Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 05
Circuit parameters of a T-line

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Hello and welcome to the NPTEL mooke on applied electromagnetics for engineers. In this module we will look at several other circuit parameters of the transmission line namely we will be considered, concerned with input impedance of a transmission line which will help us solve the problems that are concerned the transmission line circuits. Now to certain stage for that let me start with the very simple question.

So far when we have considered the transmission line we have talked about its characteristic impedance which will depends on the characteristic or the constants of the distributed constants of the transmission line that is L and C for the lossless case that we are considering, given L and C new can find out what is the characteristic impedance, given characteristic impedance and the load that the transmission line is terminated in we can find out the reflection coefficient, from the reflection coefficient we can find out what is the amplitude as well as the phase of the reflected wave voltage for the current wave at the load as well as at any other point on the transmission line.

However, we have so far not seen how to find V0+ which is the amplitude of the voltage that is propagating along the +Z direction right, we are assuming that the source is located at some Z=-L and then we have a load located at Z=0 in the typical transmission and problems the load is located at Z=0 and the source is located at value of Z that is negative okay. In that when we connect the transmission line circuit and we want to understand what is the power carried by the forward wave, what is the power carried by the reflected wave, how much power is dissipated in the load, all that of quantity is require as to understand or require as to find what is the value of V0+.

Now how do we do that, in order to set the problem, let me start with what I consider it as a paradox, because it shows up that if we completely ignore physical reality we might land up in situations where, you know we will be in kind of a trouble, we will see what kind of a trouble we will land in. The problem that I want to consider is very simple, I have a source at some Z=-L let us say, you know at some particular distance away from the load I have the source.

Let us assume that the source to have no internal resistance which means that I am assuming that the source is completely, you know internal resistance free the source is ideal and let us also assume that it is a pure harmonic source that is if you generate a sinusoidal voltage at a particular frequency ω. We will assume that the line is lossless okay, and then we terminate the line in not in load, but in our case the load will be a oscilloscope.

So why am I terminating the transmission line in oscilloscope is because I want to observe what is the voltage wave form at the load end okay. So we will take the input impedance of the oscilloscope to be very, very high compared to the transmission line, so that we can almost consider the oscilloscope input impedance to be infinity.

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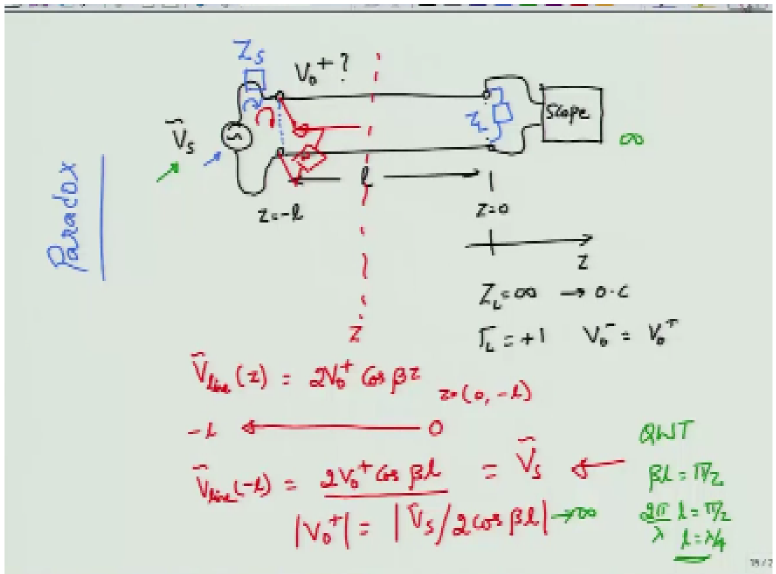

So here is our sketch of the problem. I have a transmission line okay of some length right, let me take whatever the length L be, and because I have chosen the coordinate of the load to be Z=0 and my Z axis is increasing towards the positive right side okay, this is how my Z axis is increasing. So clearly the load is kept at Z=-L okay, then I consider this output right, I take two cables of negligent physical length and then I connect this one to an to an oscilloscope okay so I consider this one oscilloscope in order to display the voltage wave from that I would be observing at this particular terminal of the transmission line so this let me connect a source okay as before I am using these kind of cruet lines to indicate that the size and the shape of these wires do not really matter to me okay.

So I consider how voltage may be a slightly better diagram could be drawn here so that disarm could be something like this okay so I have a sinusoidal voltage source here which in the phaser domain let us call this as producing some constant voltage I mean in terms of the phaser it is producing a particular sign voltage and hence as a constant phaser which we denote by V_s with a over bar or with a tilde on top of the of that. Now let me ask you what is the value of V_0+ can I find V_0+ for this particular transmission line tells out that yes I can very easily do that.

Because the input impedances of the scope is consider to be infinity the load impedance for this transmission line is actually infinity in other words this is a open circuit load for which we have already seen in the previous module that the reflection coefficient at the load will be +1 which also means that V_0 - = V_0+ and if I now consider any particular z you know on the transmission line in any particular point of the terminal points of the transmission line the voltage at that particular position which I will label as V line of course this is also a phaser.

So this voltage v line of z will be given by 2 V_0+ cos β into z, z will be going from 0 to – l okay that you should keep in mind set goes from 0 to – l being the load if you do not recall how we obtain 2V_0 + cos β z I suggest that we will do the earlier models where in we actually derived this particular condition okay now hat is the line voltage line voltage is essentially the voltage at any point on the transmission line.

And now let us start approaching the source which means that z starts to go from 0 to – l At the source point or at the z= -l plane if I evaluate what would be the lien voltage it turns out to be some 2V_0 + cos into l of course z will be –l but cosine function being an even function simply gives me cos of β l because cos (-θ) = θ for a cosine function right so this is the voltage on the line that you would be measuring if instead of connecting an oscilloscope here I would connect an virtual oscilloscope.

No I wanted to say hypothetical but I kind of say that I can actually connect a real oscilloscope so here I connect an oscilloscope of course I am not connecting this problem it is just a notation so if I measure the voltage at this terminal I would actually be able to measure the value of 2V_0+ cos βl okay but clearly KVL tells me that at this particular z= - l whatever the line voltage that I measure must exactly the voltage that the source is supplying which means that this must be equal to Vs bar.

Now here is a simple equation I was complaining to you that I cannot find V_0+ but here is a simple equation that allows me to find the value of V_0+ and what is that value V_0+ is simply give the phaser Vs/ 2 cos β into l you can clearly absorb that if I change the length of a transmission line the connection between the source and the oscilloscope the value Cos βL will also change which in turn will change the value of V_0 + okay so what would that V_0+ value b if I want to look at the magnitude of this one the magnitude is clearly given by this okay if I consider very, very special case of βL being equal to π/2 okay βL x π/2 in transmission line problems is called as quarter wave transformer.

Because β we will related later on to the wave length and it is given by 2π / λ into L equal to π/2 if you cancel π on both sides turns out to be L = λ4 where λ is the wave length which we will just talk about in few nets okay so if you consider $\beta L = \pi/2$ what will be the value of Cosine function at that point it would be Cos of $\pi/2$ will be equal to 0 which and you have a finite quantity in the numerator and 0 in the denominator clearly making the magnitude of $V0+$ go to ∞ now if you have so far not been surprised this is the point where you should really be surprised.

Because just by connecting and ideal laws less piece of water wave transformer between an ideal voltage source and you know some sort of Oscilloscope or a load with the very high input ingredients and able to amplify the voltage Vs all the way up to infinity some actually making the voltage amplifier okay just by connecting an ideal laws less transmission line of length $\lambda/4$ okay now am I violating some energy conservation principal here I will leave this in the exercise to you.

Because I have only calculated the voltages I have not calculated the current and I have not calculated the power that is generated by the source and the power that is delivered and the power that is dissipated in the transmission line because I am not taught you how to calculate power there is solution of this paradox has to wait for sometime before we resolve but the hint is that this kind of a problem arises the voltage at any particular point of the transmission line can exceed the source voltage.

But energy conservation will not be violated as we will see in one of the exercises and most importantly we have considered the transmission lines to be loss less which more or less can be okay go for majority of applications but I simply cannot find and ideal voltage source because if I could find the ideal voltage source in no matter how much current I drop I can then have the same voltage terminals maintained at the source output.

And therefore be able to obtain an infinite amount of power clearly that does not happen with any of the voltage sources in practice because in practice you always end up having the certain amount of source impedance you said yes so this paradox okay we will resolve after we understand how to calculate the power and I will actually leave it as an exercise.

Because there is couple of calculations that are involved and I will give you the steps in order to do that were so, this is what I wanted to talk to you about the paradox now problems of this nature you know I have a source the distance over here or more precisely a source impedance because the source impedance source, source resistance instead of source resistance you can actually have some reactants in the internal impedance of the sources therefore I have able this by the general impedance notation of z.

And I put a subscript yes, in order to indicate this is a source impedance okay so given a transmission line connected to the source with it source impedance at one end and instead of connecting the scope I might actually connect this one to another load okay this load could be anything it also includes some reactants to it in general therefore I denote the load resistance, instead of load resistance I denote this one by the load impedance ZL, so given this problem in this case it is not so simple to find out V0+ okay.

Indeed this problem can be solved by the method that I have described and the solution actually is slightly more complicated okay than what we have already seen over here the line voltage will have to change if I consider the original case where ZL = ∝ then the line voltage will be the same but when you start evaluating the line voltage at Z = -L you have to understand that there is some current flowing through this and that current will be the line current.

And line current flowing through this ZS impedance of the source will create a certain voltage drop here and the total voltage that you need to some will then be equal to the source voltage VS that must be equal to the line current at and multiplied by ZS to give you the drop across ZS plus whatever the line voltage okay if you are interested I can write the solution over here you know.

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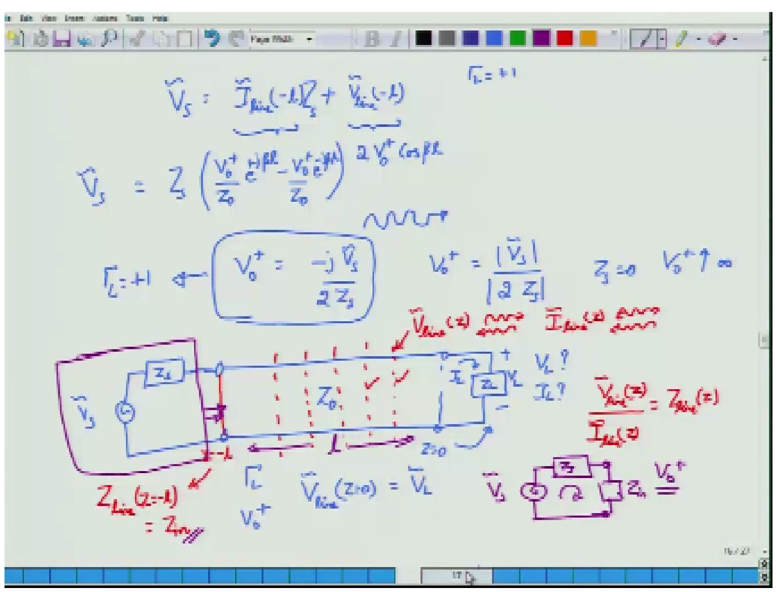

I can write the solution and that turns out to be the source voltage being equal to the line voltage facer evaluated again at –L x ZS which is the source impedance plus the line voltage measured at –L this we already know for the case of the scope connection or for the case where gamma L = + 1 this is simply 2V0 + Cos beta x L whereas this particular case that we have line at –L will then be equal to $V0^+/Z0\ e^{+jßl}$ –V0 $^-$/Z0 but V0 – is nothing but $V0^+$ therefore that could also be $V0^+$ therefore that could also be $V0^+$
And it would be $e^{+jßl}$ this entire thing multiplied by ZS this should be equal to the source impedance VS, if you actually solve this equations you will then be able to show that $V0^+$ which is the amplitude of the forward travelling voltage wave turns out to be –j $V_s/2$ Zs okay if you are interested only in the magnitude this would be equal to the magnitude of the source divided by 2Zs when Zs happens to be real then Zs can be replaced with Rs.

And you can see that the voltage is kind of split in terms of this Vs/2Rs of course in ideal scenario when Zs = 0 which of course cannot exist in practice voltage $V0^+$ will go all the way upto infinity thus giving you the same solution as you obtained earlier okay, while this problem was solves there exist slightly better methods of solving a similar set of transmission line problems.

And all of that involve the use of input impedance okay, what exactly is this input impedance that I am talking about, well consider this transmission line okay I connect this one to a general load ZL okay if I do not make any crochet shape it does not mean that I am considering the lengths here as transmission line it just that I just did not want to draws a crooked lines all over the place all the time okay.

So I consider this transmission line terminated in load ZL let the transmission line have a certain characteristics impedance of Z0 to this I connect the source having an impedance of ZS and generating a certain facer voltage VS bar okay how do I solve this problem in order to find out what is the load voltage.

And if you interested what is the load current, load current will be flowing here and the load voltage is whatever that you measure across the load. In order to solve what is the load voltage I need to know what, is a total line voltage here, because if I know what is the total line voltage at z=0 then that line voltage will be exactly equal to V_L, in other words if I know what is the line voltage phaser at z=0 this would be equal to the lower voltage phaser, okay.

So this is what I need to know but line voltage evaluating this fz=0 will require me to know what is γ_L of course that I can find out because Z_L is known Z_0 is known but this will also involve the constant v_0^+ which you need to solve or which you need to find out and the solution is not the same as this one this was a special case of solution where γ_L was equal to +1. In the general case where γ_L is not equal to +1 then this v_0^+ will also not be the same as here, okay.

So to solve this problem what we do is we develop impedance or the concept of an, impedance at every point on the transmission line. what do I mean by this, see at every point on the transmission line there is a certain line voltage right, so there is a certain line voltage that I can and this line voltage is a phaser of course so this line voltage will be there at every point on the transmission line which will be the sum of forward as well as backward travelling voltage waves. Similarly there will be a current phaser as well at every point of z and that would also be the sum of forward as well as backward travelling current. Now if I want to form the ratio of the line voltage at point on z to the line voltage at the same point on the z then this gives me what I call as the line impedance and line impedance is also changing with z, okay.

Now I can find the line impedance here I can find the line impedance and I can keep coming towards the source okay, and at some particular point where z=-l I can find out what is the line impedance here, the line impedance at the source point where z=-l is what we will call as input impedance of the transmission line, okay or input impedance as seen by this particular gentle men which is this generator will see this input impedance, okay when I terminate the transmission line of some length l, okay with the load Z_L.

In other words the load impedance at Z_L has been transformed away into and made to look as though the load impedance is actually equal to Z_{in} which is the input impedance. So what is the advantage of this, the advantage of this is very simple. Once I have what is the input impedance I can then use my standard circuit theory to simplify this scenario and then find out what is the, so this would be my input impedance, okay.

I can easily find out what is the initial voltage or v_0^+ from these expressions I will demonstrate that to you very shortly, okay. So this has the advantage of replacing this complex loads Z_L by an, equivalent input impedance that is seen at the transmission line source end. So what is that input impedance how do I calculate it.

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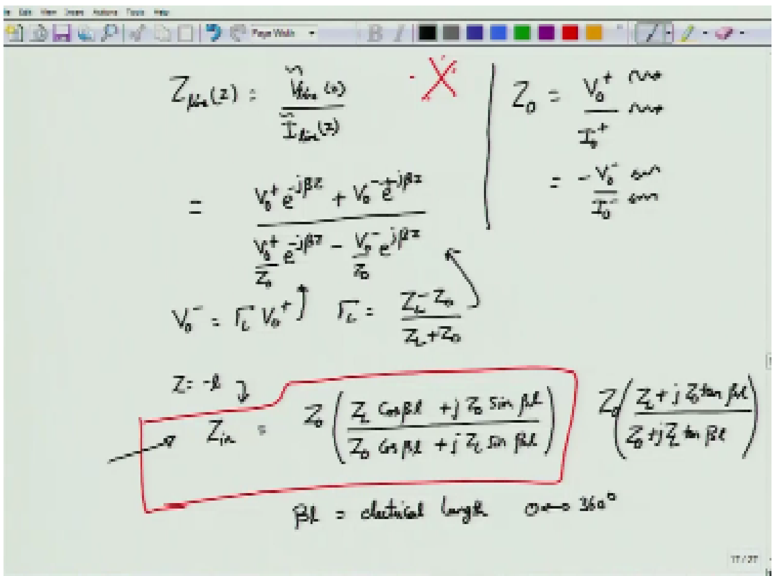

This requires that first we define what, is the line impedance, so our definition of the line impedance was that and this line impedance of course varies with at different points from the transmission line so this is nothing but the line voltage phaser to the line current phaser. Now at this point there is a bit of a caution the characteristic impedance of the transmission line was defined not in terms of the total voltage phaser or the total current phaser but this is defined in terms of the amplitude of the forward going voltage to the amplitude of the forward going current or it was defined as $-v_0^-/I_0^-$ where v_0^- was backward travelling voltage wave I_0^- was also travelling in the backward direction, okay.

So this is the characteristic impedance which and if you want to find out what is the characteristic impedance you have to assume that either there is a forward going voltage and current or backward travelling voltage and current, okay this is completely different from the line impedance because line impedance actually is that sum of the total forward plus backward voltage to is the ratio of sum of that backward and forward voltages to sum of forward and backward travelling current wave forms, okay.

So please note this difference this is very important so let me mark this important by drawing this carton okay yes now that we have line impedance of course I do not really have it any two substitute for v line I line then I can evaluate this at z = -l in order to find out what is the input impedance, I would not derive the completer expressions for this it is not there it is difficult but it is just that little bit of TDS algebra is involved.

But I will give you the step from which you have to do couple of additional algebra it steps to arrive the final solution okay, so the line voltage whether is given by v0+$e^{-j\beta z}$ remember vo+ is the amplitude of the forward going voltage +v0 –d+jβz because + represents the voltage wave travelling in the minus z direction divided by the line current which happens to be v0+ by z0 amplitude $e^{-j\beta z}$ – v0 – ω z not $e^{j\beta z}$.

You can then substitute for v0- γl x v0+ and substitute for γ l as z l – z0 /zl = z0 substitute this two expressions in to this above line impedance equation and then evaluate the line impedance at z = -l what you will end up with the input impedance expression then you will end up with the impedance expression which is given by z0 zl cos βl +jz0 sign βl /z0 cos βl +jzl sign βl okay, so you will end up with this particular expression for the input impression this occurs quite frequently in the transmission problem therefore I will mark this in a very special way.

There is an alternative way of writing this that involves taking cos βl is a factor out okay so that will go in the numerator as well as in the denominator and sign βl /cos β l can become tan βl, but the alternate form of writing this will be 0 zl + j z0 tan β l / z0 + jzl and β x l okay incidentally β x l is called as the electrical length okay, β x l is called as the electrical length and this electrical length can vary from 0 to 360^0 okay that is when β l goes from 0 to 25.

This would be the electrical length variation okay what use is this input impedance expression we will have lot more to say about the input impedance e later on okay, for now what we want to do is to solve a problem where this concept of input impedance will be required for us okay. So let us try to do that one okay the idea is that once I know what is the input impedance I can then find out what would be v0 + and then I can find v0+ from this expressions and therefore solve for what is the voltage that dissipated in the load okay, so in order to do that one let me consider a problem with some numerical values out there and I would suggest that you independently verify the solution that I am going present now.

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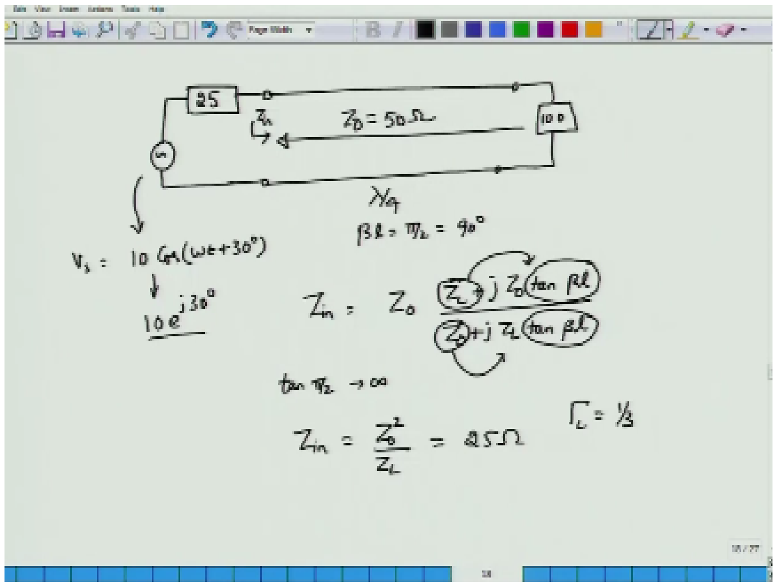

The problem is simple we have a real internal impedance of the source which is 25 volts which is connected to a voltage phaser, so the voltage that is supplied the source voltage is given by 10 cos ω t + 30^0 this can be replaced in terms of the phaser notation as 10 e^j 30^0 oaky so this is the phase and this phaser happens to be a constant and that is connected to the transmission line, so from here onwards we have a transmission line whose length is λ / 4 now λ / 4 means that β x l must be =\ π / 2 or 90^0 okay.

I am writing the equal sign here but you understand that this is ingredients and this is in degrees okay, now when I terminate this transmission line with the real impedance of 100 homes or a resistance of 100 homes and I know as what should be the voltage that is dissipated in this one I can easily find out or what is the voltage across the 100 home impedance I can easily find out that.

In order to do that I need to transform this 100 home input impedance sorry 100 home load impedance in to a certain input impedance correct in order to do that one I need to go back to the expression for input impedance which happen to be z0 x zl + zo tan βl/ z0+zl tan βl. But what is tan of π/2 it is actually. Okay our mathematician friends will tell us that when this tan βl will become very large. Then I can neglect this zl in comparison with tan βl or in comparison with jz0 tan βl.

Similarly I can neglect the 0 in the denominator with respect to jzo tan βl. So if I make this approximation I will end up having the input impedance given by zo /zl. Okay because j and j cancel z0 x z0 gets multiplied you get z0/ zl an d this value will be = 25. We can also show that the low reflection quality is 1/3 100- 50/ 100 +50. I did not mention that the character impendence was 50 over here.

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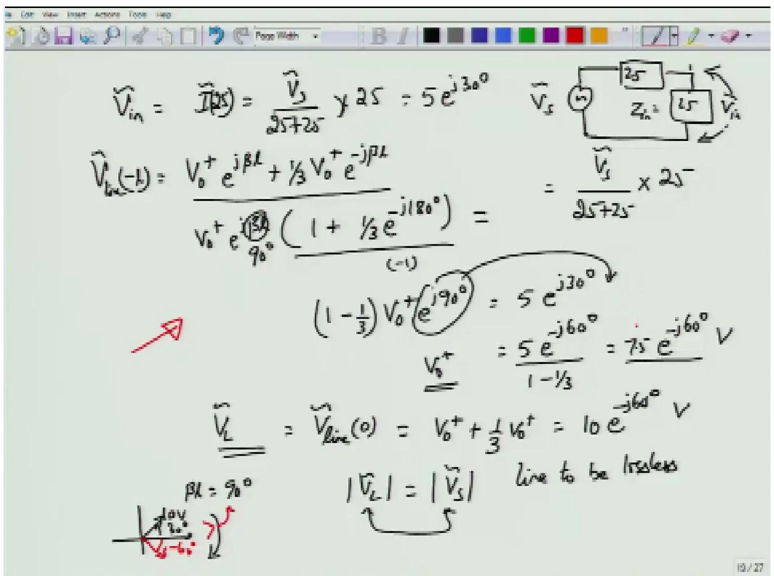

Now that I have this equation or right I have simplified this input impedance I can find out what is the initial voltage phase. The initial voltage phase will be the equivalent circuit will be the source. This is the 25 input coming in by internal impedance and this is the 25 impedance that is the equivalent impedance of the transmission lines of circuit.

The current phase will clearly will be 5ej30. Okay now I can find what the lying voltage is. I know what is the lying voltage expression which happens to be v0+e jβl +1/3 so v0+jβl. I can take this as a common factor and then end up having 1/3 b-j a $180^{0.}$ This is nothing but this should be equal to whatever the current that is getting multiplied and then this should be equal to.

Sorry this is the voltage that I have the current that I get is vs/25+25v that is the voltage and x by 25 will give me the total voltage that I seen across this input impedance. Therefore this, stand out to be print of 5 j30 but this has to be multiplied by 25. So this is the continuation of problem. I made a very small mistake.

The current that you would see is particularly will be voltage phase / 25+25 but I am not really interested in the current but I am interested in what is the voltage that has been developed across this. Which then it can be obtained ny x 25. So if x i/of the transmission line I get what is the input voltage phase.

So I call this as input voltage phase which appears in the equivalent input impedance of =5 ej30. Okay so this input impedance must be exactly =line phase that we measured at –l. we know that the –l will be in this form. This is the forward going voltage this is the backward travelling reflected voltage okay so and this backbone travelling reflector voltage is multiplied by 1/3 because what happens to the reflection coefficient for this problem.

So I can take this reason 30=per j if know βl is the 90 is okay and if I take $90^{0\ is}$ I can take this on to the and this becomes 5 spout becomes come to the – j 180 Ej-1so t6his term actually 1-1/3 v0 +E^{j90} that must be equal to 5 e_J 7.5 enjoy 60 –2 voltage what would be the load voltage I cans find out the load voltage evaluating what is the line voltage.

At 0 right but the line voltage at a z=o will be given simplify v0 +1/3vo+I have taken the liberty of the writing 1/3 so that is the in case that is the value of this particular case and this must be evaluated is very easy to evaluate because I know what is voltage 7.5 in both the $\_60^0$ it is then find the one third of its and add to the same value what I get is 10 Enjoy60^0 you might be surprise to see that the load voltage is exactly equally in magnitude thus source voltage this the load voltage this is actually vl of fazes right.

So this the vl faze so the magnitude of the vl phases is exactly=to the magnitude of the source voltage okay that magic has actually happen because we consider line to be loss less okay because we consider line to be lossless all the that the line did was to delay whatever to the load that is the essentially the reason you get the load voltage magnitude to be the same as the source voltage however the face difference between the to, right the space difference in-between the two appear.

Because of the propagation delay associate by the transmission line we had transmission line of effective length and we in tally with the source voltage that had a magnitude of 10 volt but at the angle of 30^0 with the respect to the x axis in the faze presentation but in the after delay by 90^0 which correspondence to rotation to the clock wise direction okay the value of the load voltage will be at which will be the same magnitude but it would be the at 60^0 how x axis.

The total space shift between the space voltage and the board voltage will be equal to 90 degrees and this 90 degrees phase difference is introduced by the transmission line so with we finish this module I suggest again that you look at this numbers and understand carefully as to what steps that where involved I have made a couple of numerical errors while describing a solution to you but the final answers are alright you can just verify this one once more so with we thank you.

Acknowledgement
Ministry of Human Resource & Development

Prof. Satyaki Roy
Co-ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K. K. Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

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