Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module-52
Applications Fabry-Perot cavity and Multi-Layer films

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Hello and welcome to NPTEL mook and electromagnetic for engineers. In this word we will be continue the discussion of reflections and transmission to a multi layer cavity using the S matrix approach or the matrix approach that we discussed, and before we go to the multi layer case we consider a single film having two interfaces, and this single film is actually you know widely used in spectrum analysis in the optical frequency range.

And this is known as a Fabry-Perot cavity; in fact it is also used as a fundamental design of a laser cavity. So what is the Fabry-Perot cavity? It was actually built by the scientist Fabry-Perot and the cavity essentially consists of a single medium, so maybe a glass tube having glass of refractive index 1 and surrounded by a medium of air okay.

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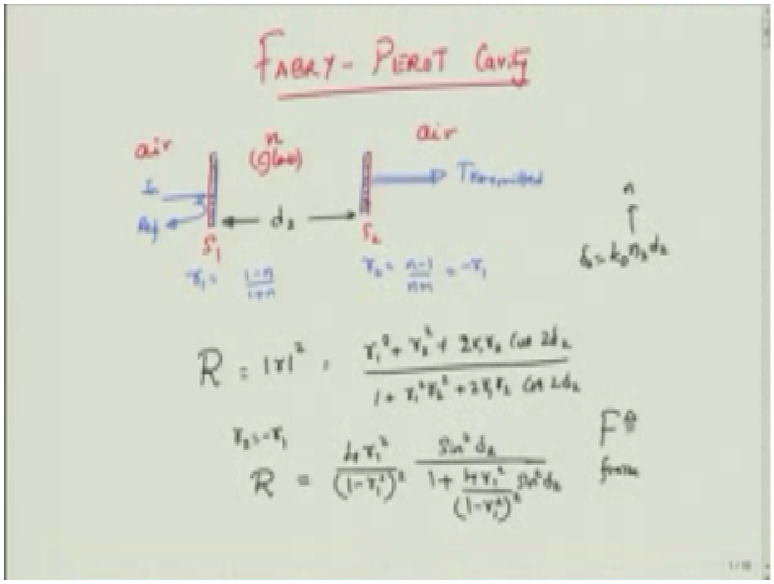

In a practical version of this Fabry-Perot cavity we can even see that there is actually a short mirror okay, so that could be a simple glass substrate on which you have coated some metal so that it will actually act as a finite thickness mirror, and then you have another mirror out there and then in between you can leave free with a medium, or you can again leave it as an empty kind of a thing.

Okay usually you fill it with a medium of refractive index which is larger than here, okay. There they can also be used in you know buried inside other media but Fabry-Perot cavity in optics and optics experiments are more or less carried out with here on both sides of the mirror. The mirror thickness we will ignore in this discussion, but one should actually consider the thickness of the mirror when defining an actual Fabry-Perot cavity.

Because it does influence slightly what we are actually about to look at the performance matrix of the Fabry-Perot cavity, okay. So you have these two mirrors okay, the mirrors actually have a very large reflection for which means if I remove the second mirror and then shine light from one side then most of the light is actually reflected. So very little of the light is actually transmitted into the second medium.

Similarly if I consider now what happens is that I actually have to bring one more mirror and then adjust the spacing between the two mirrors, okay. I am assuming the other is a very thin mirror, if I adjust the spacing between these two mirrors then if you choose the particular spacing which we will derive shortly then it is possible that although these two mirrors in general are reflecting very high reflectivity's you can see that the entire incident light could be transmitted into the second medium.

So you can see that you have two mirrors if you appropriately face them then the entire incident light can be transmitted, okay. So we analyze that by using the approaches that we have developed and the equation that we have seen in the last class, because this consists of a single film having two mirrors we will have two matrixes S1 and S2 that we saw in the last class.

For simplicity we will assume only normal incidence although we will later show you what happens when you have a obliquely incident light on this Fabry-Perot cavity, okay. So these immediately recognize that the reflection coefficient from the air to the glass interface can be given by 1-10/1+10 for the normal incidence, and from glass to air the same reflection coefficient will be given by n-1/n+1, and if you examine R1 and R2 clearly R2is – of R1, okay.

So we derive the equation in the previous module for their overall reflection coefficient or was given by, and this is the reflection coefficient given by the power, so this capital R was related to the overall reflection coefficient γ square or r square, it does not matter, so this is equal to r1 square + r2 square +2 r1 r2 cos of 2 Δ2, because I am observing that the thickness of this medium is d2, Δ2 will be equal to k0 and 2 b2 where n2 is actually equal to n.

Which we have taken class in our example, okay. So this was the numerator, in the denominator we have 1+ r1square r2 square + 2r1 r2 cos 2Δ2. We have so far not made any assumption on d2, we will do so shortly but look at this; since r2=-r1 we can substitute for this value of r2 in this expression and then simplify the expression, okay. So when you simplify the expression which I will do in an exercise for you, it is couple of steps only.

So substitute r3 is equal to –r1 so square does not really do anything to this but r1xr2 will have a negative sign now and r1 r2 in the denominator will also have a negative sign, this r1 square r2 square will be $r1^4$, okay. So if you put in all those values and then imply that a square + b square and a-b whole square kind of a formula then you can write the overall reflection coefficient as, so r1 square divided by 1-r1 square whole square, okay, you can see that.

And because that there will be a – in the sign of r1 and r2 we can show that the numerator we will have 1-cos2Δ2 and then you can use the trigonometry calculators to replace that 1-cos2 Δ 2by sine square Δ, Δ2, and then you can do the same thing in the denominator, so in the denominator we will have 1+ 4r1 square divided by 1-r1 square, whole square and sine square Δ2, this will define f as a factor which we will call as civets of the cavity.

So the larger the number of steps the higher will be the transitivity and marrow will be the angle as you can see, okay. So this civets of the cavity from the defined as 4 of square divided by $1-r^2$ to the power 2 okay, where I have taken r1 is equal to r for simplicity, okay, so instead of carrying r1 all the time I have just taken r1 is equal to r. So if I have a civets of the cavity defined in this way.

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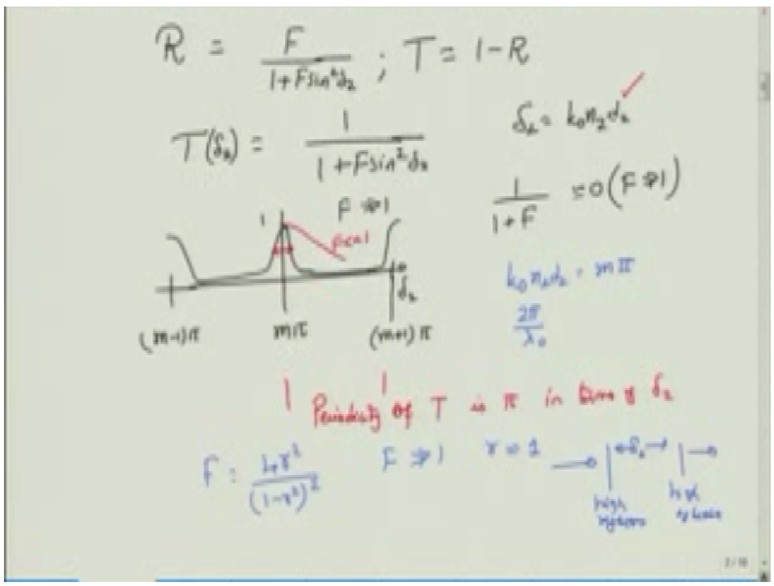

Then the overall reflectivity which is the amount of the power that is reflected back to the power of this incident is given by f / 1+f sine^2 Δ2, okay. Now I am interested in the transmitivity, that is how much power I am able to transmit out of the Fabry-Perot cavity, okay. So that can be obtained by this relationship which tells you that the total power must be conserved for a lost less system.

So 1- reflectivity must be equal to transmitivity and this will be equal to, I obtained 1/1+f sine square Δ2, okay. So this is the transmitivity that I obtain and we should understand that this transmitivity is actually a function of this quantity Δ2 okay, and we have already seen Δ2 is nothing but k0 n2 d2. In our design we have few choices, for example I can actually vary this d2 in order to make this sine^2 Δ2 go all the way from 0 to 1, correct?

So if I choose this quantity d2 for the same value of k0 and n2 and then clock what happens to the transmititvity as a function of varying Δ2 okay, assuming reasonable large value of x or the very high sign then you can see that when Δ2=0 or it is = to an even multiple of 2π right, or any multiple of π in fact, so because in any multiple of π $\sin^2$ of π = 0 and that quantity in the denominator will be = to 1, numerator is anyway = to 1.

Therefore the transmitivity at the value of a sum or multiple of π for Δ2 will be unity okay, so it could be 1 and then it quickly drops off and goes to 0 okay, then whether it goes completely to 0 or it stays slightly above depends on the sine test, okay. When Δ2 is away from n time you will see that that reflectivity when you know sine^2 Δ2=to the minimum reflectivity that you are going to get could be equal to or minimum transitivity that you are going to get will be equal to 1/1+f, right.

And then F is much larger than 1 then this quantity can be approximately 0, if f is not very large maybe f is 2 or 3 then you will see the curve to be slightly different, so it will be a curve that will look like this, but with this f much larger than 1 we will actually see that the curve almost goes to 0 and mostly it will stay flat until it reaches the next peak which happens at n+1 times π where n is an integer. The same condition applies on the left hand side as well assuming again as to be much larger will allow you to see that the reflection remains and the transmission remains close to 0 and then fix up at some m-1 into π.

From this we will see that there is actually a periodicity, the periodicity is of the, in terms of periodicity of the transmission coefficient is about π in terms of Δ, so okay. Now we have already seen that f must be much larger than them so that the peak here which denotes the transmitivity around this particular value of mπ should be as sharp as possible, so if you want a very sharp peak transmissions then you want to keep f to be much larger than them.

Now how you will you make f larger than 1, remember f is nothing but $4r^2$ /1-r^2 to the ^{2} so clearly for f to be much larger than 1 or f must be approximately= to 1, okay. So this is a very interesting phenomenon where you actually take a very high reflectivity mirror okay, and then you somehow put one more high reflectivity mirror and the result is the total transmission, so almost your reflection and an entire transmission into the, or outside this cavity, okay.

Of course for that to happen you should choose $\Delta 2$ such that this is an odd multiple of $m\pi$. So what does it mean to choose $\Delta 2$ as an odd multiple of $m\pi$? That means k0 n2 d2 must be = some odd value of n into π. I know k 0 to be the free space way that is to given by $2\pi/\lambda 0$ where I give this $\lambda 0$ as a special peak or the wavelength in which you are choosing this condition to be true, okay. So please note that if you take a different wavelength okay, this condition in general will not be and if at the back wavelength the transmission will not be true but in fact there is a large amount of reflection, okay.

So in fact the phenomenon wherein the transmission speeds at certain wavelengths can be used to revolve those wavelengths, okay. If you have a slightly broad band wavelength then you want a separate, only the particular wavelength $\lambda 0$, you will pick only that $\lambda 0$, then if you choose $\Delta 2$ appropriately and the value of the finals then you will be able to just remove that portion of the wavelength, or that portion of the spectrum around the wavelength $\lambda 0$ and leave all the other reflected back.

So you can actually pick wavelengths from this or you can separate or resolve wavelength from this, okay. So that is major use of a Fabry-Perot cavity, it can be used as a printer; it can be used as a resolving element, okay. So coming back to what should be the difference d2, do not be surprised if you actually see a familiar expression now.

So $k0=\lambda 0$ into d2 will be = sum π times π, π on both sides can be cancelled out and what you have, so sorry, you actually had $\Delta 2$ here, $\Delta 2$was 20 into π so this is alright. So we have actually written this one correctly. So if I now look at what is d2, d2 will be = to $m\lambda 0/ 2n2$, okay, and remember that this $\lambda 0/n2$ is actually the wavelength in the medium okay, it is a wavelength in the medium and then wavelength divided by 2 if you choose for this choice you will see that the transmitivity actually peaks okay.

So this is the distance between these two is twice the quarter wave transformer distance okay, so that is something that I wanted to prove point out, okay. I would actually ask you to solve the same problem not in terms of these matrices but in terms of the transmission analogical that we already talked about, in this case the transmission logic is very simple, so we have a film and then with the incidence of $\theta 0$ which is the higher incidence and inside this material or inside the film is actually captured as a transmission line in which a distance d2 okay, with a distance d2 having a characters to incidence of $\theta 0/ m2$ okay.

So if you actually look at what is the overall effective incidence and then calculate the correspondent value and then set the required value for d2, calculate the reflection coefficient or and then a transmission coefficient is set the value of d3 such that to achieve this condition we will end up having the same condition as we have derived from the latest approach okay, but when you go to multiple layers make its approaches easier to program and therefore that is the approach that we are going to take care of.

Before we leave this subject there is one small case that we need to talk about, we have so far assumed that in the film on the Fabry-Perot cavity light was incident normally to the surface, what the light is incident at an angle. Well if you are incident like that an angle then we have to consider two cases, one is the transverse electric case and the transverse magnetic case, right that is TM polarization and in place of that small r that we wrote we have to replace that r by the appropriate reflection coefficient corresponding to TE or TM, and in case of Δ2 it would not be this Δ2 which is = to k0 into d2.

But rather the corresponding component which is normal to the surface which means there has to be a cos θ angle right, so that would mean you replace this r with either r_{te} or r_{tm}
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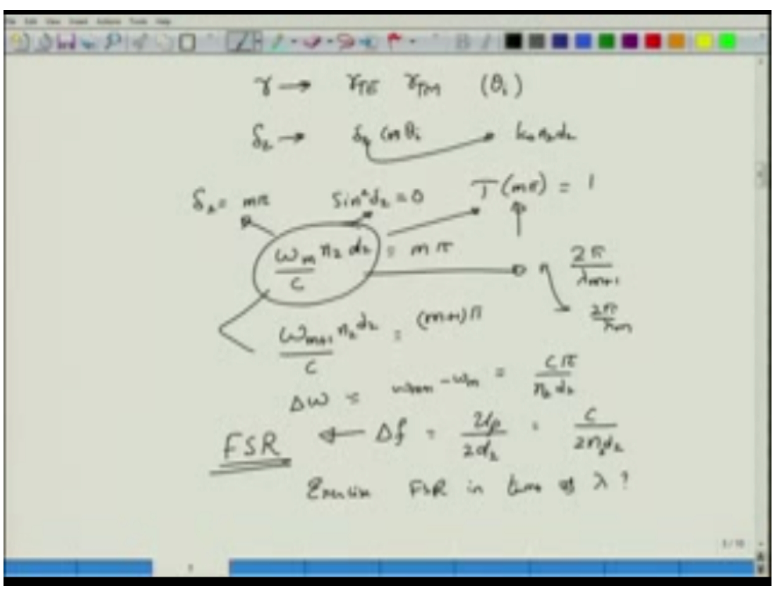

Appropriately which both of them on the angle of incidence and then you replace Δ2 / Δ2 cos θ Δ2 by itself is given by k0 and x dt2 okay, d2 being the thickness of the particular layer okay. I would also like to introduce a couple of additional terms that we talk about, so we have already seen that when $\Delta 2 = m\pi$ right so we had π^2 $\Delta=0$ and the transmitivity at this particular $m\pi$ value where $\Delta 2 = m2 = 1$ okay, and we seen that this can be achieved right at a particular wavelength in fact, okay.

So wavelength can be related back to frequency right by the phase velocity so instead of writing k0 as $2\pi/\lambda 0$ I can write this as ω/up where up is the phase velocity of the waves inside the fabry perot cavity okay. And because there is a number m here or the substitute m will also subscript the value of ω okay telling as that then $\Delta 2 = m\pi$ that actually happens at a specific frequencies of ωm okay, so the condition $\Delta 2 = m\pi$ means that ωm into d2 divide by well, I wrote C but in general it could be any phase velocity right, so it could be phase velocity well, but because I have already written n2 over here so this is actually correctly written as c itself okay, if not I could combine n2 and c and write this as up the phase velocity.

So this could be = to $m\pi$ okay, so when your frequency is satisfies this condition then $\Delta 2$ will be $m\pi$ $\sin^2$ $\Delta=0$ and consequently the transmitivity will be = to 1 okay, the same condition can actually occur where $\Delta 2$ can be = to m+ 1 into π that would be the next peak in the transmitivity so at that peak let say the frequency is ωm+1 and remaining everything is the same and now if you look at the difference between these two frequencies so there is a difference between $\Delta\omega$ which is given by ωm+ 1-ωm and this will be equal to c/n2 d2 times π.

Okay so rewrite ωm as 2π into fm and then obtain the frequency difference Δf and you will see that it would be up/2d2 or you can write it as c/2n2 d2 where up is c/d2 and then the frequency Δf which tells you how periodic in terms of the frequency that the fabry-perot cavity is, is called as the free spectral range okay, so this is all about the fabry-perot cavity actually I will leave as an exercise what to do about SSR in terms of wavelength.

Okay you find out what is the expression for SSR in terms of wavelength well, you just have to replace these conditions by $2\pi/\lambda$m+1 and appropriately adjust or find out the value of Δm one of the important uses of the fabry-perot cavity is so I have this fabry-perot cavity like this if I am able to move in the second medium, second rm right on the medium second rm slightly then on the fly I can change the value of d2, so as I change d2 and if the input sequence of the input wave length remain the same where the fabry-perot cavity was resonant at the frequency earlier would not be the same resonance frequency because I have changed d2.

So earlier d2 before the resonant at some ωm now when I change it slightly the same ωm the fabry-perot cavity would not resonant and hence would not allow the transmission of that particular frequency it will allow transmission into the new frequency so in fact if I can slightly not scan this second arm I can go over a broad wavelength picking up each frequency individually or in the group of frequencies with very sharp transmission peaks I can pick it to up individually so this device which makes use of movement of one of the arms in order to resolve the frequency or find out all the frequencies in the supplied optical spectrum is called as fabric scanning interfere metric spectrum analysis or simply fabric Ferro scanning interferoity okay the reason for interfere meter will become clear shortly.

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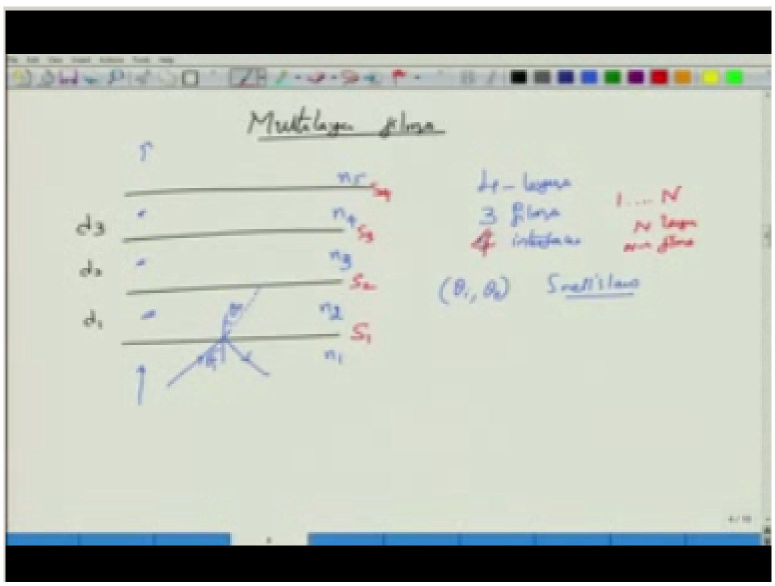

Where we now consider the case of a multi layer films are no more difficult to deal than the single layer films the general structure would involve something like this in general you can have the distance to be very different so let us say D1, D2, D3 and so on you will come in with lights at an oblige incidence right and there will be a reflected light as well and into the second medium this light would propagate and hit at a certain transmission angle θt so you have an incidence angle θi from the incidence medium.

And then in the first film right you will have a certain transmission angle how do I calculate θ and θ t or other how do I calculate or from θi the relationship between these two at any interface is given by snells law right so I apply snails law over here where I am also assuming that these reflecting index are n1,n2 or other sorry outside is n1,n2,n3,n4 and n5 right so you notice here there are actually 1,2,3 and 4 total layers so this is an example where we have considered 4 layers and how many films are there only three films right.

So there is film 1 film 2 film 3 incidence medium and the transmitted medium are especially the same okay or you can actually have a subtract down here and outside would be different medium but you will have four layers 3 films and there will be 5 interfaces to work with and therefore you have to play one matrix or the transmission we have talked about at each of the interface so s3,s4 I am sorry there is only 4 interface matrix where as there are actually 1to 4 interface for this is 4 layer actually I mean by 4 interfaces.

So this is actually 4 layer 3 film 4 interface the number of s matrices require for this one will be one to n in general for a layer and n-1 films okay if you consider n film there will be n+1 layers and therefore there will be matrices okay and what are the electric fields here so this would be e1 incidence e1 reflected or other e1+e1- there would be because light went inside here there would be an angle of incidence which if actually related to θt in this second layer then there will be reflection similarly there will be incidence on to every medium so this is even plus even minus so this would be e2-e3+e3-e4+e4-e5 and finally you have e5+usually because I do not have any incoming radiation from e5- okay.

So you can write down a matrix for each of them so at the interface or the i^{th} interface matrix is will be equal to $1/ti^{ej\Delta i}$ $ri^{ej\Delta i}rie^{-j\Delta i}$ and $e^{-j\Delta i}$ where of course I is given by k0ni in the i^{th} layer or the ith film that we were talking about so I could be k0 ni di distance times cos θi so please note that this is not the incidence angle which the light makes into the approximate film okay from the film this would be the value of Δi in the most general case this multi layer films are used especially what if called as optical coating in order to make mirrors.

And filters and reflectors and the theory of optical coding is wide it is involved optimization it involves lot of mathematics not just the matrix approach that we have consider and it is an extremely important design problem the basic idea is to use this matrices or you know in cases doing cooperate polarization you have to modify this two by two matrices into 4 by 4 matrices but that is essentially a big estimation problem which we do not have time to cover okay but very quickly I will tell you what are the types of these multi layer films that are used.

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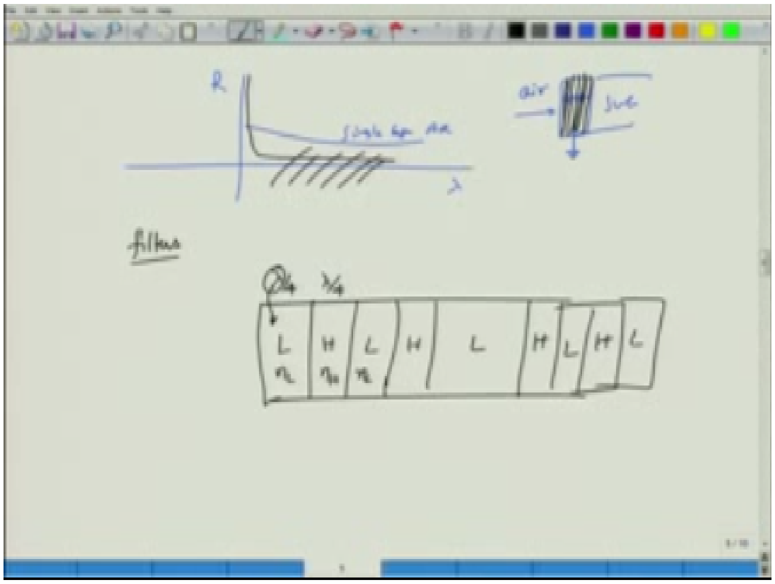

For more details I will give references which you can refer to okay, if you build five layer tag or a four layers tag okay, to eliminate reflection what would happen is if you look at the single layer reflection you would see something like this so this is the single layer AR coated material so you will have a substrate out there and there will be a coating material here and this is air let us say, so when you incident light is this coating material is of single layer then this is the kind of reflectivity that you see as a function of wave length, okay.

So clearly you are not doing a very good job of removing reflections, on the other hand if you actually do a multi layer coating with appropriately chosen the angles which I would not tell you because that is, it is extremely difficult problem to talk about that, then what happens is that the reflectivity is actually very low which means that in this entire region there is a complete transmission so this is the job of an AR coating to transmit completely and that it does with very low reflectivity and over much broader wave length than a single AR filter can do, okay.

In optical WDM components you come across what is called as filters, okay we will have more to say about this filter technology when we talk of WDM component thing, but these filters are called as interference filters what this interference filters do is that you have an alternate stack of high low, high low indices in between you will have either for a low or a high does not matter so I just picked arbitrarily at low here, but you could in fact have a high refractive index each of these layers are of λ/4 thickness.

But please remember the λ that we talk about is the medium λ so you will have to look at the corresponding reflection sorry, corresponding refractive indices to actually find out what is the corresponding λ and then do that λ/4 and then again follow up with the stack of alternating H and L, the stack of alternating H and L will cause the reflection sign to differ so which means there will be a phase of π.

And depending on whether you, you know adjust the phases to be in such a way that they can constructively interfere in the incident medium or interfere destructively in the interfere in the incident medium you can have a highly reflecting stack or you can have a highly transmitting stack, okay both type of filters are used one is the transmitting filter one is the reflecting filter and these interference filters because the reflection sign changes +-,+- because of the high low, high low thing and because of λ/4 thickness each one way propagation will be a π/2 phase two way propagation will be a π phase.

So you can adjust the phase difference and because of the partial reflections and the interference among the partial reflections you can have a complete transmission or very near complete transmission or very large reflection over very broad wave length regions, okay, so these interference filters we will have more to say in the WDM components, thank you very much.

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