Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course title
Applied Electromagnetics for Engineers

Module -47
Reflection of uniform plane waves

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Hello and welcome to NPTEL mode on applied electromagnetics in this module we will discuss wave reflection and transmission in the scenario what is called as the normal incidence from one medium to another medium in the previous module when discussing skin effect whether it was on the flat conducting material or the region or whether it was a round wire we had seen that one of the ways to excite the surface currents on imperfect conductor was to actually net a plane wave okay be incident on that one in the previous model I showed you a picture where the region below this was all a conductor.

And the electric field actually had an X component in this way so the electric field was in this way this was caused by some way if we can actually take a plane wave and then make it fall on the imperfect conductor these electric field lines will therefore generate a corresponding surface current and we have seen what happens when there is a surface current at the at the surface between you know yesterday in the in the case we considered was between air and an imperfect conductor so these surface currents will then eventually has to propagate down they will attenuate and I know generate I mean they will attenuate.

And lead to what is called as the skin effect in this problem that we discussed in the previous lectures we never really bothered as to what happens to the fields in the incident medium is no because these are the incident medium and this is the second medium so this wave was supposed to be incident from the first medium on to the second medium and we know what happens to this wave in an imperfect conductor as it propagates down it simply gets attenuated but what happens to the fields in the first region that we have not investigated and that is precisely what we want to do in this case.

Okay we also consider you know because we have already analyzed the imperfect conducting case or for imperfect conductor or a good conductor case we will assume a perfect conducting material so for all Z greater than 0 right so Z equal to 0 is the interface plane that separates the interface between air and a perfect conductor and by perfect conductor we mean if $\sigma=\infty$ okay so in that case and assuming in the region one is an insulator we can have for example air as an insulator with its impedance of 377 ohms or in general you will have a certain value of μ naught and an ε I am assuming that the first medium or the first region you know where the plane wave is traveling is also non-magnetic.

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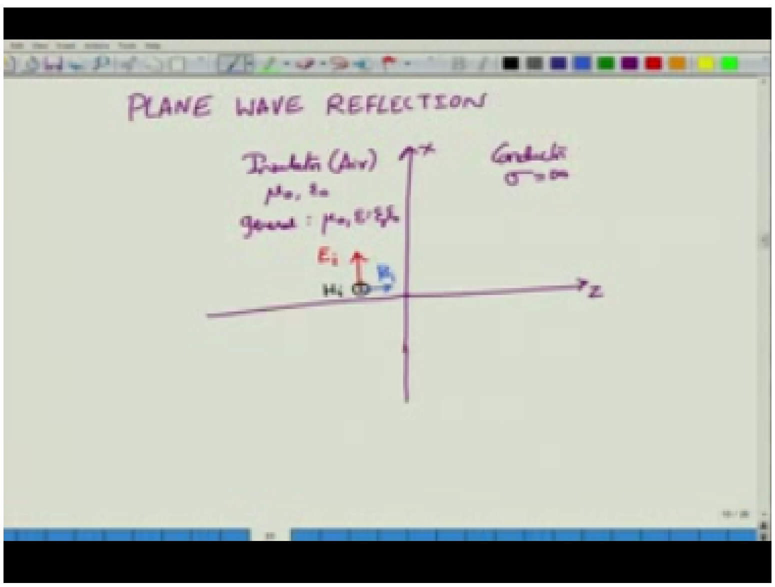

And therefore μ is equal to μ naught okay so this is the problem that we are looking at today you have an electric field incident and the magnetic field incident together of course EI and H I form an electromagnetic wave so this is a uniform plane wave which is propagating along z direction it is propagating along z axis and the propagation coefficient is given by β I remember what β is β I is √μ 0 ε if the medium is in general insulator or it would be equal to ω into μ naught ε naught if we are talking about air okay and of course 1 by √ of μ naught ε naught is equal to the speed of light or we will call this asup1 where you P denotes the phase velocity and that one denotes that we are in the region 1 okay.

So this is the region 1 which is insulator region 2 is a perfect conductor with the value of conductivity being equal to infinity so now this wave starts to this wave is propagating from Z =-∞ and then it approaches the Z= 0 interface this is the Z= 0 interface okay and the vertical axis is along the x and y axis will be coming out of this particular plane so this is how the y axis would be coming out okay so that is what we have what happens to this uniform plane wave as it impinges on the good conductor okay in order to answer that we need to know what is the boundary condition that is necessary that these fields have to satisfy okay what is the boundary condition between a perfect insulator such as air and a perfect conductor.

So with the conductivity going all the way to infinity what is the boundary condition well the total tangential electric field okay must be equal to zero right so you have et1being the tangential electric field there that one must be equal to zero and the tangential magnetic field must be equal to the surface current density or the sheet current density that needs to be there clearly in a perfect conductor there cannot be a et2 that is tangential component of the electric field now you will have the magnetic field component so these boundary conditions must be satisfied but our problem is not a static problem.

Okay because our electric field E I if you write it in the complete vector notation will have some amplitude let us call this as X hat even okay and it is propagating along the z axis so you have to the power J Ω t minus β I into Z what is Ω here Ω is the frequency of the plane wave okay so is the frequency of the plane wave and β is related to that particular frequency by the expression that we have written similarly the magnetic field for the incident wave will be along the Y direction because the wave is actually propagating along the z axis Y cross H must point in the direction of the propagation which is Z.

And the amplitude here will of course be even divided by X 1 where X 1 is the medium impedance given by √μ naught divided by ε naught for air or it is in general given by μ naught divided by ε 0 or ε R the product of ε 0 and ε R is what we normally call as ε okay so this is the medium impedance it also has the same time and space dependence as does the z-component okay now if this is all that is incident okay.

Now at Z =0 is the incident plane of inter phase which actually separates the insulator and conducting materials right so all this region is conductor all this region is an insulator so at Z =0 if I try to apply the boundary condition what will that boundary condition tell me fore I it will tell me that x hat e one or rather X hat I can drop because X hat indicates that to the medium the electric field is tangential.

And in this case it is tangential right so if this is the medium so you can look at this one if this is the medium and the wave is propagating with the electric field along my thumb direction and once it hits this one here right so once this hits the interface which is this hand right then the electric field amplitude at z=0 this my hand is that equal to zero the electric field because it is tangential see this thumb is tangential to this interface right.

So because of that is the boundary condition tell me that tells me that this tangential electric field μ must vanish now how can the tangential field vanished for all values of time it is one thing that it can vanish at a particular time but the boundary condition is not dependent on the time it is independent of time for all times the tangential electric field must vanish.

And the magnetic field which is this finger let us say okay such that X cross Y will be along the z axis the magnetic field H will have to generate a sheet current a surface current on this interface so to the left of this hand is the air and to the right of this hand is the conducting material and this one separates the air and the conducting medium and therefore forms the boundary.

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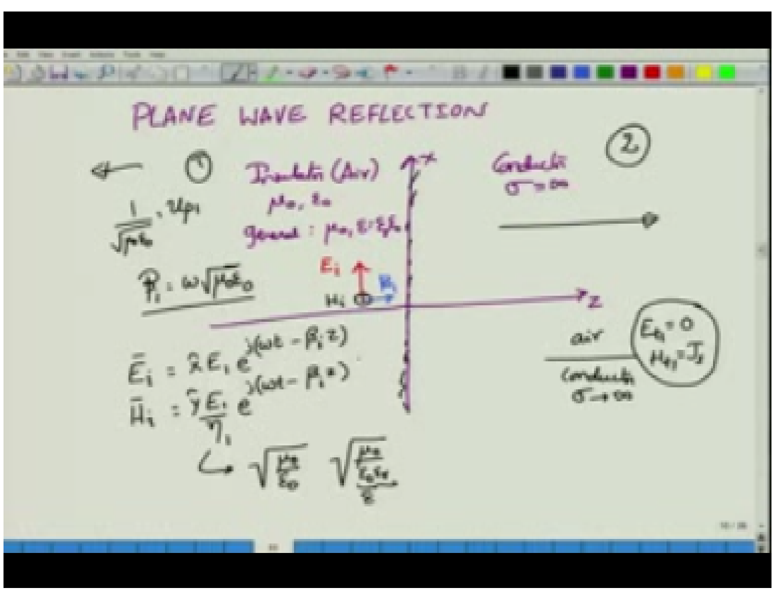

So clearly if I just have the incident wave or the incident electric field and the incident H field I cannot satisfy boundary condition for all time correct because evaluate this one at Z =0 do you get the tangential electric field in the region one will be even e^{j}ωt okay or you can say that this is e1cos ωt because fields have to be real okay and this if it is equal to 0 because there is no e T to here there is no tangential component of the transmitted field there is nothing to be transmitted on the second region so we have the condition that says that e1cos ω T=0 and if this has to hold for all values of T then there is only one trivial solution which makes even equal to zero obviously that's not what we want right.

So with only incident waves you do not satisfy boundary conditions so what should happen then if you want to satisfy boundary condition well you will have to have a reflected wave at this point okay so you will have a reflected field this reflected field must propagate along minus Z direction because of course the incident wave is propagating along plus Z direction this wave must propagate along minus Z direction okay.

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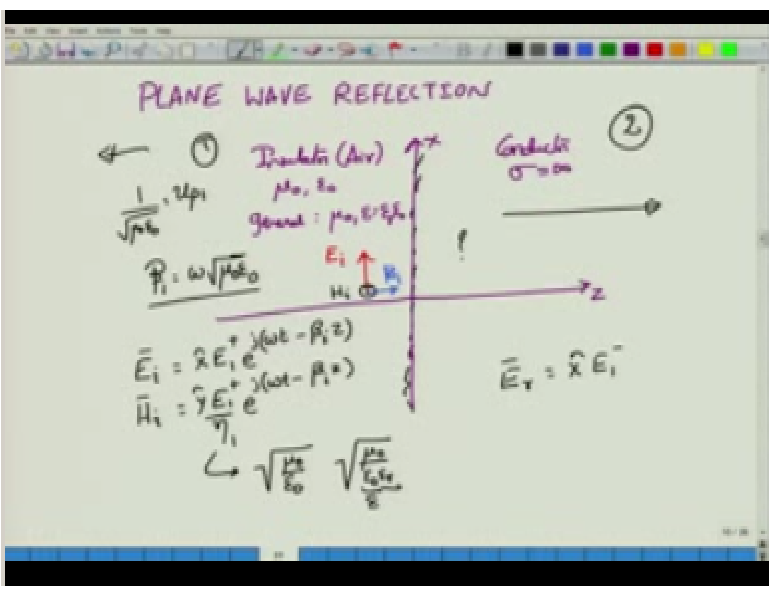

So let me tentatively write the electro reflected electric field corresponding to the reflected uniform plane wave as having the same direction as the incident wave so X hat and instead we will write this one else even - okay so minus indicates that this is a reflected wave amplitude okay so let me also go back and adjust the amplitudes for incident fields I will write them as even plus and even plus so that I know plus indicates the wave propagating along the plus Z direction and a minus indicates of a propagating along the minus Z direction.

So what will be the frequency of the reflected wave well because you know you can show by the application of the boundary condition that suppose you postulate that this will have a different frequency let us say okay and just show you that one so you have e ^{J}ωt-β Rz is the reflected field so in this region now you have a reflected field which is propagating along the minus Z direction so this is the reflected free our component and the propagation is along the direction along - z direction okay.

And the magnetic field of course will have to have a minus y direction why should I have a minus y because x cross y will give you the Z direction of propagation whereas X cross minus y will give you a minus Z direction of propagation which is exactly the direction in which the reflected wave is propagating therefore the reflected magnetic field HR will be along minus y direction and then it will have even -/e1e$^{j\omega t}$+β RZ because the wave is propagating along minus Z direction okay so these are the fields that I have written the question is can I make abhi different from Ω Prime and what will be the relationship between β I because β is the magnitude of the incident wave vector.

And β R is the magnitude of the incident wave vector or the propagation constant right so what should be the relationship between the two in region one which is where the action is now taking place because nothing is propagating inside the second medium so in medium one where the action is taking place the total tangential electric field okay will be the sum of incident and reflected fields right evaluated at Z=0 so in the Z equal to zero interface the total tangential field let me write this as e total X e1+e$^{j\omega t}$+xe1-e$^{j\omega t}$ and this total electric field must then be equal to zero why should it be equal to zero well boundary conditions tells you that this must be equal to zero.

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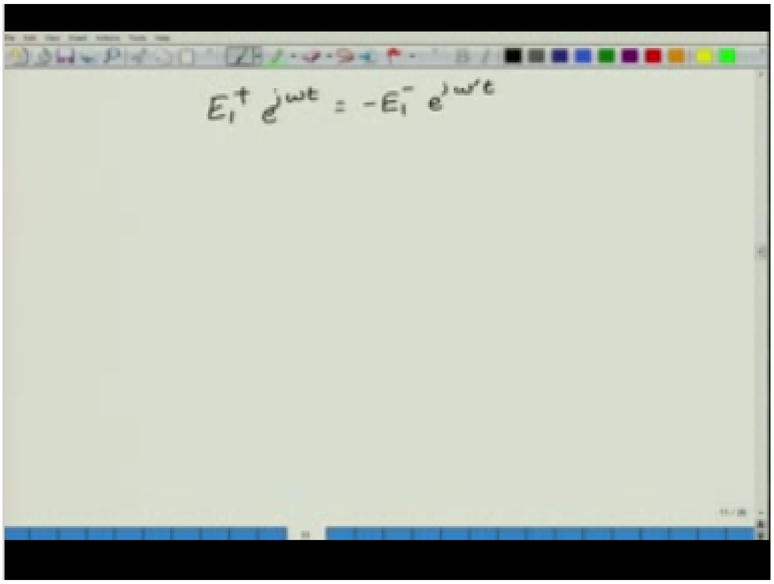

Then you have a relation which now tells you $E_1+e^{j\omega t}=-E_1-e^{j\omega t}$ so on the left hand you have a sinusoidal wave you should take the real part of it on the left hand you have a real cosine wave on the right hand side also you have a cosine wave and these two frequencies if they are different then their amplitudes will be equal only at a few points in time okay but you want this relation to hold for all values of time so clearly if that has to happen the frequencies on the left-hand side of this expression must be equal to the frequency on the right-hand side of this expression okay therefore ω' must be equal to ω. So there reflected wave will have the same frequency as the incident wave okay.

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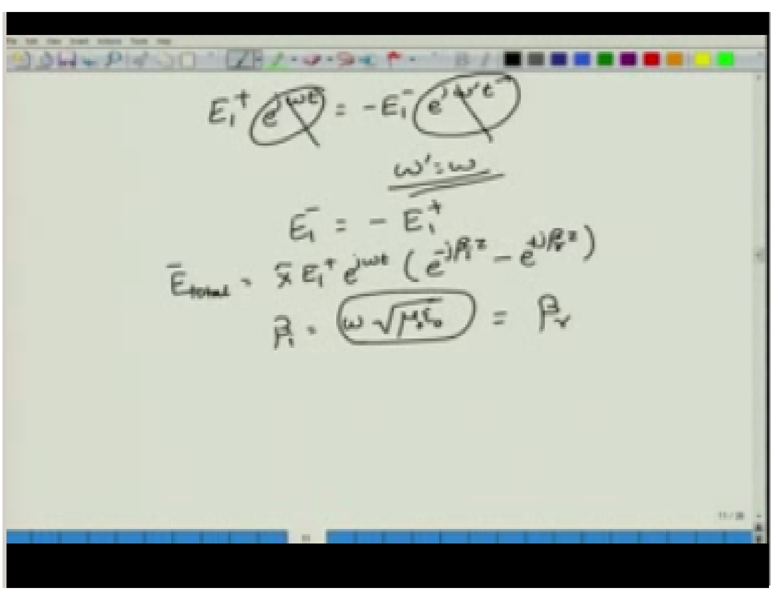

So the only conclusion now that I have left is to tell you that even - which is the reflected wave electric field amplitude must be equal to minus even plus okay so even - is equal to minus even plus therefore I can go back and rewrite the total electric field as Xe1+$E^{j\omega t}$ is also the same because the frequencies are the same so you have $E^{-j\beta iz}-e^{j\beta rz}$ said minus E power minus J sorry plus J β are said now what about β I and β are should they be different or should they be equal look at what is β I β I depends only on the frequency Ω and in we have shown that β I mean the reflected field or the reflected wave also has the same frequency Ω.

And √με or μ naught ε naught because we are considering air in this example so none of this actually depend on the direction right the propagation constant the magnitude of the propagation constant depends only on the frequency and on the Constituent parameters μ naught and ε naught of the medium right clearly this must also be the case for β R because β R corresponds to the propagation coefficient of a wave which is propagating in the same medium as the incident wave its β R magnitude must also be equal to Ω √μ naught ε naught so β R is equal to β I.

Okay because βR=β I I can replace this R subscript here with I and what is this expression in here exponential of $e^{-jx}-e^{jx}$ is 2J sine X correct this is a sin X expression and 2J gets multiplied out there therefore the total electric field in the region 1which is now the sum of incident and reflected waves is given by it of course has the same direction as X but this is given by even plus we will write this as minus 2J okay sine β I Z right.

So β I is equal to β R therefore it does not matter whether I use β I said or β are said okay I have not written what the complex exponential factor $e^{j\omega t}$ because I want to write only the phase or form of these expressions therefore I have simply dropped $e^{j\omega t}$ I can drop it please remember I could not have dropped that one in this expressions because I have assuming that they are different.

So two phase I mean when I talk of a phase or when I sum the phase or components they all have to have the same frequency so because of this condition that we have proved I am able to remove the $e^{j\omega t}$ factor from this expression and whatever that I am left out is the phase or of the total electric field so you have a phase or electric field over here which we have shown that should be equal to -2jsinβix I will leave the exercise we leave this as an exercise for you to find out what will happen to the total magnetic field of course on the surface the magnetic field will have to be such that the sum of the incident amplitude.

And the reflected amplitude must be equal to the total surface current out there but away from the interface what happens so on the interface at Z equal to 0 I know that H I plus HR must be equal to the total surface current j s but far away from the interface what should be the total magnetic field right I will leave this as an exercise for you to find out what should happen to this one and you will probably not be surprised but you will see an appearance of cause β I into Z somewhere over there I leave this as an exercise for you to find all right now we have looked at the fields that are present at the region in region one away from the interface do these equations surprise us fortunately these equations are not surprising to us.

Because very similar thing happened in one of the earlier modules when we talked of short-circuit termination of a uniform loss less transmission line so you had a lossless transmission line wherein you had an incident wave but then when you terminate that transmission line with a short-circuit what happened there was a reflected voltage our reflected current and the reflected voltage amplitude was- the incident voltage right so V R is equal to minus V I in other words with a short circuited termination the reflection coefficient that you obtained was equal to minus one right so that happened earlier and we know that the fields actually form the standing wave and in this case there is no difference the fields do form a standing wave.

Because the incident wave is coming in the reflected wave is going back at the same time but with a face of 180 degree opposite with respect to the incident wave they both will interfere together to form a standing wave pattern in fact if in this particular case a standing wave pattern will be the maxim because the reflection coefficient will actually be equal to minus 1 right the magnitude of the reflection coefficient is equal to unity in this case.

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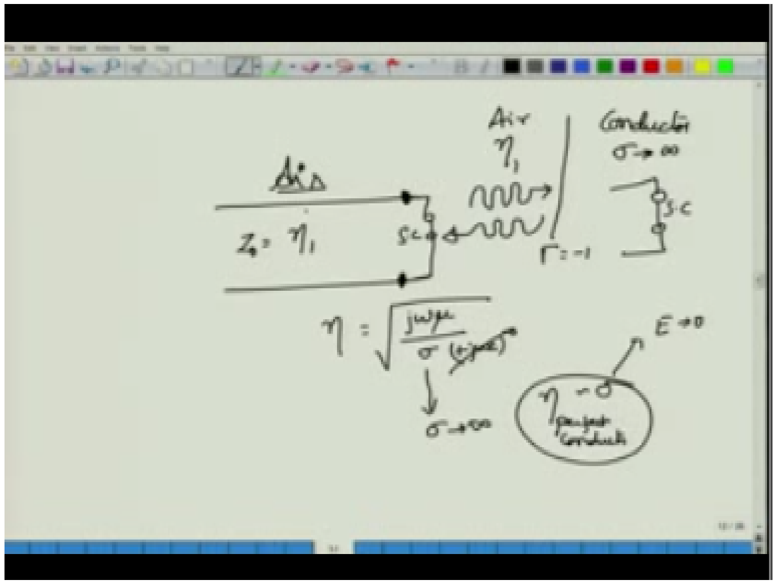

So what is our lesson we can in fact think of the space problem so you had a air right you had air with the impedance of X 1 and you had a conductor and we found that for a perfect conductor where Sigma tends off to infinity right whatever the incident electric field that incident electric field was completely reflected correct with a reflection coefficient of minus 1so because of this reflection coefficient being minus one this conductor is equivalent to a short-circuit termination so this is a short-circuit termination of a uniform lossless transmission line as a uniform lossless transmission line and then terminated when you consider a perfect conductor terminated by at its load or you can consider conducting material as its load.

And the impedance of that conducting material perfect conducting material is a short-circuit now does it also make sense yes we have already seen that the impedance of conducting material right is complex and it is given by $\sqrt{J\omega\mu/\sigma}$so we have seen this one earlier of course there is a plus J Ω ε term but we neglect this plus J ω ε term assuming that this is a good conductor but now we are not even talking of a good conductor we are talking of a perfect conductor for a perfect conductor Sigma goes off to infinity and therefore X goes off to zero so χ of a perfect conductor the impedance of the perfect conductor goes to zero of course this also is true because the electric field in a perfect conductor goes to zero.

So you do not have any tangential electric field and therefore because of that reason this electric field also goes to zero the impedance of a perfect conductor is a short circuit so whenever you terminate this one you do obtain a short circuited kind of a equivalent circuit of a transmission line so the problem that we had in terms of waves can be re-written in terms of the transmission line problem okay now let us very quickly jump to another scenario where I have a second medium of impedance X to the first medium having an impedance of X one which means I am assuming that this is also insulator again I will assume that this is a perfect insulator.

So which means that Sigma will be equal to zero the same condition of Sigma equal to zero is true even in the region one okay so this is region one and this is region two as before I have an incident electric field which will have an amplitude of even okay propagating along the z-axis this would be my z-axis as before this vertical line is along the x axis and y axis will be coming out of this plane okay so I have the magnetic field which has an amplitude of h1 plus and is along the y axis of course h1 plus is nothing but even plus divided by X 1 ok we know in this case that there has to be reflected field otherwise you will not be able to satisfy the boundary conditions okay you can show that by trying to assume that when there is an even plus.

And h1plus there of course be some waves which are transmitted into the second medium as well right so the second medium wave is also traveling along the plus Z direction and the amplitude of the second transmitted field can be written as e 2 plus the subscript 2 indicates that this is a region 2 okay and there will be a magnetic field we will not I mean magnetic field in the same direction because electric field we have assumed to be is in the same direction for incident and transmitted fields so this would be H 2 plus of course H 2plus will be equal to e 2 plus divided by X 2 you can show by applying the boundary condition that only even plus cannot be equal to e 2 plus right because the values of X 1 is usually different from the values of X 2otherwise there is no boundary that we are actually talking about

Because this is not equal there has to be some reflected field right there cannot be additional transmitted fields you can show that because it only means that you can try to increase this one but please remember these have to be valid for all frequent I mean all times and at all points along the Z= 0 plane okay because of that you can clearly show that there has to be a reflected wave and that reflected will be at the same frequency as the incident wave the transmitted wave will also be at the same frequency as the incident wave okay.

And what would be the reflected field looking like the reflected field will we will assume that it will be in the same direction as the incident field the electric field of that one so you have an amplitude of even minus for the reflected and it is propagating along minus z direction and the magnetic field will have a minus y direction to conform to the fact that the reflected wave is propagating along the- said direction okay now applying the total electric field the tangential electric field boundary condition at Z equal to zero I am not going to invoke a power J Ω T obviously because we are assuming that they are all phase and it is true they are all phase in this case all the frequencies are same the incident reflected.

And transmitted okay so in the region one the total tangential electric field has an amplitude of $e_1+e_1-=e_2$ electric field in the region- which is e2 plus correct what will happen to the magnetic field well magnetic field amplitudes h1+ h1 - okay I have not yet written them in terms of electric field here so this must be equal to h2 plus okay there is so you might ask what happened to the surface current density well we do not have any surface current density because this is the interface between two insulating materials so no surface current density and replacing h1 + H 1- as well as H 2 plus with respectively with appropriate values of e1+/ η1+e1-/ η1=X

And the direction of the magnetic field is of course along the minus y direction therefore there will be a minus sign here and then you have e2+/ η 2 please remember that H will be e 2/H will be e by the medium impedance η2 now what is that we have we have a simple no matrix equation so that will have a matrix elements of 1 by1/ η1 by X/ η1 times e1+e1-=1/ η 2 okay so I have this simple equation now it is only a matter of calculation little bit of an algebra for me to defined what will be the ratios of even - 2 even plus this ratio of even - 2 even plus of course is what is called as a reflection coefficient and the ratio of e2+2+e2 is the amplitude of the transmitted field to even plus which is the amplitude of the incident field this ratio will be called as a transmission coefficient right.

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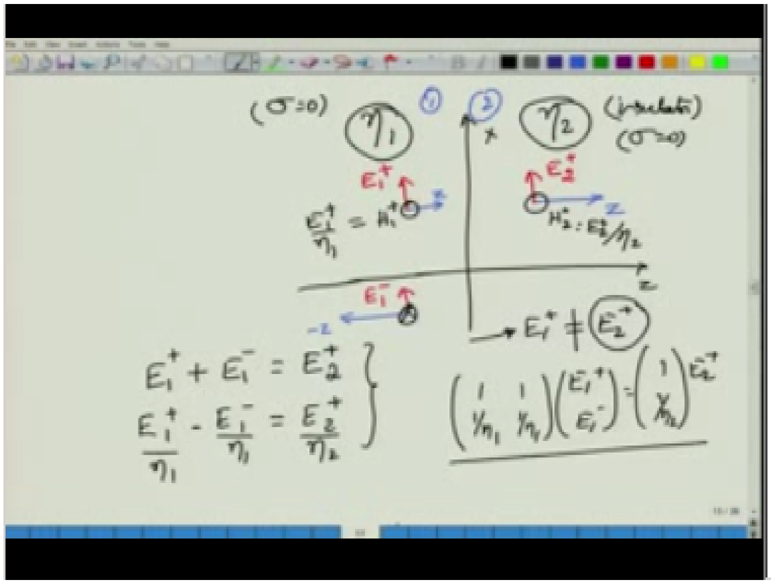

I will leave this as again a small exercise for you just have to sum and then difference the equations on the left and the right hand side.

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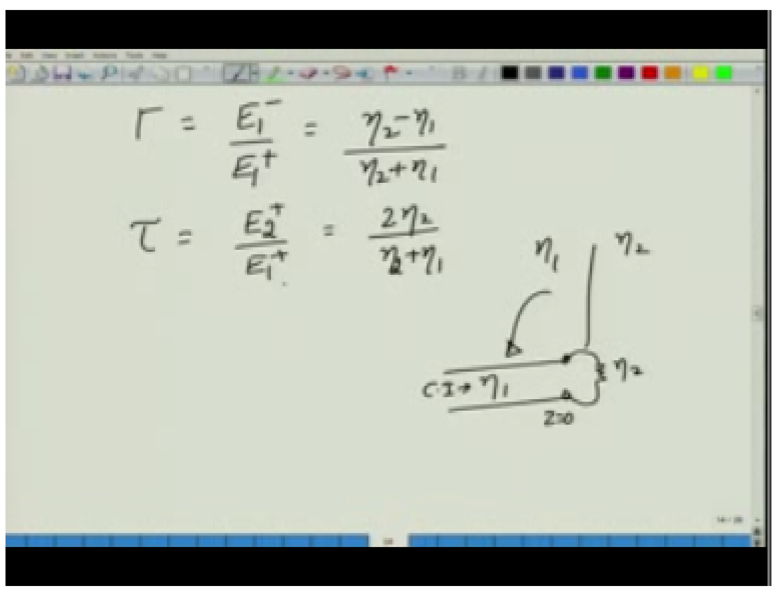

To show that the reflection coefficient for the electric field in this particular case okay if you want you can put a subscript of electric field E here but I am not going to do that this gamma which is the ratio of the reflected field amplitude to the incident field amplitude is given by $\eta_2 - \eta_1 / \eta_2 + \eta_1$ and similarly you can show that the transmission coefficient which would be the ratio of e 2/e1+ this is the incident this is the transmitted this would be equal to 2 η 2/η2+ η1 in fact this medium that you had η 1 and η 2 can be considered to be a transmission line.

Okay with the load the second medium acting as the load with an impedance of η 2and the first medium being a uniform transmission line having a characteristic impedance of η 1 okay so at Z equal to 0 which is where the interface is separating the two medium the problem of the waves can be know quite easily in an analogy shown to be equal or equivalent to the problems of a transmission line okay so we will have more to say about this in the next module until then thank you very much.

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