Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 56
Planar optical waveguides

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Hello and welcome to NPTEL mook on applied Electromagnetics for engineers in this module they will look at an intuitive picture of waveguides, waveguides of a different type we have seen wavelets of metallic type that is you take two nice metal plates put them parallel with each other forms what is called as a parallel plate waveguide, or you take a metal and then make it into a hollow box either it is a circular box or a rectangular box then you obtain what is called as a rectangular waveguide okay.

And what was the characteristic about this rectangular waveguides or the parallel plate waveguides was that we analyzed the propagation of the modes by assuming this multiple interview no multiple reflection from one surface to the other surface, so you had an incident mode or an electromagnetic wave hitting the top surface of the metal plate then depending on whether it was a T- mode or a TM mode there was a reflection down there of course both modes reflect but then the characteristics are slightly different.

So they would reflect then at the bottom plate again you have so there would be one more reflection right and what was happening to the direction which was perpendicular like this so if this is the direction of the waveguide coming towards you what was the nature of the field in between it was actually a standing wave right, so you had a Sine KX Sine KYY when you consider two dimensional wave words and so on, so that was something that we explained using the multiple reflection model and it turns out that you can use the same model except that the details will change in order to understand propagation of modes through this plane or optical waveguide first of all what is the structure of a planar optical waveguide well it will actually have a slab okay.

You can think of a cement slab kind of a thing so I just take that slab and place it there okay usually you place that slab over what is called as a substrate the substrate has a lower refractive index compared to the slab refractive index on top of this slab you can either cover it or you can leave it open whatever you do that top surface area is called as a cover okay. So you have a cover you have a slab, slab is called as a film because it is kind of a small in film out there and this is resting on a substrate okay.

The confinement mechanism the propagation mechanism of the modes in this planar optical waveguides is not really the you know the vanishing of the electric fields at the at the metallic walls but rather what is called as the total internal reflection okay, what is this total internal reflection when you have an interface between higher refractive index and a medium of lower refractive index when you send light at a certain angle which is greater than the critical angle if you send light at an angle less than a critical angle it will just pass through only some amount of light would be reflected back okay.

So there is no propagation happening right the reflected component what might still go back but you would have lost much of the energy in this transmission and you would lose much of the energy after a few reflections, instead if I start increasing the angle of incidence and I come to a certain point where the angle is actually just greater than the critical angle then what will happen then we know that when light goes from a higher refractive index material to the lower refractive index material then the material will act.

I mean the light will actually get reflected in back into the first material or the first medium and there will only be even assent waves generated over here the even if in waves will actually attenuate as you move away from the surface in a manner of attenuation coefficient that we have calculated and they will propagate along the surface in the in you know in the form of an even ascent wave as we have seen right and depending on whether it was a T mode or a TM mode the wave as it incidents reflects back okay with the total internal reflection the magnitude of reflection coefficient is the same but the angle which it will pick up the phase angle upon reflection will be different, we have calculated this phase angle upon reflection for the TE case and that will play an important role over here, so in the total internal reflection when I send light.

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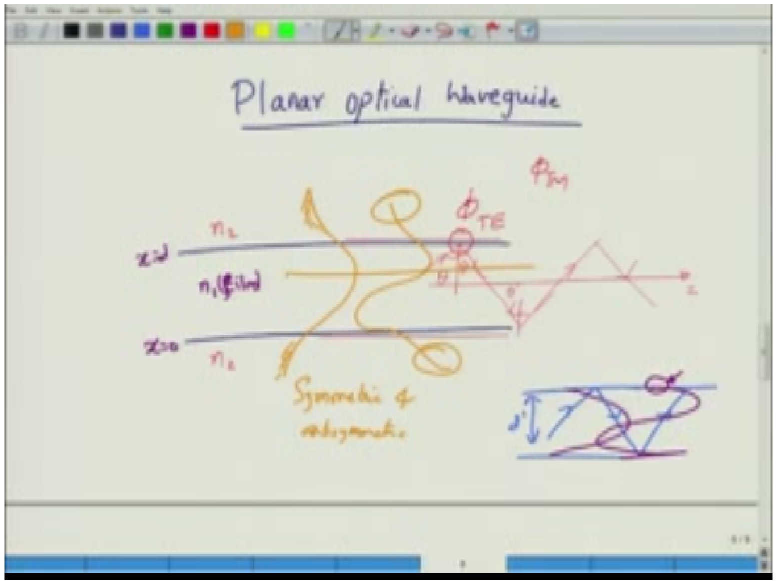

At an angle of some angle of incidence let us say θ okay then there will be complete reflection of the light provided you are satisfying the that θ must be greater than the critical angle and once this light gets reflected it would reach the other region or the lower boundary and if this angle theta is also the you know θ Prime let us say if this angle θ prime is also larger than the angle of critical angle then it would again be reflected back okay, so this way you will see multiple reflections happening and the wave effectively propagates along the axis of the waveguide and this axis is what we call as the z axis.

Okay the film itself or film itself is bounded by two surfaces one at x = 0 and the other at x equal to d okay so one at Xx = 0 and the other at x = d and in between you have this multiple bounces of the light and when light reflects back it will actually also accumulate an amount of reflection so if this is a TE mode then it will accumulate a face upon reflection of ϕ TE if it is the TM mode it will accumulate a face of ϕ TM okay, so this is pretty much the basic idea of this one but let us dig a little deeper into this one.

When we dig a little deeper we will also assume that in 3 or the substrate and the cover have the same refractive index which we will denote by n2 this is done just to simplify the formula there is no other reason why you should keep them to be the same in fact they are not the same most of the times, but if you keep them same then the equation that you write will be slightly simplified and that is the reason why we actually do that, now before we proceed further let us look at what other differences do we find from the metallic waveguide okay we know that if I were to consider a metallic waveguide right.

So if I have a metallic waveguide let us say of the same distance maybe or maybe a different distance because the wave frequencies are different, so let us say we have a compatible metallic waveguide of some d-prime thickness right, so you had this multiple reflection model as inhere as well this was not total internal reflection this was simply reflection of a perfectly conducting material right and if you examined the modes the shape of the modes how do they look like for example in the TE 10 mode you had red component along X it would have a cosine kind of a waveform right.

So the EY component was going as Sine ϕ / A into X so for the TE 1 0 mode this is how the ey component was looking like right the higher-order modes obviously will be very similar but they would be in the form of a full cycle of sine or multiple cycles of sinusoidal signals right, so this was how the electric field was present and please note here, that the field was completely 0 in contrast to that the modes of a dielectric waveguide unfortunately do not vanish outside okay the modes so here if you see in the metallic waveguide.

There was no field but in the case of dielectric waveguide we will actually see that there will be considerable amount of field presented the outside, so inside it might look that it is kind of similar to this Sine ϕ / A but then outside the field actually extends however the amplitude of the field outside starts to reduce as you move away from the interface indicating attenuation of these waves okay, however there is considerable of energy outside and the reason why this happens is because on a directed to dielectric interface.

The boundary condition is that the tangential electric field component simply be continuous across the boundary it is not going to 0 between two dielectrics right if it was a dielectric and metal the field was going completely to 0, but here it would not be going to 0 so similarly you will have the higher-order modes maybe I did not draw it properly but you should actually consider this is one complete cycle and then extending all the way, so you will actually have modes which are either symmetric about the center of the film or you will have more which are anti symmetric about the center of the film so accordingly you call them as symmetric and anti-symmetric modes and be you know observed that they field outside of the film or outside of the waveguide .

Will not be 0 they will extend in terms of nonzero values outside as well okay, so this is what the major difference between the two is now let us carry forward this analysis of the waveguide.

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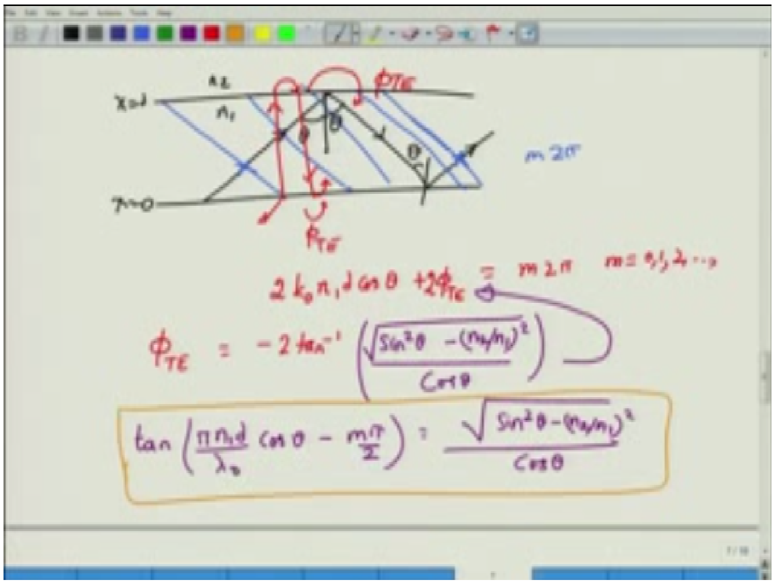

In terms of those Rea pictures that we have drawn right so we assume that light is incident from the lower interface and fix the interface from n1 n2 over here and we have chosen the angle of incidence θ to be greater than the critical angle okay, so if it is greater than the critical angle then this entire light will be reflected back ,okay again the light will start to reflect right, so the angle of reflection back here is θ and the angle of incidence on to this one is also θ while the Ray has you know traveled from the bottom x = 0.

Boundary to x = d boundary onto the top and got reflected there it also picked up a certain amount of phase since we are going to analyze the TE modes we keep the ϕ TE the phase upon reflection from the total internal reflection to be of the TE type okay not of the TM type but you can extend analysis for TM case as well now as the wave is incident and then comes back and the wave begins to propagate again right, if you were to look at what happens to the phase fronts right the locus of all the points which are actually at this common phase point right, so if you look at them the common phase points if they have to interfere again or if they have to be consistent and then travelling.

Again they have to interfere between the two or they have to have a face multiple of phase of 2 ϕ amongst the two components right, so because the phase front here and the phase front here if they are I know different only by a factor of two ϕ or an integral multiple of that then the point here will be in phase with the point here and then this would continue the reflection process multiple reflection process okay, so in performing this what kind of total phase change has happened please remember that the phase change that has happened.

Would have happened along the direction as well as the transfers to the direction okay this is exactly same as the rectangular waveguide there also we looked at what was happening to the first component or the face in the transverse direction, so you had e^{-jkiz} and you had e^{-jkrz} or rather + JK RZ and ki component along Z direction was some ki Cos θ k said kr component along the Z direction for the reflected was again KR Cos θ and ki and kr are equal to each other right, so in the same manner over here the transverse component which we can write like this.

So this is the transverse component right and if you look at from one of the angles the total phase difference that you are going to obtain for the wave that just begins to propagate again will be the transverse direction of propagation here along from x = 0 to d and then there would be a reflection with a total phase of ϕ TE here then it comes back there will be one more reflection of ϕTE while it spends the same propagation distance right and this vertical line is clearly given by K_0 which is the free space propagation constant k0 given by 2ϕ / λ_0 times the refractive index n1 here.

And the angle of incidence or the cosine of the angle of incidence which is Cos θ times d okay so this is the total face that you are going to obtain from one jump from the bottom to the top and there will be one phase ϕ TE added while it starts to return but since you are looking at only the phase difference even when the wave propagates from top to bottom it is only getting delayed further right it is not getting a -2 K Sine θ and therefore gets cancelled out it is only getting delayed further so which means this K0 n1 d Cos θ appears again with the same sign and we take that into account by simply multiplying this 1 /2 similarly the phase upon second reflection over here is also adding to the overall phase of 2.

So therefore we change ϕ TE 2 twice ϕ TE and this should be equal to m2 pπ with m = 0 1 2 and so on okay indicating the number of integral multiples and each value of M then indicates a different mode that is propagating inside the optical waveguide okay, it further I want to know what is the face upon reflection fie de and you can look at one of the lectures that we have put this expression up this was given by- to tan inverse of $Sine^2$ θ - n2 / n1 whole square divided by Cos θ okay.

So n2 by n1 whole square Cos θ I think there is a square root of this one I suppose so you can find out what it is right so I believe there is a square root of this I forgot whether there is or not but anyway you can find out the expression from the previous lecture okay, so this is the total phase angle okay so which you are going to obtain and you can actually put this phase angle into this expression okay put that phase angle into that expression and then rearrange the equation so that you obtain Tan of π n1D divided by λ 0 Cos θ – m π/ 2, so I am just bringing some of this back into this one.

This should be equal to square root of Sine √ θ - n2 / n1 whole square the entire thing divided by Cos θ okay, so this equation is important for us so let me put a box around the equation and this equation is also not very easy to solve because this equation is what is called as a transcendental equation okay not easy to solve analytically so there are two options one you can solve them graphically the other option is to actually solve it numerically okay on a computer you will do it on a numerically.

But for a long time people did not have computers as easily as we had so people used a graphical approach in order to solve this particular equation what they did is a very simple thing you have two equations right, on the left hand side you have one equation on the right hand side you have one equation when do you say that you have a solution in those values when the left hand side is equal to the right hand side you have a solution, so sketch the left hand side separately as a function of θk right hand side separately as a function of θ.

Wherever they meet those are the solution points and I would not go into the details but if you do that sketch.

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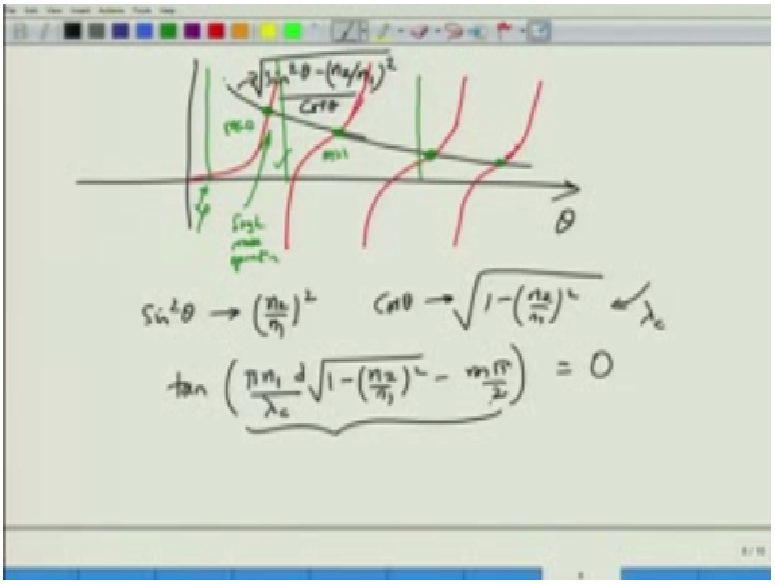

If you actually sketch it out you see that as you change the angle θ okay the term on the right hand side starts to look something like this, so this is the term which is $Sine^2$ θ - n2 /n1 whole square under root divided by Cos θ whereas the left hand side term must go as tan right, so it would go in this particular way and then it would come back here so it would see that so these are all going all the way to ∞, so these are all going to ∞ wherever they are meeting wherever these two meet that is the solution.

So the lowest one for which that happens will be when m equal to zero then the next one will be m = 1 m = 2 m =3 and so on okay in contrast to a metallic waveguide where for each mode you had a lower cutoff frequency only when the frequency exceeded that particular cutoff frequency did that mode propagate in this case there is no such lower cutoff frequency all modes will propagate at all frequencies that is good that is bad it is bad because when multiple modes propagate.

There is a chance that these modes interfere with each other resulting in what is called as intermodal dispersion and which will further distort the pulses, so if you send in a nice rectangular pulse the pulse will actually expand out go above its or go beyond its original bit boundaries leading to dispersion and leading to lower data rate okay, but it can be avoided provided I can actually make it into a single mode condition okay so I can perform a single mode condition.

I will come back to that one shortly before we go to that let us look at what is the cutoff wavelength okay, so we want to understand what is the cutoff wavelength beyond which the particular waveguide will be a single motor it is not that below this one the waveguide does not propagates it will propagate only that beyond this cutoff wavelength that particular mode will be or the propagation will be single mode or multi modal alternatively, if I choose the value of θ such that you know I pick the value of θ at this point.

Which is the vertical green line then this waveguide supports only two modes okay it still supports propagation there is no lower cutoff for example I can keep moving this green line back all the way over here even here there is a solution because this curve would have you know would can you extend this curve and then this curve is actually meeting this particular point okay, so you can actually see that there would always be some solution and of course you cannot really pull θ down all the way here.

Because you have to keep this θ greater than the critical angle otherwise the light would not even propagate through this one right, so you cannot pull this down as it is but if you can come very close to this value okay so this is not allowed whereas this is allowed when you have when you do this one that is when you restrict theta to only this region then there is only a single mode operation okay single mode operation over here so what is that critical angle or what is the angle at which I can try and make this one well when you approach critical angle $\text{Sine}^2\theta$ approaches n2 by n1 whole square which clearly means that the right-hand side should be equal to zero right and Cos θ approaches 1 - n2 / n 1 whole square under root okay.

So when cost it approaches this one the $\text{Sine}^2\theta$ approaching n2 by n1whole square means that the right hand side of this term is actually equal to 0 and you can rewrite this equation, so let us say this actually happens at some $\lambda = \lambda$ C so that I can say Tan of π n 1 d / λ Cos θ is 1- n 2 by n1 whole square under root okay this - m π/ 2 must be equal to 0 okay, so if this is 0 then one possible solution is to make this angle itself equal to 0 right so when I do that what I obtain.

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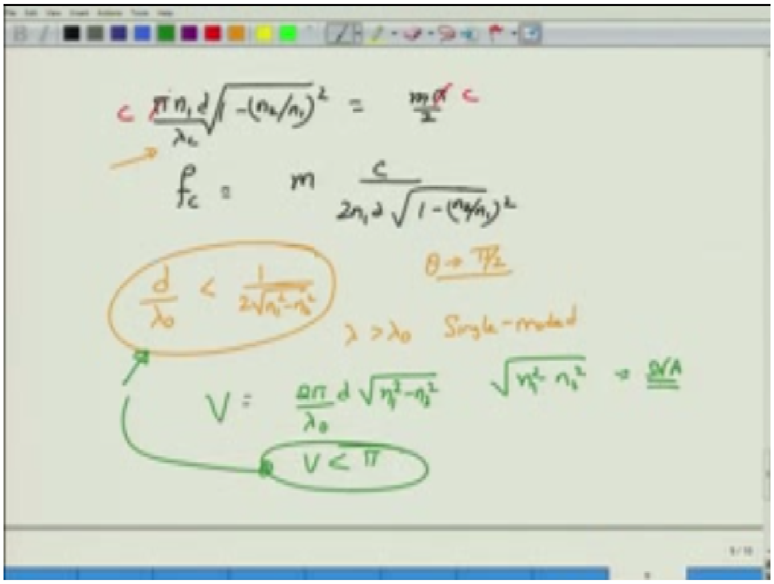

Is πn1d / λC 1 - n2 / n1 whole square under root equal to M PI by 2okaythis determines the cutoff frequency for the m okay no matter whatever that is there will always be one more propagating below this cutoff wavelength okay, so you have or above the cutoff wavelength so in terms of the frequency you have FC = mC / 2 n1 d square root of 1- n 2 by n1 whole square you can show that this is you know you can actually obtain this one by multiplying both sides of this equation by C and then canceling pi on both sides and then rearranging the equation.

So C by λ C will become FC and therefore you can rearrange the equation okay so indifference with metallic waveguides directed waveguide will always support at least one mode of propagation, so no matter what the frequency F that is there will always be one it is the frequency increases or the θ increases then there will be multiple modes but at least there will always be one mode carrying the data from it okay in fact single mode condition as I told you can be imposed if you select d and λ 0 to be in the fashion.

That is if you put θ of the angle of incidence to be less than Pπ / 2 right so then you are actually ensuring that this is a single mode condition there will only be one branch of an meeting the right-hand side right, so when that happens you can impose this condition and that condition actually tells you that d by λ_0 should be less than 1 / 2 into square root of $n1^2$ - $n2^2$ this is precisely this equation earlier that you have seen here which I am just rearranging them okay.

So you have for the lowest fellow that I am actually trying to find out when this condition is satisfied ok then for every λ that is greater than λ0 the waveguide will be single moded if λ is less than this particular wavelength or the cutoff or the wavelength λ0 for which this condition holds then that wave guide will be multi moded okay, so that is the difference between the case of a dielectric and a metallic waveguide in fact we normally don't discuss them in terms of these ratios of d by λ.

We introduce another parameter called as V number okay this V number is given by 2 π by λ0 d square root of $n1^2$ – $n2^2$ under square root is sometimes called as the numerical aperture okay, so this $n1^2$ - n^2 square under root is sometimes called as the numerical aperture or the sine of this one is sometimes called as a numerical aperture there are two different ways in which we normally define that do not worry about that so the condition that you have here can actually be rewritten.

After multiplying appropriately the condition can be written as V to be less than fine so as long as we is less than π you have a single mode operation for every frequency that you have considered if V is less than π it would be a single mode operation in an integrated optical waveguide you are just for the single mode operation as much as possible by adjusting the thickness of the waveguide or the film thickness, if you adjust it you will be able to adjust to the single mode operation okay.

Typically what happens in an integrated optical waveguide is that the index difference between the film and the cover or the substrate is usually much larger okay which means V number is actually larger so you can actually make, we know go if you do not control then it is very easy that we might actually exceed π and then you start getting multimode operation for a given wavelength right, so if you want to avoid that one either you reduce the wave so either you increase the wavelength because increasing wavelength means V is actually decreasing V is inversely proportional to lambda so you increase the wavelength alternatively if you want over the design wavelength of operation to be single moded.

You actually reduce d the film thickness because V is directly proportional to d in fact the product of d and $n1^2 – n2^2$ under root the numerical aperture more or less determines what is whether your waveguide is single moded or multimodal because λ or the range of λ over which these need to work is usually fixed okay all the analysis that we have carried out including the cutoff frequencies for TE case can be considered or can be read arrived for the case of TM modes as well okay.

You can know again find that TM modes will have a symmetric and an anti-symmetric mode it would be essentially the same kind of expression that you are going to obtain the cutoff frequencies for TE and TM modes are the same although the actual propagation values will be slightly different okay, but the cutoff wavelengths are all the same cutoff frequencies for different values of where also same okay so in the next module we will look at more you know interesting optical waveguide much more popular one called as the optical fiber thank you very much.

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