choose N = -1, so when I choose N = -1 I get 2βZ I am finding the maximum this would be = -2 sorry -2 Π.

Since β is 2 Π/ λ the maximum occurs here if I expand β to 2 Π/ λ and maxima occurs here at -2Π, so 2Π cancels on both sides and maxima occurs at λ/2 – λ/2 which means that from the load okay so this is the open circuited transmission line from the load at a distance of λ/2 I will get a maximum of course in this case there is 1 maximum right on the load itself so I forgot about that one so of course in this case that is 1 maximum right on the load what it means I that there will be voltage maxima through a minima and one more maxima.

And the distance between the two maxima happen to be λ/2 clearly the distance between maxima and minima will be λ/4 as you can easily verify.

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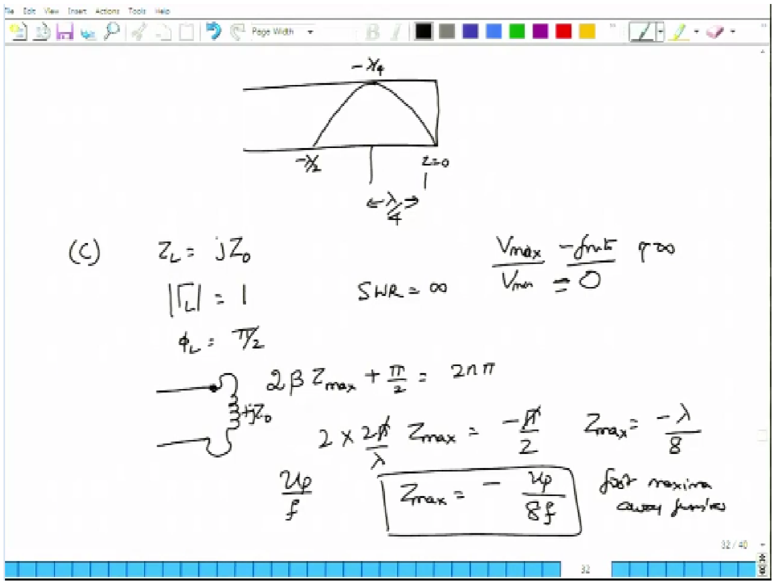

What about the short circuit case in the short circuited transmission line first there will be voltage minima then there will be maxima at a distance of λ/4 from the load okay so this is the load location again there will be one minima, at λ/2 away from the load okay as before maxima and minima are separated by λ/4 with respect to both okay, now I consider a very special case of pure reactants termination.

Suppose I consider the case where Z L is either an inductor or a capacitor, suppose this is an inductor with + JZ 0 okay, so this an inductor you can clearly see that the magnitude of the reflection coefficient will be = 1 which indicates that SWR must be = infinity why will this happen because V max could be some finite value but V min on a transmission line which lost less and in this case will be = 0.

So till because V min = 0 the ratio of finite value to 0 will go unto infinity okay so that is about the magnitude, but what about the phase angle the phase angle here ϕl will be equal to Π2 so where will the location of maxima be, so look at this equation again 2β Z max + Π / 2 must be = some 2N x Π, suppose I consider N = 0 which is a valid situation in this case so in that case what will happen 2 x 2 Π/ λ x Z max will be = - Π /2, so Π cancels on both sides and Z max occurs at – λ/ there is a 2 here in a denominator.

There is a 4 numerator so that would be 8 right so the maximum occurs when this one is equal to – λ/ 8 okay you can also write down what is λ is nothing but Up/ frequency, so the location of the maxima is given by – Up / 8 x f okay, this is the location of the first maxima away from the load okay away from load for the case of a termination which is a pure inductive termination, okay so this pure inductance had the load value = + JZ0 I believe this as an exercise for you to find out when will the for the case of a termination with a pure capacitance that is when ZL = -J0 where will the first maxima be located it turns out that the first maxima will be located at a distance of λ/4 from the previously calculated value.

So in that way inductance and capacitance they will shift the maxima voltages and the minima voltages values by about λ/4 these concepts are important because in general of course you will not I mean you will have to find out the maxima and minima location based on an experimental value and from then back calculate what the load but in a you know in question where we give you what is the load value then it is easier for you to find out where the maxima and the minima are located okay more over these value that we have found are quite okay for simple cases but when you consider a general case where the load happens to be something like RL + JXL the it will be very difficult for you to keep doing this algebraic manipulation.

So there is a graphical way in which you can solve these problems this graphical way involves the use of what is called as smith chart which is what we are going to discuss in the next class okay so until then please revise your concepts of no load terminations standing wave ratio reflection coefficient and the relationship between on this impedance reflection coefficient and standing wave ratio thank you very much.

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