Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL.)

Course Title
Applied Electromagnetics for Engineers

Module – 48
Total internal reflection

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Hello and welcome to NPTEL mook on applied electromagnetics for engineers. In this module we will continue the discussion on the phenomenon on reflection, this time we will look at a concept called as total internal reflection which is very important for optical waveguides and optical fibers. So propagation of light inside optical fibers is governed by the phenomenon of total internal reflection.

You might be familiar with the concept of total internal reflection from in our earlier physics courses or optics courses. But this time we will look at it little more detail and to see what happens when light is totally internally reflected onto the first medium.

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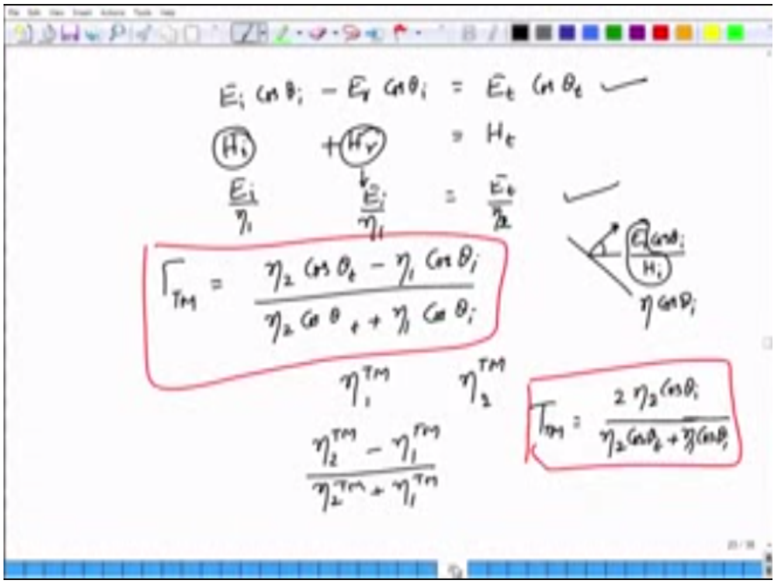

So we can work with either TM polarize lights or this TM polarize light for this, we will arbitrarily choose the TM polarized light okay in order to show you the concept of total internal reflection. But please keep in mind that the total internal reflection phenomenon is independent of which polarization that is both polarization exhibit the phenomenon of total internal reflection. But before we can discuss that we have to know to a little bit of a preliminary study to clearly bring out the notion of total internal reflection.

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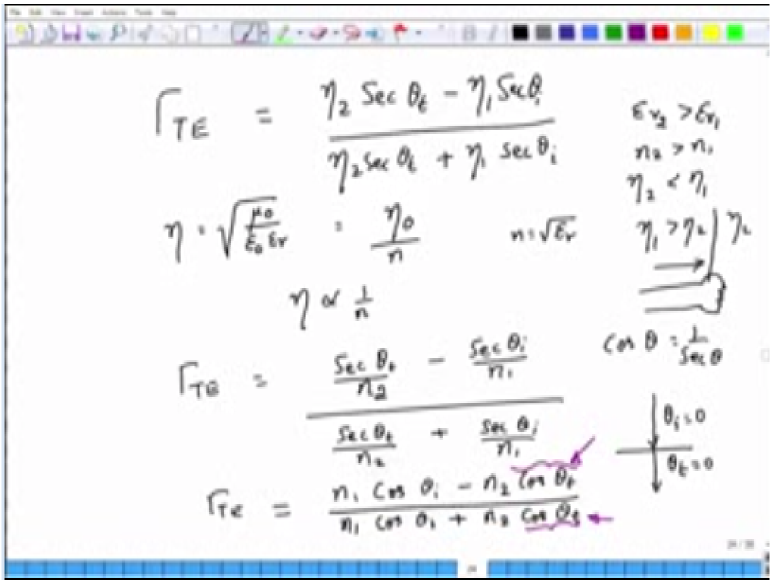

We begin by looking at the reflection coefficient of the T modes okay. So we know that the reflection coefficient γ of the transfers electric nodes we have shown you the expression in the previous class is related to the medium impedances $\eta 1$ and $\eta 2$, so this is obviously the medium with the impedance of $\eta 1$ and this is the second medium of impedance $\eta 2$. And light is incident at a certain angle θi and exists into the medium at an angle of θt.

There is of course the reflection also, and that is the ratio of the reflected light to the incident light is precisely what this γt is telling us okay. So the reflection coefficient expression for the case of TE polarized light is given by $\eta 2$ sec θt where θt is the angle of transmission into the second medium with respect to the normal to the interface $\eta 1$ sec θi okay. So since these are predict low let me raise this, but I hope that you have the picture of reflection in mind that we are talking about.

So divided by $\eta 2$ sec θt + $\eta 1$ sec θi okay. I want to rewrite these equations in terms of the refractive index. Now I can do that provided the magnetic, provided the media that we consider both medium 1 and medium 2 are non magnetic in nature. And since we know that for any medium which is non magnetic η is given by $\sqrt{}\mu 0/\ \varepsilon 0\ \varepsilon r$ this can be rewritten as $\eta 0$ which is the impedance of the free space medium divided by n.

The n is the refractive index which we have seen from the previous module to be related to the relative permittivity as $\sqrt{}\ \varepsilon r$. So this η therefore is inversely proportional to the refractive index of that particular medium in the expression for γt therefore, I can rewrite this as sec $\theta t/n2$-sec$\theta i/n1$

and do the same thing for the denominator as well replacing η/ η0/n and cancelling it all the sides will give me this expression, sorry this is sec θi/n1.

Now I can readjust the equations and knowing that cos θ = 1/secθ, I can write this expressions in terms of cosine of angle, whether it is the angle of incidence or the angle of transmission, I would like to rewrite these equations in terms of cos θt or cos θi. So I will leave this as a very small exercise for you to carry out and the resultant will be very useful for us, and will be written, the reflection coefficient will be written entirely in terms of the refractive index n.

So please note that this is the refractive index n that we are talking about of the first medium. So n1 cos θi – n2 cos θt/n1 cos θi + sorry, no it is very easy to get confused between η and n, I am sorry for the poor choice of notation, but this is a standard notations so we will have to deal with this so n to cos θt anyway does this equation make senses so does it make sense in the case of a normal incidence then angle of incidence = to 0 therefore the angle of transmission is also = 0 so if you substitute θi = 0 θt = 0 you get n1 – n2/n1 + n2 okay and this is something that you should know.

If ∈ r of the second medium is larger than ∈r of the 1st medium this means that the refractive index n1 will be larger than the refractive index n1 okay and η being inversely proportional to you know the refractive index means that η2 will be less than η1 because the impedance is scaling inversely proportional to refractive index the 2nd medium will have lower refractive index than the 1st medium so this equation certainly make sense because if you consider the scenario where η1 is larger than η2 and then you go from 1sr medium to and other medium or rather if you go to the case where η1 is less than η2 okay and you from 1st medium to the other medium or rather we consider the same case.

So η1 is > η2 and we are in this 2nd medium having an impedance of η2 and when you shine light at normal incidence on to this one the corresponding reflection coefficient will be negative because you are now kind of terminating a transmission line with a higher characteristic impedance with a load impedance which is less than the characteristic impedance okay, so because of that the reflection coefficient will be negative and that is precisely what this equation is telling us.

Therefore this equation as if now look fine and we will continue to use the equation now before we can use the equation there is one small thing that we need to do I do not want to use this θ t I do not want to keep θt in this expressions I want to replace this θt or equivalently cos θt in this expressions in terms of the angle of incidence because I would lie to vary the angle of incidence and then try to see what happens to the reflection coefficient of this t wave okay.

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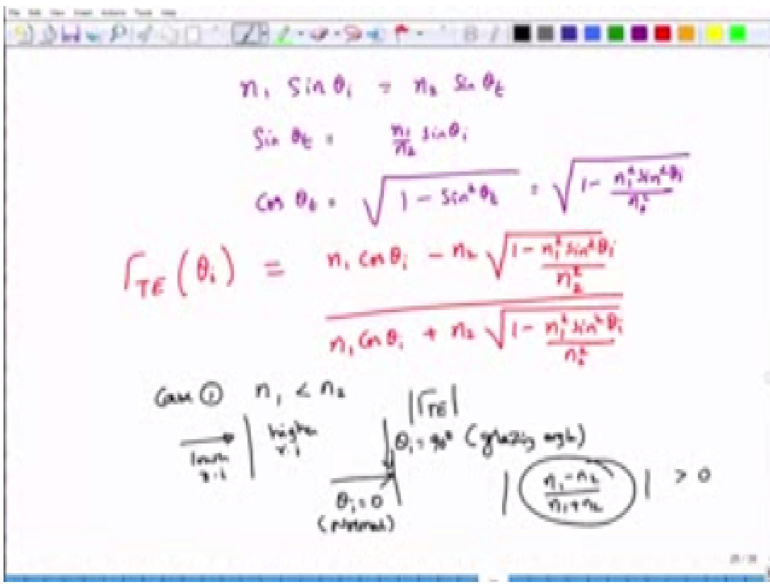

In order to do that I will employee shells law which tells me that refractive index n1 times sin θi must be equal to n1 sin θ t therefore sin θ t sin of the angle of transmission is given by n1/n2 sin θi and since cos θt = $\sqrt{}1-\sin^2$ θt this can be rewritten as 1- $n1^{2-}\sin^{2}$ θi/ $n2^2$ okay this is an extremely useful relationship to keep in mind and we plug this relationship back into the expression for γte therefore my new expression of γte which is a function only of θi the angle of incidence only assuming that n1 and n2 are constant of course is given by n1 cos θi – n x cos θ t which is nothing but 1- $n1^2$ $\sin^2$ θi / $n2^2$

This whole lot in the denominator will be n1 cos θ I + n2 x √ of this quantity okay so the √ actually covers the entire are given to this so please note that one now we consider 2 cases in case1 we have situation where n1 is less than n2 that is your medium of incidence as a lower refractive index than the 2nd medium which as a higher refractive index this is a common situation that you would encounter if you take a tube of water and put a pencil in that and then look from the top because air as a lower refractive index than water this is the situation that you commonly see on a daily bases kind of thing right.

So let us see what happens to the magnitude of the reflection coefficient right so what happens to the magnitude of the reflection coefficient as we now vary the angle θi what is the limit over which I can vary the angle well this is 1 limit where $\theta i = 0$ which is of course what we call as the normal incidence and at the other end you might have the you know light incident or the electromagnetic wave incident just in the grassing angle or in the surface along the surface of the interface okay so this we will have $\theta i = 90^0$ and this is called as the gracing angle okay, so you are just creating the surface along like that.

So in this two extreme cases your γt magnitude should vary okay obviously when you consider $\theta i = 0$ and n1 less than n2 the actual value of the reflection coefficient will be negative but it would still be a real number and negative real number is what you get when $\theta i = 0$ we can easily verify that from this expression in the normal incident which is n1 – n2 / n1 + n2 n1 is less than into.

Therefore this will be equal to some number and that number is n1 – n2 / n1 + n2 whatever that number is that number is less than 0 but we are not interested in the actual value we are only interested in the magnitude therefore the magnitude of this quantity is which is the lowest part of the value that you can have this magnitude will also b you know quit approaching one because n2 will be much larger than n1 so you can kind of into remove ne some numerator and denominator.

Relation to the n value of n2 there and the value of γt will be approximately equal to 1 this kind of makes sense right, so if you putting a very high refracting material out there or material then most of the light that you would incident would actually have to come back right, why would that be I mean you have a large value of the refractive index which means there must be an even larger value of εr which is relative permittivity which means the impudence η would be very, very small so very small impedance value mean it is almost like a short circuit in terms of the transmission line they feel like short circuit termination for the short circuit termination the value of the refractive index will be sorry the reflection coefficient will be 1 in magnitude at least right.

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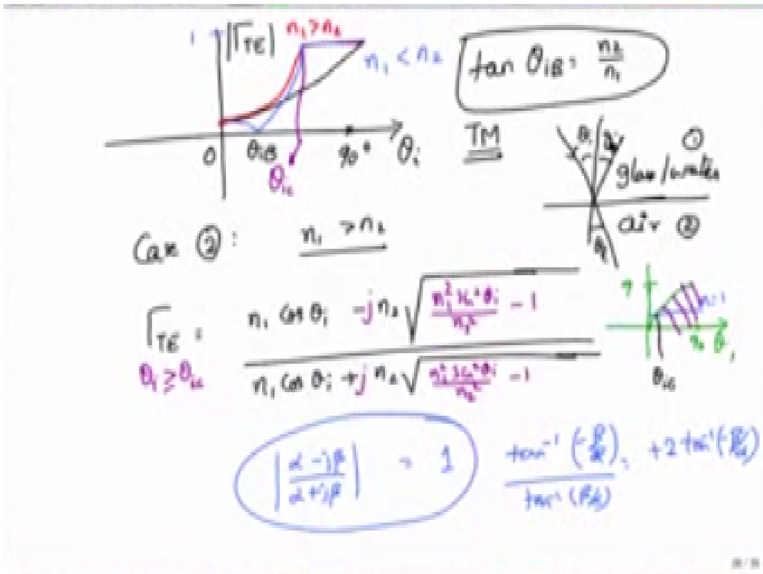

So we would not go to the extreme cases w keep the values of n1 and n2 reasonably separate I mean reasonably close to each other okay, so magnitude of γte as the function of the angle incidence angle of incidence okay, if you plot γte looks something like this what θi = 0 it begins with some small value but a positive value because you are looking at the magnitude and what will happen it θi = 90^0 when you go back to this expression when θi = 90^0 Cos of θi will be = 0 right, so this term is gone similarly this term is gone.

What happens to Sin^2 of θi that this would be equal to 1 this would be equal to 1 okay so this is 1 this is 1 so now we have – n2 $n2^2$ – $n1^2$ and √ / n2 the whole thing / n2 x √ of $n2^2$ – $n1^2$ / n2, n2 cancels on numerator and denominator and further more this will be equal to – 1 and if you actually look at this your absolute value would approach 1 okay it make sense that this would be the case so your if you plot it at θi = so this was that 0 so this was that 90^0 and expressing in terms of degrees here.

The corresponding value here would be = 1 so the magnitude of the reflection coefficient goes something like this okay it is kind of monotonically increasing and of course reaches 1 at θ = 90^0 this is the so called gracing angle insert you can show what the gracing angle is if you take this piece of paper and if you look from the top what you are doing is your actually looking at it in the normal incidence so this is what you are doing in a normal NPTEL but if you actually hold up the paper.

And then look at your eye level what your doing is essentially the gracing angle and you can actually feel that almost nothing is seen you cannot see much of it if you keep it close to the eye because most of the light is actually getting reflected okay, so this gracing angle thing is something that you can actually tried out is a piece of paper but the absolute value of γte fro the case when n1 is less than n2 will monotonically increase in this way.

So far there is seems to be no surprise who at favor what is happening now we come to the interesting case in case 2 we consider n2 to be greater than n2 right so you have maybe glass or you have water or you have some other material as the first medium and then as a second medium we have here okay, now light is incident at some angle θi and we want to see what happens to the transmitted and reflected angles sorry this is θi as well okay so there will be a reflection there will be a transmission but now.

Let us start varying θi from 0 to 90° and see what happens, okay. We go back to the expression that we had I will rewrite the expression for your convince so this is n1 cosθi-n2 $\sqrt{1-n^2 \sin^2\theta i/n2^2}$ okay, so I will just write it in this way in the denominator also you have n1 cosθi+n2$\sqrt{1-n1^2 \sin^2\theta i}$ this expression, right.

Now as I start increasing where θ=0 there is no problem right, you will get the same n1-n2/n1+n2 but this time at θi=0 the magnitude of γTE will be greater than 0 but the actual value of γT will also be greater than 0 that is it is real faraday's positive okay. Now as I start increasing θi something very interesting happens to the argument in this square root, right. Please remember we are considering the scenario that n1 is larger than n2 okay, and this $n1^2 \sin^2\theta i$ let me sketch that for you, okay.

If I just sketch $n1^2 \sin^2\theta i$ for you okay, let us say n1=3 and n2=1 okay, so three times $\sin^2\theta i$ or rather three square which would be nine, nine times $\sin^2\theta i$ as we start sketching this at θ=0 this would be 0 the quantity $n1^2 \sin^2\theta i$ will be equal to 0 and you go only up to 90° right, so start at 0 and then you keep increasing at 90° what is the value here this would be equal to 9, correct θ=9. However, let me draw a line of n=1 for you let us say this is the n=1 line, what do you observe here for this region as long as your less than some critical angle which we will call as some θic.

The product $n1^2 \sin^2\theta i$ will be less than the blue line, since it is less than the blue line then 1- of that quantity will also be since the numerator is less than the denominator right, so this quantity in the brackets will be less than 0 I mean sorry, not less than 0 that could be less than 1 and 1- some quantity less than 1 will be some non-zero value and everything seems to be going okay, right. So the absolute value of γTE starts let us say we consider the same n1 and n2 difference here it will again start with the same way but it would slowly increase here, okay.

So maybe let us say this is what it would increase okay, now at some angle θic which we see calculated just now right, I am not showing them to be on the same line just to kind of bring out what is happening with the difference out there okay, so I am just trying to show you that these two lines are different that is just for our own purpose okay, so it starts at some point which is positive but then it kind of starts to increase right, at this point θic something happens to this $n1^2 sin^2θi$.

The green curve is no longer below the blue curve okay, in this region this entire region this term $n1^2 sin^2θi$ becomes greater than $n2^2$, so when this is greater than $n2^2$ what will happen to this square root thing this will become an imaginary number the integrate, the radical under the square root sin becomes an imaginary number okay, which makes this γTE the value of γTE to be a slightly different quantity in that case what I can do is, I can since I know that this quantity will be greater than 1 I will readjust the quantities over here okay, I will readjust the quantities by writing as $n1^2 sin^2θi/n2^2$- sorry, this is actually 1, 1 but because I put this – and + over here there will be a complex number j there, right.

So I will have a similar complex number in the denominator again I will have $n1^2 sin^2θi$/n22-1. Of course $n1^2 sin^2θi$ is larger than $n2^2$ only when θi is greater than θic, so only after the so called critical angle then $n1^2 sin^2θi$ is greater than $n2^2$ and which makes us to rewrite this γTE, okay so this is only when θi is greater than or equal to θic, okay only in this region when the green curve lies above the blue line. Now does this equation not tell you or remind you of the form let us say α – j β / α + j β where β is this quantity and α is just n1 cos θ I yes it does right and what is the magnitude of this one the magnitude will be equal to one but there will be phase relationship here right so the phase will be tan inverse of – β / α right tan inverse of β / α.

So you actually get – 2 or rather 2 tan inverse of – β / α so there will be a phase relationship when θi greater than θic but the magnitude of γ t will be equal to one okay. so the curve does something very interesting the curve after θ I = θ ic breaks up and suddenly shoots up in magnitude and remains equal to one okay, so it suddenly shoots up in magnitude and remains equal to sorry this was not clearly written let me write it let me rewrite this line sorry again so this is what happens at θ ic okay.

So this is what happens at θ ic whereas the other case started of smoothly and eventually kind of very gracefully approached one okay this was the case when n1 less than n2 and the red is case is when n1 is greater than n2. Now what is the meaning of the reflection coefficient becoming unity and staying unity for all the values there that means the entire power is actually getting reflected back so the light instead of getting transmitted in to second region is now completely getting reflected in to the first medium.

So no light at least at this point seems to be the there is no light in the second region right so this phenomenon in which once the angle of incidence becomes greater than the critical angle that there would not be any change in the I mean the entire light would actually reflected back in to the first medium is called as the total internal reflection okay, so far we have written for te case thus this θ ic depend of tm case well no right.

Because cos θ t cos θi never dependent on whether you are sending initially at e light or tm polarized light even when you learned the concept of critical angle Snell’s law none of it was discussed with respect to a particular polarization yes the way γ tm changes as θ I changes is completely different from the way γ te changes, but in terms of the critical angle and what happens beyond that both polarization will lead to the same condition that light is actually reflected back.

At this point let me just write down what will be the scenario for the case of tm γ tm condition as I start varying θ I okay so let us assume again that this point you know we are on this one actually as θ I changes at some point the reflection actually starts to go down eventually reach as 0 okay so which means that is the very special thing that is happening when reflection is 0 that means there is complete transmission in to the second medium and then suddenly you know after this point shoots up to one this is very interesting scenario right.

So why is that in the tm case this reflection coefficient is going to 0 well there is a very special angle called as the Brewster angle and this Brewster angel is given by the tan of Brewster angle is given by n2 / n2 of the medium refractive index n2/n1 we would not derive it but you can use this you know result and this is true in general only for the tm case because we are assuming okay, so for such a two non magnetic media only tm polarized light exist this phenomenon of Brewster angle where the angle of incidence equal to the Brewster angle the entire light is transmitted in to the second medium and nothing actually comes back in to the first medium.

This phenomenon of Brewster angle is used in many applications one application is what we call as polarization upon reflection okay.

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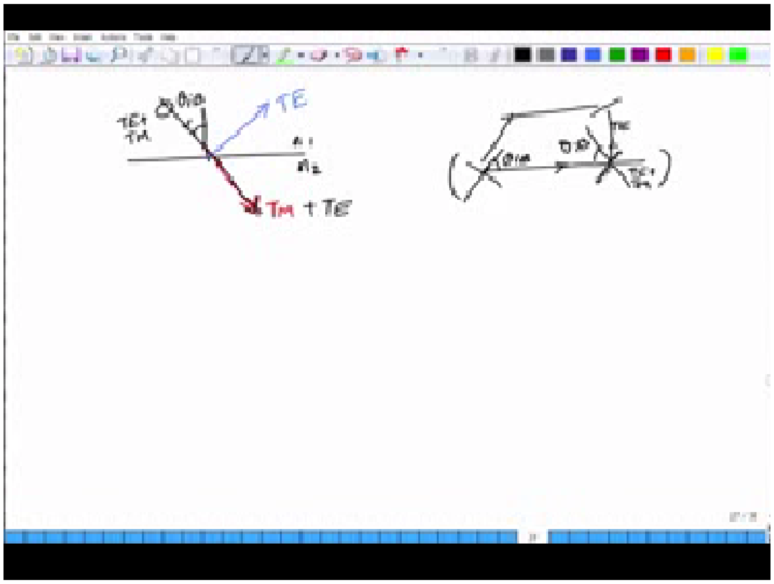

Suppose I incident light at the Brewster angle okay let us say this light has a combination of te as well as tm so this is coming from some unknown source okay and I as do not know what is the amount of te presenting that what is the amount of tm present in that in general it contains both components let us say. Now if it is possible for me to vary the angle of incidence right and I make it fall on to the second medium having a refractive index of n2 the first one having a refractive index of n1 if I decide or if I determine what is the Brewster angle and then let light come in to that angle of incidence.

Right and I make it fall on to the second medium having the reflecting index of the first one having the index of n1 if I decide or determine what is the brief angle and coming to that angle of incidence then what will happen because of the angle the TN component completely transmits whereas the reflected component will have only TE components where as the transmit component will not only have TN because of the complete transmit but that is also partial transmission of TE therefore in the transmitted case you will have both TN and TE but in the reflected case the light is uniquely only polarize along TE.

So in fact what you have done it is kind of separate out the TN and TE folder right they separated the TM folder by they are actually employing reflection phenomena and whatever is reflected will be polarized only along filed if we inflect that is either gravities if you have in laser okay instead of having this mirrors if let us say we keep a mirror here before keeping the mirror like this such that the line that is incident okay is at an angle of restart angle so this is with normal to the line it is the mirror actually it is the light or maybe I keep one more mirror at this point okay do not worry about so this is the normal.

So you have as it is going and coming back impact then what happens the light that is lifted right only be polarized along TE directions transmitted light will be TE direction the transmitted light will be TE and TM but the reflected light will be TE if this light is further with transmitted and reflected back okay I am not showing the entire line which is here but if you do this in this and therefore what happens after a million propagation is that the TE component they are selectively amplified because that is the one reflected and it is coming back and TM component is keep transmitting the second medium and may be user 6 amplitude potentially what we have done is to obtain a very nearly single polarization light from the laser gravity okay this windows are.

Kept at the angle tested all the incoming lights it is angle is called the brushed window and this are extremely useful in separating out the polarization in selecting the particular polarization and in suppressing the particular polarization so in all cases suppressing the TM polarization while selectively taking out the TM polarization.

So this is what we see at least you know in terms of you know from the total internal reflection and the phenomena of total internal reflection and from the phenomena of angle we have not derived we have not shown that this angle is to be the same for both people Tm and TE to be obvious we have to replace this expressions for reflection coefficient in terms of TM and carry out the very similar analysis okay thank you very much.

<u>Acknowledgement</u>

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