Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 44
Skin effect in conductors

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Hello and welcome to NPTEL MOOK on applied electromagnetic for engineers, in this module we will continue the discussion of wave propagation inside a good conductor by a good conductor we mean that the conductivity of the material or the conducting material is much larger than the product ωε, where ε is the permittivity of the material ω is the given frequency of the wave that is propagating inside the conductor.

We will touch upon a very practical and important topic of skin effect which tells us under high frequency conditions what will happen to the current flow inside these conductors, whether the current will be uniformly distributed throughout the conducting medium or whether it would be not uniformly distributed through the conducting medium, this has no practical importance because if you are designing systems at very high frequencies you want to know what is the current flow inside the conductor so that you can actually use the right amount of conductor.

If for example the conductor seems to have been there enough flowing entirely near the boundary then there is no point in having a 10 meter thick conducting wire, because most of that conducting material will not carry any current. However, if it turns out that the current is uniform so then the entire 10 meters of thickness of the conductor will carry current and therefore you do not have any savings on the conducting material okay.

So that is determined by what is called a skin effect and we will talk about skin effect, we have already talked about how a wave would propagate inside a material which is a good conductor right, we have seen what will happen to once you start from Maxwell's equation what will happen to the propagation coefficients the propagation coefficient will turn out to be complex with the real part having the attenuation component and the imaginary part having the phase coefficient right, so just to refresh your memory.

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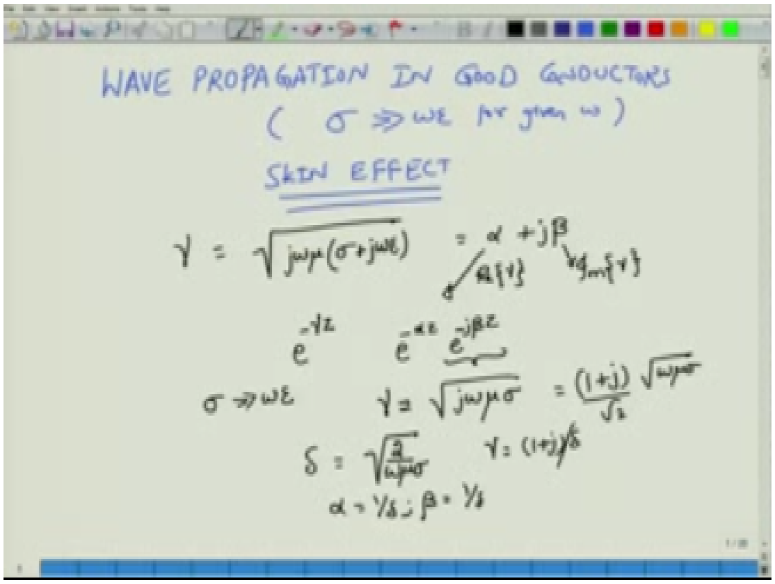

In the case of a good conductor material propagating insider in general a conductor first then the propagation constant γ will become or the propagation coefficient γ will become complex it is given by √Jγμσ+jωε if you split this itself into its real and imaginary components you will obtain some α and β, where α will be the real part of γ and then β will be the imaginary part of γ, this α tells you how the waves are attenuating because we have assumed that the waves as they propagate along the z axis will propagate as $e^{-\gamma Z}$ okay.

So in place of γ if you now substitute for α and β we will have $e^{-\alpha z}$ and $e^{-j\beta Z}$ this factor we already know simply allows you to determine what is the phase delay that is the wave is experiencing with respect to a certain reference phase whereas $e^{-\alpha z}$ corresponds to the attenuating factor the amplitude of the electric field or the magnetic field attenuates as the wave propagates inside the conducting material okay.

For the case of a very good conductor is much larger than ωε therefore γ can be rewritten as jω approximately can be written as jωμσ and we have already shown that this can further be written as 1+j/√.√ωμσ we also denoted by the variable Δ okay, we denoted this variable Δ√2/ωμσ therefore γ=1+jΔ okay so1+j/Δ okay which indicates that α is 1/Δ and β is also equal to 1/Δ okay, so the wave experiences phase shift as well as the wave experiences attenuation.

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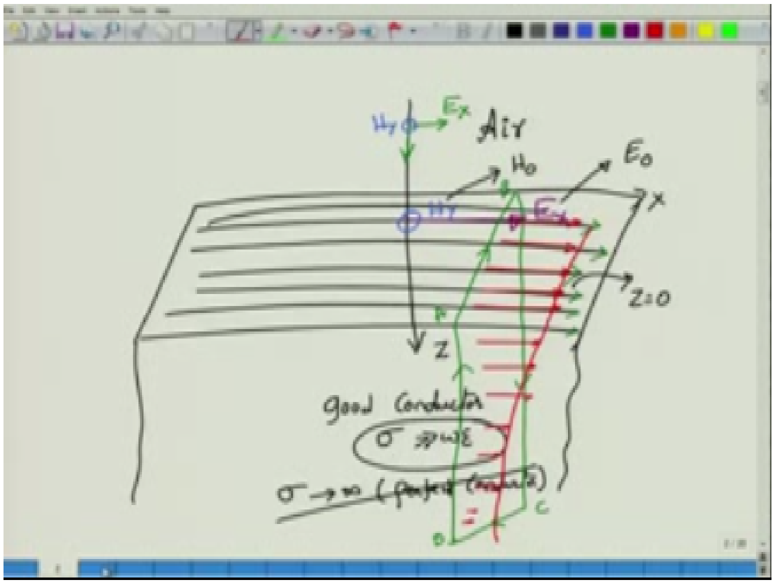

Now consider this kind of a scenario where we assume that the direction of wave propagation is along z axis and we take this one to be the x-axis and we have this plane which extends all the way from -∞ to +∞ but this is located at z=0 separating two regions below this z=0 the entire medium below this particular plane is made out of a good conductor okay, so this entire thing is made out of a good conductor where σ is much larger than ωε and above is a perfectly insulating material as an example we will simply take this one to be air.

Now when there is a plane wave propagating so it has a certain electric field component and the direction of propagation which is along the z axis so the wave has an electric field which we will assume to be having the x component clearly the magnetic field would then be along the Y component correct so the pair Ex and Hy will have a certain ratio of the medium impedance inside the air, okay.

Now imagine this wave know steadily approaches the interface and at some particular time which we do not really worry about at what time this happens because we are only going to look at the steady state, we have the situation that the electric field or the wave has actually landed and is now incident on the conducting boundary, so it z=0 any value of Z below or greater than Z equal to 0 right, will then be completely a conducting surface and on that conducting surface the electric field is oriented along the x direction the magnetic field is oriented along the Y direction the wave has now incident on the conductor.

At the surface we will assume that the amplitude of the electric field is given by E0 okay and the amplitude of the magnetic field is given by H0. Please note that we are only considering a good conductor we do not let σ go to ∞ at this point which represents a perfect conductor okay, we do not have a perfect conducting situation here we have an imperfect conductor but that imperfect conductor is also a very good conductor in the sense that its σ is much larger than ωε.

Now what really happens in this case, well you should first of all raise an objection saying that we have not really considered the scenario where a plane wave is impinging on another conducting material and what happens as a result of that, you will be right we have not really addressed the situation of what happens when a plane wave hits a particular medium and what changes happen inside the first medium and inside the second medium as such okay.

But we will disregard that problem, because we are not really looking at what is happening to the medium or the fields inside the medium in from in the air side we are interested in what is happening once the plane wave hits and has a value of the electric field at the surface as E0 and the magnetic field is at 0 what happens once that wave starts to propagate inside the conducting medium, of course you know that that wave has to decay in amplitude okay.

Now let me show you what is known kind of the picture that I want you to carry forward for the next few you know minutes so that you really understand and appreciate what is happening to the skin effect. Imagine that this is the conducting media of course this should exist all the way to ∞ and all the way to ∞ this medium is air and the wave now appear I mean the wave is now incident and lands on this conducting media so this surface which you can see on the top will separate the media air and conducting material below, so all this wooden block you can think of this as comprising of the conducting material.

Now where is the what is the direction of the electric field on this surface well the direction of the electric field is along the X direction, so this is the X direction of the electric field that we have which is indicated by my fingers pointing to that you cannot really see but my thumb or now you can see the thumb indicates the direction of propagation okay, so on the surface these of course my fingers are not uniform.

But you have to assume that all the fingers should have the same length because you are actually looking at a uniform electric field, the electric field amplitude does not depend on the value of X okay, so on the surface it has a constant value of E_0. Now what happens as this wave begins to propagate well, you can imagine that as the wave begins to propagate because of skin effect the amplitude of the electric field starts to reduce as a function of z right, so it goes as exponential of $-Z/\Delta$.

So this is the length on the surface the length reduces the reduces, reduces, reduces, reduces eventually but I think an exponential decay at ∞ the amplitude would be equal to 0 so this is all the electric field. Now in a conducting material the relationship between electric field E the conducting conductivity of the material σ and the current density j is well known this is the point form of Ohm's law that we already know J equals σ times E.

So if this is the variation of the electric field inside the material so should be the variation of the current density J on the surface you have the current density J_0 which is given by σ times E_0 and this J vector or the amplitude of the J vector starts to reduce, reduce, reduce eventually going towards to 0 at $Z=\infty$ okay, this was all about electric field, what about the magnetic field now magnetic field will be along the y axis, so in this case assume that they are coming towards you as you look at the screen so this is how on the surface you have a uniform magnetic field H_0 and then this reduces in amplitude as it goes down and eventually goes off to 0.

So at Z equal to ∞ you do not have an electric field you do not have a magnetic field nor you have the current density because all the amplitudes of these quantities have gone down to 0. Now if I ask you what is the total current that is passing through a particular plane which I will obtain by cutting through this material in this way so you have to imagine that I am cutting the material through this way and exposing this particular cross section okay along Y I go a certain distance W okay, so maybe this is y equal to 0 where the point I am showing this is $Y=W$ and then you take this edge you know you keep going inside right inside the conducting material you go all the way to ∞ come back up parallel to this loop or to this line and then comeback on to this edge to meet back at the surface okay.

So along Z you go to ∞ then go and on -y direction then come back and then do this now clearly there have to be some amount of current that is flowing through this open surface because my J vectors are all crossing this okay, on the surface the J vectors are uniform but then they reduce as you go down to the conductor. Let us calculate what is the amount of that current that is obtained by cutting the plane, so I am I will show you a kind of a way to you know this is the loop that I am but I have drawn and I have actually illustrated that one to you by this example of the duster okay.

We call this loop let us say we name this loop as ABCDA okay, the J vectors are all coming out perpendicular to this loop because the J vectors are all along the x axis and while these are the J vectors that come out on the surface they are uniform but as you keep going inside the conducting material you will see that they are all coming but then the amplitude actually starts to decay exponentially, so the amplitude is decaying exponentially. So what is the total current enclosed.

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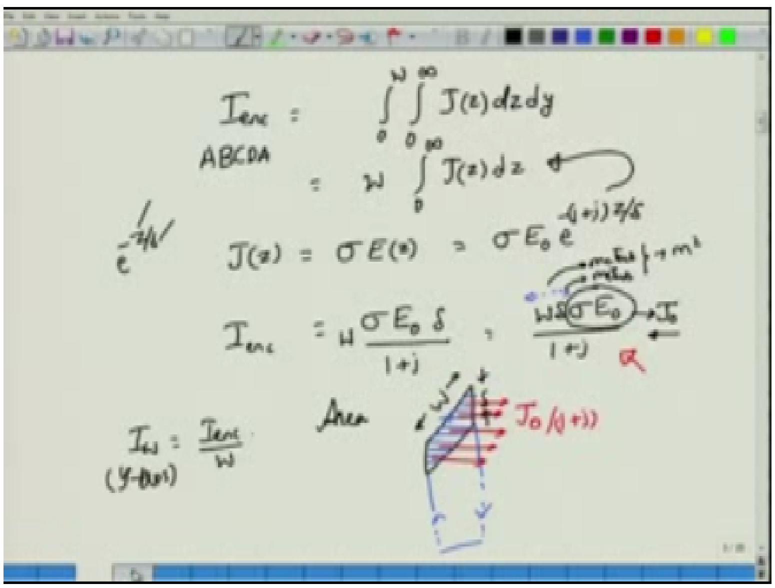

Well, the total current that is enclosed over the loop ABCDA will be integral over 0 to W because I have assumed that along the Y direction they have taken a distance of W along Z we have taken all the way from 0 to ∞ right, and the way the current density would be varying with respect to z dz dy here the J vectors are all independent of Y therefore there is no problem of not simply removing the integration with respect to Y so with respect to Y what would happen it would get multiplied by W.

And here you have 0 to ∞ J(z) dz what then should be J(z) well we know that J(z) the way the current density would change with respect to z is given by σ times E(z) and we know that electric field is along the X direction but its amplitude is z dependent and this is going as σE0 which is the amplitude at the surface and $e^{1+j-1+jz/\Delta}$ right, where α=1/Δ β=1/Δ so here you have $e^{-\gamma Z}$ and that is essentially what I have written.

If you now put this one back into the expression and carry out the integration I will leave this as an exercise you do not get ∞ okay, because the amplitude keeps on reducing although the loop area seems to be ∞ and in fact it is ∞ the total current carried by that open loop ABCDA is finite and you carry out this integration you will see that this will be given by σE0/1+JΔ so did we get this one correctly yeah, so this is the total current that is enclosed by putting this J(z) back into this one sorry, this is multiplied by W actually so you get WσE0Δ/1+J.

Now I can instead of writing this w here I will do a small trick I will multiply W with Δ or I put W and Δ together and I still have my σE0/1+J okay, now look at this term W into J okay, what is the units of W well, W is the distance unit so it is measured in meters what is the units of Δ, Δ again has to be measured in meters why because you have $e^{-Z/\Delta}$ Z in meters therefore Δ also has to be in meters okay, so you have this also Δ measure in terms of meters so what will happen when you take the product of these two this will give you meter square and what is the units of meter square or what is the quantity which has a units of meter square that is area correct, area has meter square.

What is this area, this area has a width of W along the y axis and along the z axis it is not ∞ but the z axis width is just about Δ okay, and if you know or you already know that this σE0 is the current density J0 which can now be considered to be uniform over this cross-section so over the cross-section which is W wide and Δ thick you can assume that the current density you know passing through perpendicular is uniform and has a value of J0. But if you really want it you can also consider this to be a complex J0/1+J but otherwise the point is that over this cross-sectional region the J vector is uniform or you can think of this current coming off as a result of the uniform current flowing only in this thickness.

Rather than the current that actually obtained by going all the way to ∞ coming back right, so this was the original loop that we had considered but rather than finding this original loop where the current is changing exponentially you can obtain the same value of the current if you assume the entire current flows only in this small cross section of thickness Δ and a width W. In fact one also defines sometimes what is the current per width okay, please remember current per width must be taken along the y-axis okay.

So current per width is given by the total current enclosed by the loop divided by W and obviously this can be obtained from the above equation or from this equation by simply dividing this one by W. Let us not come analysis yet, let us go back to this example that we have talked about okay, now notice that we had considered this cross-section right so we had considered this cross-section now instead of considering that cross-section let me move a distance L along the surface of this particular interface right.

So on the interface I move along a distance of L then I move a distance of W then go back by moving a distance of L and then come back to the starting point okay, so I will move from here to here then go along the direction of W then come back and then go back right, so I actually form a loop right. Now on this loop now if I ask you on this loop if I just consider these two planes okay, so this plane is that some say X=0 and this plane is our sum X=L okay, what is the potential difference between these two planes.

The potential difference between these two planes must be given by the integration of the electric field along the length L correct, so on this plane and this plane the potential difference between these two planes is integral of electric field along the length L but luckily for us this electric field is uniform on the surface has a value of E0 the length is L and E is along the x axis integration is also along the x axis therefore e.dl which defines my EMF will simply be given byE0 into L.

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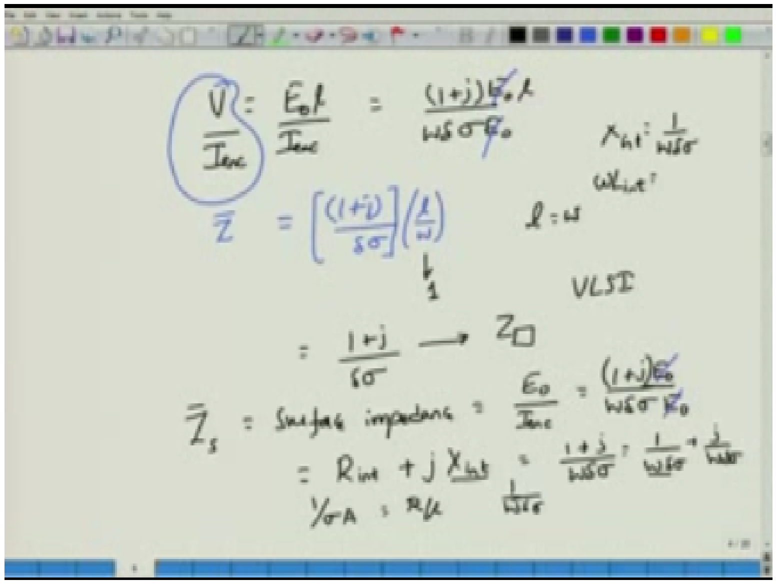

Alright, now E0.L is the potential difference that I see on the surface if I now consider the ratio of potential difference to the current enclosed okay, what do I get I know what is the current enclosed this I know from the previous expression given by WΔσE0/1+J so I need to go back and put that expression into this i get 1+J/wΔσE0 okay, times E0 into L clearly E0 cancels on both sides so I can remove E0 from the equation so E0 is cancelled on both sides so what I obtained is 1+J/WΔσ or rather I will rearrange this one.

So let me say 1+J/σΔL/W okay, and what is the ratio of voltage to the current potential difference to the current this has to be the impedance and in this case the impedance turns out to be complex right, so in this case the impedance turns out to be complex but notice we are not done yet so notice one thing suppose we consider L equal to W that is we consider the potential difference between two planes which are W apart and the current per width or the total current of the same width W right, so if I consider that one and then take the ratio since L=W this quantity will be equal to 1.

And what I obtain is a complex quantity 1+J/Δσ and this is called as the square resistance and the square resistance is very important in VLSI or micro electronics areas where you can actually calculate the resistance or the complex impedance of the materials nor the of the materials or the interconnects by this idea so you divide whatever the overall region where you are computing the impedance into small squares and of each of course they are all squares so on each square you calculate what is the impedance and then put them together in order to form the overall impedance.

We are not yet done this impedance that we have defined is defined by the potential difference to the current enclosed right, there is another impedance which also turns out to be complex in this case this is called as the surface impedance okay, the surface impedance is defined as the field amplitude to the total current enclosed okay, so it is not the potential difference but it is the field quantity so E0/I enclosed and this will be equal to 1+J dividedby now because there is a E0 on the top here WΔσ here this is times E0 is also present.

So this luckily again E0 cancels on both sides and the surface impedance that I obtained can be thought of as having two components that is the real part and the imaginary part we will denote the real part as R int which will be the resistive part and the reactive part by X in which of course is a function of frequency right. So this will be equal to 1+J/WΔσ okay, so there again you can rewrite this one this is 1/WΔσ+J/WΔσ okay, what is this W into Δ this is area and then you have 1/σ times area right.

So 1/σ times area is resistance by end of along tube which has an area of a right the resistance of such a tube as you know is given by L/σa or resistivity or resistance of that one is given by L/σa so this component R int can be thought of as resistance per unit length. Similarly X int will be equal to 1/wΔσ and X int is equal to ω times so X into let us write it over here so X int is equal to 1/wΔσ again W into Δ is the area right, but X int can be written as ω into L in where L is the inductance it is not actually an inductance that is there but it kind of an inductive reactance okay.

So this inductive reactance and has a certain inductance associated with that one and that equivalent inductance and int will be given by 1 by, so L int will be given by 1/WΔ which is the area σω okay, so this is the inductive converse of what I have trying to tell you is that if I consider a good conductor and have some way of inducing a current either it could be a plane wave that has been applied or I have actually taken this piece of conductor and applied a potential difference right.

I can apply a potential difference on the conductor between two planes now right, and those two planes will not be at the same potential because this is not the time invariant or the static case this is a case where there is time variance in the form of a sinusoidal variation so if I induce a potential difference and cause a current to flow this current effectively flows only through this small thickness, so you can imagine again to this one that if you take if you take this dusting part thickness as Δ right, the current does not really go all the way through the ∞ you can imagine that most of the current or the effective current is lying only in this small patch of thickness Δ okay.

So the phenomenon in which the current actually starts to flow very close to the surface just below the surface and over a thickness of Δ is called as skin effect and this is an important artifact or an important effect in high frequency systems. We will not leave the skin effect at this point we will go to an even more interesting way of the skin effect by calculating what is the skin effect in a different structure which is again very important practically and that structure is a simple round wire, okay. So we imagine that there is a round wire okay,

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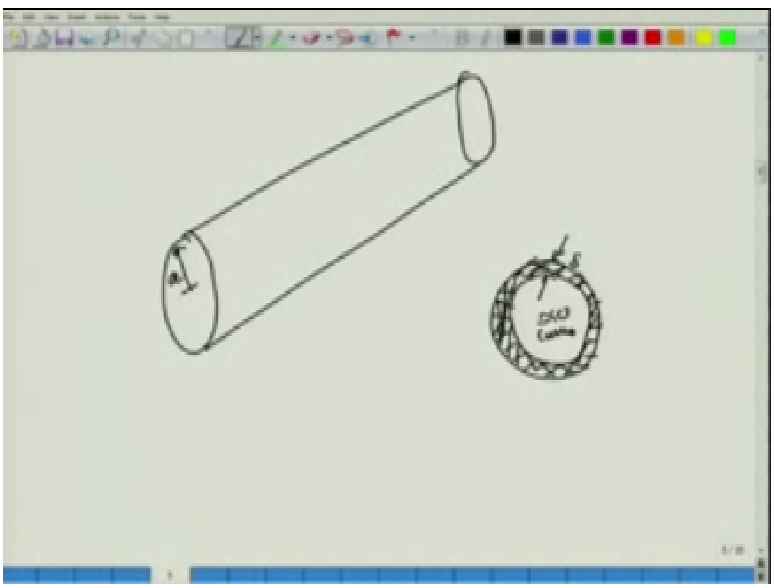

So I have a wire of some radius a here okay, this is made out of a good conductor again so I would not write it here but this is made out of a good conductor okay, and what I want to find is what is the skin effect what is the surface impedance would it be the same as the previous case would it be different from this case well before we can even answer these questions we have to understand something that is it is not slightly important.

We have not really talked about the fields inside a wire we have not even talked about the fields inside a wire when we are in oh we have talked about the fields in a wire only for the static case and we have calculated the inductance of this single wire right, so which we computed to be something like μ/8π you know in the coaxial case that we had computed there was an inductance corresponding to the inner tube which was carrying the current I, and this current I was a static current at that time the corresponding inductance contribution was μ/8π.

We have already seen this one however what happens when the current is varying with time and the frequency is very high you can again imagine that most of the current will then be flowing in the thickness of Δ okay, so there will be essentially approximately no current here but because most of the current and almost the entire current is kind of concentrated on the thickness of Δ near the boundary of this wire okay.

But what is the value of that how do we calculate that these are not simple to calculate from the plain wave analysis that we have done so we will do a slightly more rigorous analysis wherein we will employ the quasi static analysis by employing amperes law and Faraday's law and we will see how the fields inside a wire okay, or how the current distribution inside a wire can give rise to skin effect. So we will consider these quasi static analysis of the wire in the next module, thank you very much.

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