Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 49
Oblique incidence of plane waves

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Hello and welcome to NPTEL mook on applied electromagnetic for engineers in this module we will consider reflection of waves and transmission of waves but this time we will assume that the angle of incidence is not equal to 0 right, so the previous case that we considered is what is called as normal incidence they have already talked about what normal incidence is suppose this is my interface and this is the way in which the incident fluid form plane wave is approaching the medium so when it approaches straight in this way right.

Then there is a certain perpendicular you know there is a perpendicular vector to this interface, so this is called as the medium or the plane of interface which separates out the two regions so this is region 1 and this below is the region 2 and if I say what is the, so if I consider this these are the z-axis then red equal to 0 can be considered as the interface plane and the normal to that interface plane will be along the z axis so this would be the normal to the interface plane and my electric see my incident β vector which is you know incident on this one is parallel to the normal to the interface.

So the angle between this electric sorry the propagation vector incident propagation vector and the normal to the interface is actually equal to 0 so this is called as the normal incidence. Now there is no reason in infer it is much more common that the incident propagation vector will not be in the same or will not be parallel to the plane of to the normal to the plane of interface what I mean is that if this is the propagation constant now the propagation constant can actually come at a certain angle over here okay remember in the normal incidence case you had the electric field to be perpendicular so it is little difficult for me to hold and show you both so assuming this way.

So this is how the propagation vector you know for the normal case was and the Mac in the electric field quantity was along the thumb direction that I have considered and this direction is what I considered as the x-axis okay the magnetic field was coming towards you so this was the magnetic field my index finger is the magnetic field okay which was along the y-axis now clearly when the angle of incidence is not zero or the normal incidence things change a little bit now this is your propagation vector the electric field you know in you cannot lie in the entirely along the x direction now because it has to be perpendicular to the K vector.

So essentially this vector when it rotated by an angle of θi as measured with respect to the normal to the interface me means that the electric field will now have both X as well as the Z components similarly the magnetic field can or cannot have the components. So in one case we consider the magnetic field to be along the Y direction okay, so electric field has two components X and Z but the magnetic field has a single component along the Y direction okay and the propagation vector is at an angle of ninety at an angle of certain angle θi with respect to the normal.

So this index finger is now normal to the interface and these two are now at a certain angle okay so this case is what is called as the oblique incidence case and I have shown you only one example of this oblique incidence in which the electric field was in the same plane as the plane of interface as well as the known the normal thing that is in the XZ plane which is called as the plane of incidence which contains the normal to the interface as well as the propagation vector okay in this case the it was the XZ plane that we considered then the electric field having both components X and Z was in the plane of incidence.

So this particular you know component or this particular case is what is called as parallel polarization parallel polarization here indicating that electric field is in the plane of incidence and look at the magnetic field it was actually transpose it was along the y-direction it was transferred to the plane of incidence and it was lying in the plane of interface. So this magnetic field hedge which is parallel to the electric field vector as well as the propagation vector β okay is therefore transfers to these two vectors and therefore this polarization the parallel polarization is also called as transverse magnetic polarization okay.

Now what will happen once the wave hits you know the interface at a certain angle well there has to be a certain reflection because the medium impedances are not matched, so there is an impedance sorry there is a reflection out there plus there will be some transmission okay so there is a reflection then there is a transmission as before in the same in the case of a normal incidence and our job is to determine how the reflected fields look and how the magnetic fields or in the transmitted fields look and what is the ratio of the electric reflected to incident electric field and transmitted electric field to the incident electric field okay.

And in this case we will see that this ratio has to be taken with a specific electric field components only okay this has to be taken only with the tangential components you should not take the ratios of EZ reflected to EZ incident or EZ transmitted to EZ incident okay those are not allowed. So all these things can be made more clearer if you examine this picture that I have drawn here so you carefully look at the picture.

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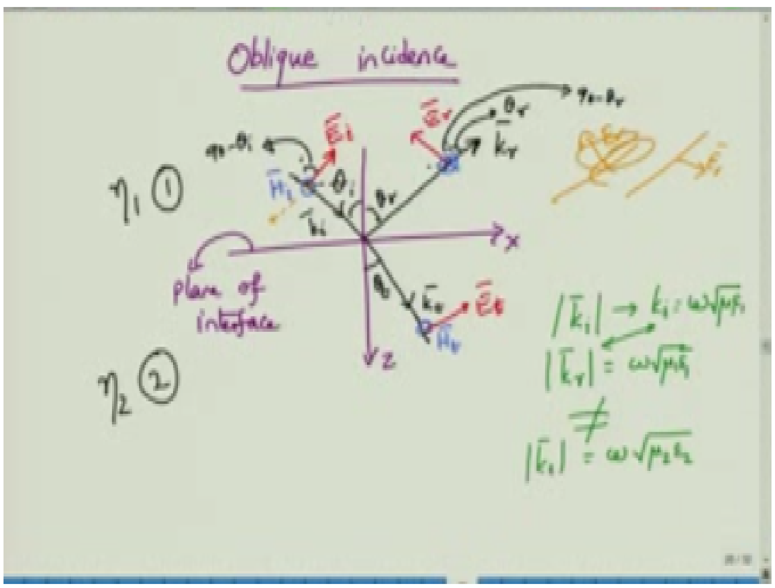

There is a X and a Z plane the Y plane of course is coming out of this page let us say or it is going into the page does not matter which way it is but it is certainly perpendicular to the screen okay in region one which has a certain impedance of let's say ETA one and a medium of two medium to which has an impedance of η two you have an incident wave vector I have departed from the convention of taking β as the propagation vector in favor of taking K as the propagation vector this small change is because most of these concepts are applied at the optics community it is common to consider K as the propagation vector rather than beta as the propagation vector.

So because of this small you know change in the notation I hope there should not make the lecture itself be difficult to you just mentally substitute k with β whenever you feel like okay. So you have an incident wave vector I denote the incident fields by a subscript I okay so you have Ki eI and H I, H I will be along the Y direction okay and E I is the incident electric field which is in the XZ plane as is the K I incident wave vector because of the reflection there will be reflected fields and there will be transmitted fields the reflected field or the u reflected wave will have an angle or will make an angle of θ R okay as it propagates away from the plane of interface the angle of incidence which is the angle between the incident wave vector K I and the normal to the surface of the plane of interface which is z axis is θ I.

So θ I θ R and θ T respectively give you incident reflected and transmitted waves let me clear up one point over here the frequencies of the incident wave will be the same as the frequency of the reflected wave and is the same as the frequency of the transmitted wave okay the frequencies do not actually change you can actually rigorously show that this is indeed true because the fields have to satisfy boundary condition okay but there is a problem with the boundary condition as well that I will come to in a short while but let me give you up front that frequencies remind the same.

So we will not worry about the frequencies being different let me also give you a second note which is important in this context okay because the incident wave vector has a propagation incident fields or incident plane wave has a propagation vector k I the magnitude of this ki if we denote this one by just ki without a bar on top of it must be related to the frequency of the incident wave and the Constituent parameters of the medium. So this must be $\omega \sqrt{\mu 1 \varepsilon 1}$ okay, because the reflected wave is also in the same media the magnitude of K are the reflected wave vector you know the reflected wave vector magnitude must also be equal to $\omega \sqrt{\mu 1 \varepsilon 1}$ these two therefore must be equal in magnitude of course in direction they are different but in magnitude they have to be equal.

But these two are certainly not equal to the magnitude of the transmitted wave vector because the transmitted wave vector will be related to the Constituent parameters of the second medium okay so they will be equal to $\omega \sqrt{\mu 2 \varepsilon 2}$, so please keep these relationships in mind we will have use for these equations shortly okay so this is the structure that I have and you can see that the magnetic field is along the Y direction and the difference is that the magnetic field changes its direction of the electric field changes its direction depending on the appropriate you know changes that are required so that the field actually starts to move in the minus Z direction okay also let me tell you that the way I have drawn ei and the way I have drawn er they are completely arbitrary okay.

I can you know imagine that electric field ei can be drawn this way and the ER can still be written in the same way I can chain that one the equations will tell me later on whether I have used the correct directions or I have used in correct direction so in other words the direction of the reflected fields and the transmitted fields well the reflected fields and transmitted fields are more or less determined by the equations so if you get confused that you know instead of writing ER in one of the exercises or somewhere you actually chosen to write this as ER in this way do not worry the equations will tell you which way the electric fields are directed okay. So do not worry about this direction of the incident and reflected fields okay.

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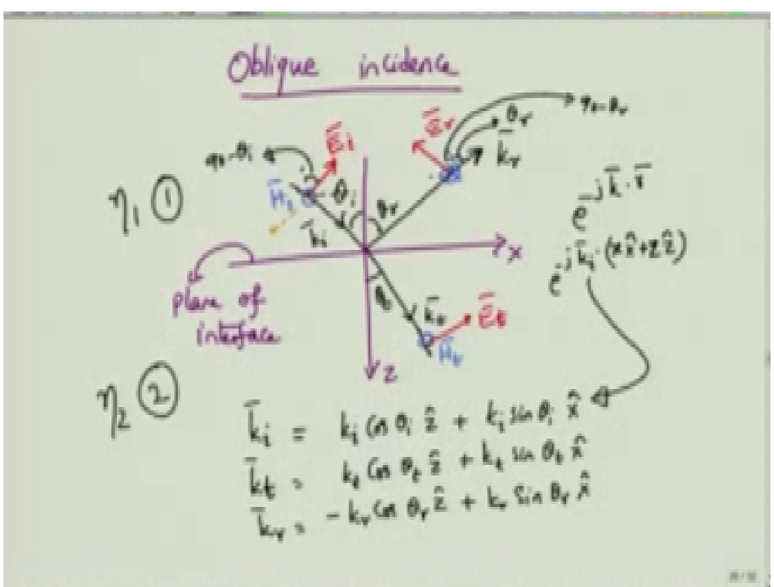

Now first I would like to write the corresponding wave vectors for the transmitted and reflected I mean incident reflected and transmitted field so I want to write down the propagation vectors ki kr and ke okay luckily for me ki and kt are oriented with the same X and z kind of components of course the actual values will be different ki the incident wave vector of the incident propagation constant of propagation coefficient is given by the magnitude of ki the incident wave vector times cos θi along $\hat{Z}$ + K I sine θ I along x axis.

So clearly if you decompose the ki vector you will see that this would be ki cos θ I along the $\hat{Z}$ downwards and ki sine θi in the horizontal direction similarly KT will be equal to K P magnitude cos θ T $\hat{Z}$ + K T sine θ T $\hat{X}$ and K R will be equal to magnitude of K R times cos θ R with a minus sign for the Z component because the wave is moving away from the plane of interface correct so the wave is moving away from the plane of interface but it will have the same component or the component in the same direction along the X direction in this case so it will be K R sine θ R $\hat{X}$ okay.

We will assume that the fields are described by their you know phase factors which hire in the form of e power -J K. R which for the incident wave vectors means that this is ki vector dot R vector is nothing but X $\hat{X}$ + Z $\hat{Z}$ okay, so this would be e-ki.x$\hat{x}$ + z$\hat{z}$ you can substitute for ki from this expression so you will see that this would be ki cos θ I z + ki sine θ I x X okay, so please do make this substitution and actually write down the phase factors for each of the three waves okay.

Now look at the electric field over here right so this electric field can be decomposed into two components one component will be along the x axis the other component will be along the z axis the component along the z-axis is definitely the component that is normal to the plane of interface and therefore of no help for us when applying the boundary condition similarly e R can be decomposed into the Z component and the X component in this case it would be - Z and -X the way we have written them but again whether it is plus side or minus said that particular component is perpendicular to the plane of interface having no use for us in the boundary condition same case for ET as well okay.

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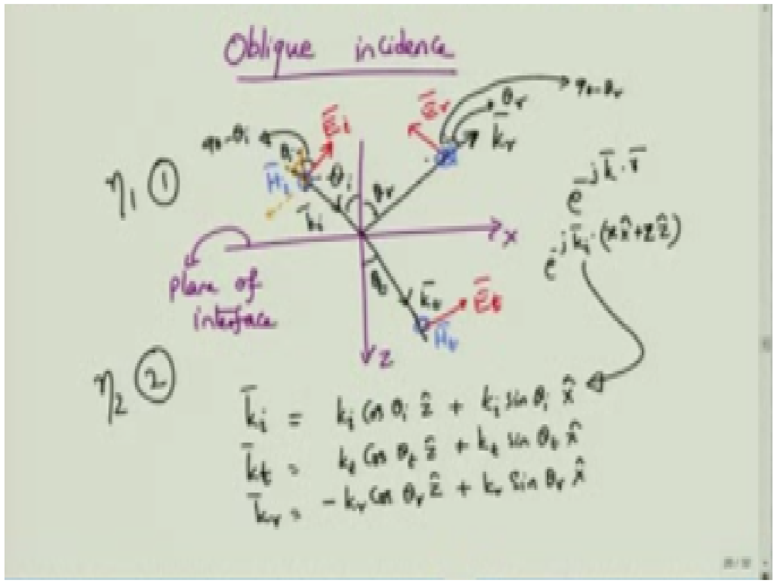

So my first job is to write down what is the electric field incident wave vector and if I go back to this you know picture I can see that this ki line makes an angle of theta I with respect to the perpendicular line over here so at this Junction so maybe I am you know not very nicely showing it to you so between these two the angle is θ I so the one that I showed you with the orange now but I do not want that angle what I want is the angle that ei makes with respect to either the vertical line or with the horizontal line but because K and E are perpendicular the sum of the angles here plus the sum of the angles in the next one should be equal to 90^0.

So luckily the angle E I with respect to the vertical will therefore be equal to 90 - θi and if I decompose the electric field incident vector I will have e I cosˆ or cos of 90 – θi along the minus z axis and E I sine 90 - θi along the x axis so let me write down that.

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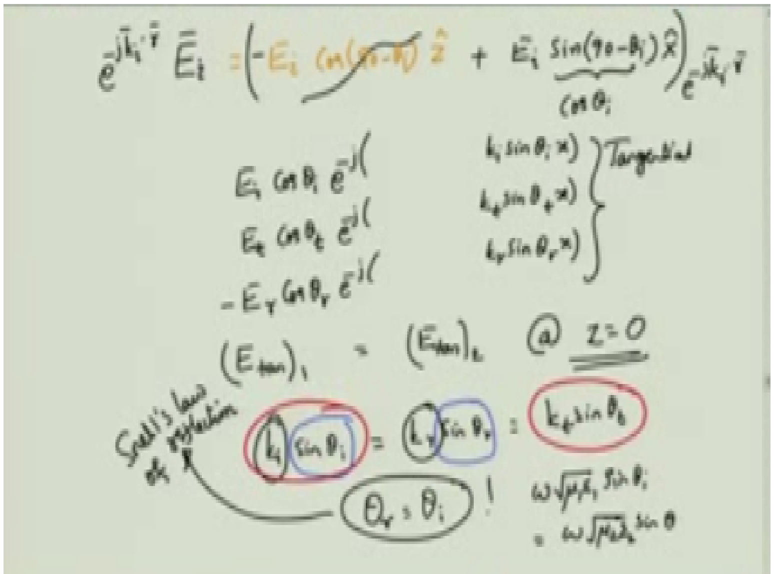

So ei is equal to the magnitude of ei, I do not know that I mean it is given as part of the problem but right now it is just a constant for us ei cos 90 - θi along the -z direction so this would be along - Z direction okay + e I sine 90 - θi along the x axis right but this is not of my interest so I would not really bother that one of course I have not also written the phase factor I should write the phase factor as E power minus j k I . r here so again the entire thing gets multiplied by e power minus JK I . r okay for the incident phase factor.

So here I have sine of 90 - θ so this is nothing but sine 90 cos θ therefore this is cos θ is what i am looking at so I have e i cos θi e^{-t} let me write on ki dot r completely so I have K I cos θi Z + K I sine θi X okay so this is the tangential component of the incident electric field what about the tangential component for the transmitted field well that would be e t cos θ T following the same argument it would be et cos θ T E^{-j} k t cos θ T times Z + K T sine θT X okay.

So that is coming from this relationship so here you have ki as ki cos θ I and ki sine theta I the electric field of the transmitted and the incident wave vectors make the same you know or in the same kind of a direction so therefore they are easy to write this one for ER I need to find out the angle so the angle made by K R with respect to the vertical line will be θ R therefore the angle made by the red line E R with respect to the vertical line will be 90 - θ R but this time the electric field will have a component of - Z right.

So it would have a component along - Z and a - X component so I can go back to this one and rewrite this as - er which is the reflected field amplitude times because there is also a 90 - θ R associated there will be cos 90 - θ R along the z axis but sine 90 - θ R along the -x axiswhich again means that this would be a cosine wave only so this would be cos of θ r and let us write down the complete you know expression for the phase factor this would be K R cos θ R Z this would be a minus direction plus K R sine θ R into X, so these are the tangential fields that I have for the medium 1 and medium 2 and boundary condition tells me that the tangential electric fields in medium 1 must be equal to the total tangential electric field in medium 2 at the boundary and what is the boundary intercepts that we have boundary interfaces set equal to 0 right.

So I can substitute for Z equal to0 which allows me to simply remove all these Z dependent terms okay so I remove all the Z dependent terms all the plus terms also even remove and I just have a phase factor which is now dependent on X now this is not what we expected for the case in the normal incidence when we applied the boundary condition at Z equal to 0 right the phase factor just became a constant they did not depend on X Y or Z right in this case they depend on X Y or Z.

So it does it matter that I satisfy the boundary conditions at one point but does not satisfy the boundary currents at the boundary conditions at the other point you cannot have that so these relationships have to be valid at all points of X ok and the only way that you can actually have them valid at all points of X is when you know when you make the phase factors be equal to each other ki sine θ I = K R sine θ R which must be equal to K T sine θ T okay otherwise you won't be able to satisfy these are different I mean at all points of X okay.

Now there is no surprise that ki must be equal to K Rafter all they both are in the same media the incident field and the reflected fields are both traveling in the same media therefore these amplitudes are the same but now look at this very beautiful relationship sine θi = sine θ R right and θ I and θ R in our case can go from 0 to 90 and if your range is only from 0 to 90 and sine θ = sine θR then the only condition that you can you know draw from this one the conclusion that you can draw from this is that θ R = θi okay this is a very important thing and in literature or in your 10 standard 8 standard you might have known this as Snell's law of reflection right.

The angle of reflection is equal to angle of transformation that so-called law is just a consequence of boundary condition that's a little bit of a surprising second law that you would have studied it will would have related the angle of incidence refractive index angle of this one transmission out or the refraction and we would call that one and that comes by equating K T sine T to K I sine θ I okay so what is that relationship ki I know is given by ω x √ μ1 ε1 and sine θ is just sine θ I this must be equal to ω √μ2 ε 2 sine θ T.

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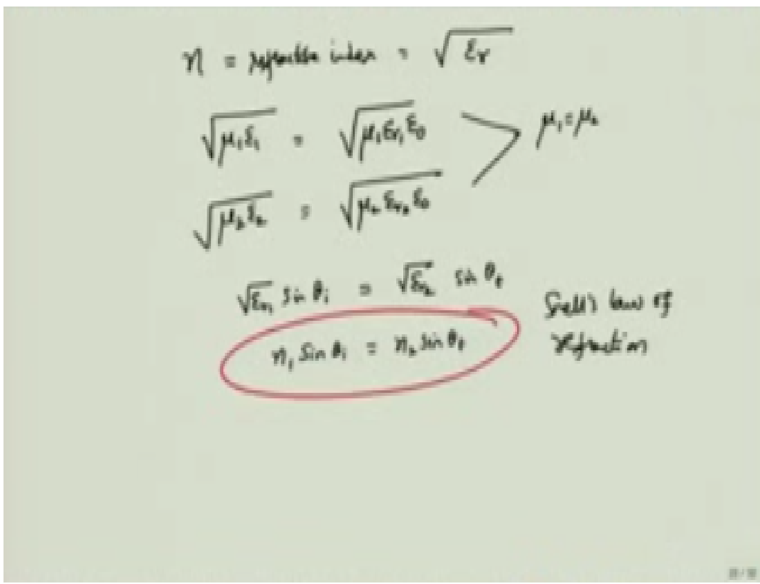

And if you know if you do not know this one the refractive index n which is the refractive index is defined as the √ the relative permittivity of the medium okay, so in the first medium you had √ μ1 ε 1 which can be written as √μ1 okay ε R 1 ε 0 okay and for the medium two you had a μ 2 ε 2 √okay this can be written as √μ2 ε r 2 ε0 if I assume same magnetic media therefore having μ1 = μ2 then the equation that we have written here so this equation can actually be rewritten as √εR 1sine θi = √ ε r2 sine θ t okay this identifying √ε R 1as the refractive index of the first medium N 1 you have N 1 sine θ I equal to n 2 sine θ T.

And this relationship is what is called as Snell's law of refraction being another word for transmission is not it beautiful that we actually obtained the Snell's law of reflection wherein the angle of reflection is equal to angle of incidence furthermore the incident wave the reflected wave and everything lie in the same plane while we also obtained the second law which is Snell's law of refraction okay which tells you that N 1sine θ I = n 2 sine θT the angle of incidence sine of angle of incidence 2 sine of angle of refraction must be some ratio of n2 to n1 and therefore that must be constant as long as n 2 and N 1 are constants themselves.

Are we done yet well we are kind of done but we still need to understand the reflected field amplitude to the incident field amplitude and transmitted field amplitude to the incident field amplitude in other words I want to derive the reflection coefficient and the transmission coefficient for the amplitudes only the ratio of amplitudes in order to do that one I go back to the boundary condition well from the boundary condition that I had I had AI cos θ ie T cos ε T - Yi R cos θ R.

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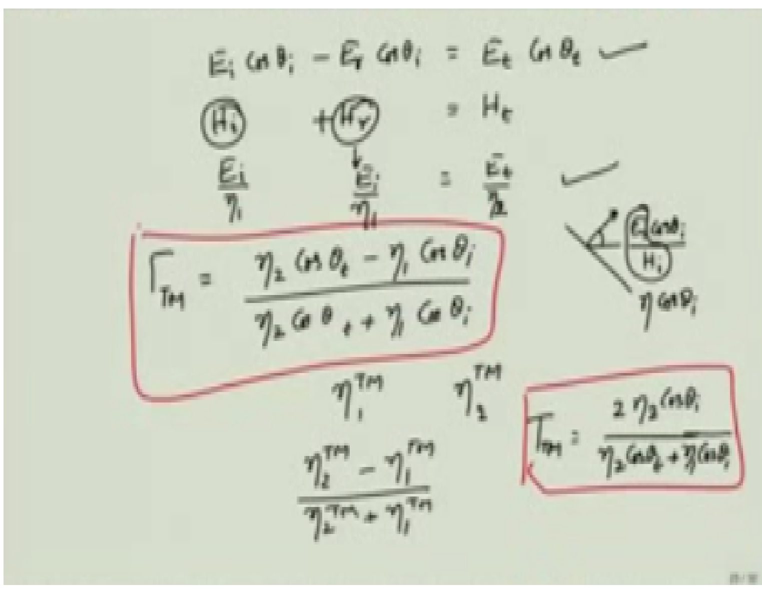

In the medium one we have e I cos θ I correct - eR cos θ R but cos θ R is nothing but cos θ I so I can rewrite that one as θ I itself this must be equal to e t cos θ T are we done yet well we are not done here because we still have the magnetic field relationships okay the magnetic field relationship is that the magnetic field is along the Y direction in the in the region one it is along the Y direction in region two Y direction in the region 3 right.

So because they are all in the y direction and y direction happens to be tangential to the plane of interface z equal to 0 you can you can write the amplitudes of the incident field as H I+ H R must be equal to HT okay now H I can be rewritten as e I by ETA 1 H R can be written as e I by ETA 2 which is the pity ratios right. So this must be equal to ET / η 2 sorry this e HR is ei / η1 only because it is in the same medium now you have two equations two unknowns you know you can combine the equations and obtain gamma which is the reflection coefficient for the TM case TM is what we have considered the magnetic field is actually perpendicular right the magnetic field is perpendicular to the plane or this is also called as a parallel polarization.

And this reflection coefficient for the TM case therefore if you solve the equations will be η 2 cos θ t - η 1 cos θ I / η2 cos θt + η 1 cos θ I this is a slightly simple way to remember that this is η cos θ T see you had the electric field in this way right what is the tangential component of the electric field the tangential component of the electric field was e I cos θ I if you divide this one by H I EI / HI will be η meter of the first medium that is getting multiplied by cos θ I so if you define the impedance of the medium as η TM okay η TM for the first medium right which would be η 1times cos θ I.

Similarly you define η T M for the second medium which would be ETA 2 cos θ T right then you can remember this one γT M as simply the second load impedance η 2of T M - η 1 T M divided by η 2T M + θ 1 T M okay the transmission coefficient for the case you know the transverse magnetic polarization case is given by 2 η 2 cos θ I this is not the same so there is a slight change here so do note that one η 2 cos θt + η 1 cos θ I okay. So this is the transmission coefficient and what we have obtained here is the reflection coefficient.

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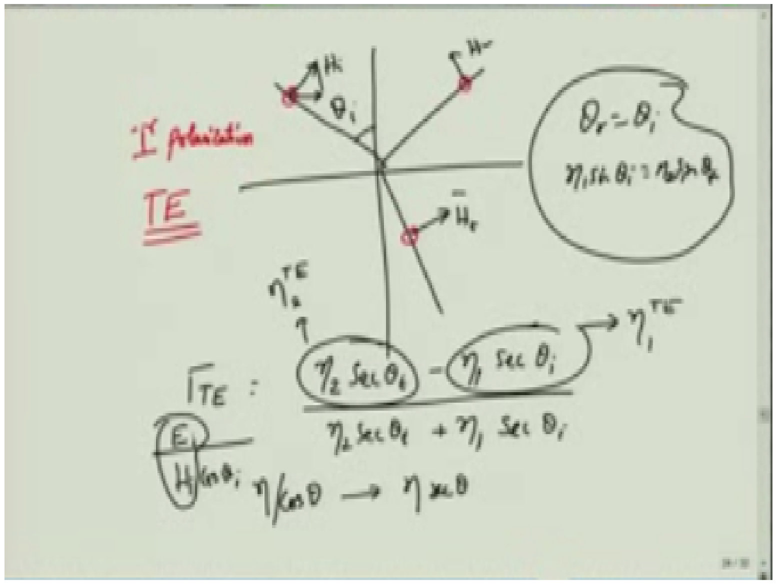

There is one more case which we have not discussed and in this case what you have is the you know oblique incidence at an angle of theta I reflection and transmission but here the electric field lines will be perpendicular to the plane of interface and the plane of incidence so in other words the polarization here is what is called as the perpendicular polarization perpendicular because the electric field happens to be perpendicular to the plane of incidence and the intercept this case is also called as transverse electric polarization okay in the transverse electric polarization you will have the magnetic field H in the same plane as the x and z planes okay.

Whereas the electric field will be perpendicular to it again you can define or you can derive γ te which is the reflection coefficient and even for this case θ R will be equal to θI and n1 sine θ I will be equal to n 2 sine γT okay. So these relationships are independent of what polarizations that you have in fact when you studied this Snell's laws you never were told that this is applicable for TE or applicable for TM modes or TM polarizations okay you can derive the expression for γ te as well γ te will be you know you can actually obtain this one as η 2 secant θt -η 1 secant θ I / η 2 secant θ T + η1 secant θ I okay.

Again 1 no reasonable way to think of this one is because the tangential H component here will be cos θI right so it will be H I magnitude times cos θ I the medium impedance can be obtained by looking at the tangential electric field which would be e I along the Y direction now E I / H I is η, η / cos θ will be η times secant η. So you can define this η2 secant θ as the effective impedance for the te case of the second medium this would be effective impedance for the te case of the first medium and so on.

So this way you can remember only one formula for reflection coefficient which is Z 2 -Z 1 / Z 2 + Z 1and replace Z 2 by appropriately TM case η2 TM or η 1 TM and similarly for TE case you replace that set by η times T η of T case so this you know completes our discussion of oblique incidence please review all the relationships carefully the surprising aspects of oblique incidence and the appearance of total internal reflection is what we will do in the next module, until then thank you very much.

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