Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

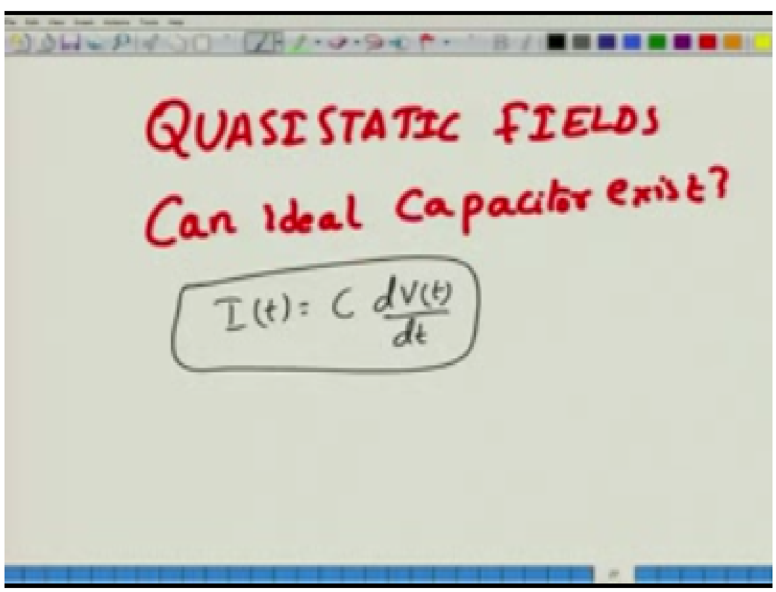
Course Title
Applied Electromagnetics for Engineers

Module – 43
Can an ideal capacitor exist?

by
Prof. Pradeep Kumar K
Dept of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to NPTEL mook on opted electromagnetic in this module which is more for an entertainment, of course you will have to follow the arguments to be really entertained by this module, we consider what is called as a quasi static field okay, we try to answer can an ideal capacitor exist. What do we mean by an ideal capacitor? I mean from circuit theory we know that if I have a capacitor of some value C then if I put in some current I and maintain a voltage V(t) across this capacitor, then I know that the relationship between V, I and C is given by.

(Refer Slide Time: 00:50)



I (t) is = C dv(t) / dt right, we do not worry about what is the shape of the capacitor what is the capacitor made out of if there is an ideal capacitive element we expect the relationship I of t to be

$= C\, dv(t) / dt$ where C is the capacitance of the structure okay but can really this equation be justified physically I mean do we actually have an ideal capacitor let us see why this equation is not complete picture, why this is only an approximation and how does this approximation break down as we increase the frequency of the operation.

That is we assume that the potential V (t) that is maintained between the capacitor plates is of the sinusoidal form and as we increase the frequency what happens to this equation okay. The real in which we are going to operate is what is called as quasi static field, we have already seen one region of operation when we talked of a transmission line where the length of the transmission line or the dimension of the transmission line along its length was much larger compared to wave length okay.

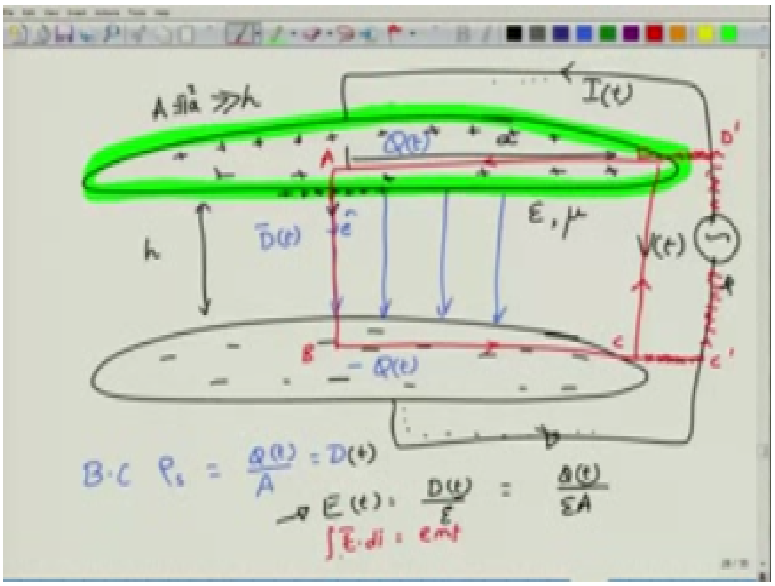
So wave length was actually only part of the transmission line so the land transmission line usually was a very large, very long length compared to the wavelength of the light that we were using okay, that is one extreme then there is another extreme as we discussed which is known as a geometric limit where the dimensions of the devices are very small, whereas the wavelength of the light is very large. That dimension is the circuit theory somewhere in between is what we have in the form of quasi-static.

So here charges potential will be varying with time current will be varying with time but not so fast that one has to consider the effects of ∇ay between when the source changes its distribution and the field correspondingly responds to the changed distribution okay, something that you will be seeing later on as we talk about waves okay. So for now we understand that charge distribution could be changing, the current distribution could be changing but they are not changing so fast that if the charge Q (t) is kept here and it is varying very slowly with respect to time.

So that you have a Q (t) the electric field still remains more or less like a Coulomb electric field so you will have $Q(t) / 4\pi r^2 \varepsilon$ okay so we are so this kind of a variation is what is called as asynchronous variation, in a synchronous variation we assume that the fields are tracking the changes in the distribution of the current or the charges in a synchronized manner okay. No phase ∇ay or no phase lag appears.

So Q(t) changes t also changes / the same thing okay of course this is an approximation because in reality you will have to consider, the fact that the source distribution changing will take some time to be affected to the electric field okay.

(Refer Slide Time: 03:58)



So let me go back and write down the plates over here, so these are the plates that I am considering and to this plate I connect an external potential or a voltage generator V(t) or an EMF generator V(t) which will supply a current I of T to the plates we assume that the plates have a radius of a and therefore their area a which is πa^2 will be much larger than the separation between the plates H okay. I have shown you in some sort of a weird geometry but you have to imagine that these are actually some nice circular plates okay.

So I have a nice plate okay slice circular plate and then these plates are separated / a very short distance H, so a of the plates that is area of the plate is much larger than the separation H. So current is being pumped in what happens to this current if you remember is the flow of charges so the charges keep flowing and then they now come to the conducting plate top conducting plate and of course these charges are removed or essentially negative charges are deposited because the current has to return back to the voltage generator V (t).

So there will be positive charges on the top plate there will be negative charges on the bottom plate there will be no charges in the region in between which is characterized / having a ε and a μ okay. So there is no charge in between so what happened to the charges? The charges will uniformly distribute it okay they will get uniformly distributed over the entire plate okay, so that is what is going to happen and we will assume that okay. We will also neglect any fringing that might occur on the outside edges of the capacitor.

So there will not be any fringing fields over here at the edge, so all the charges are deposited uniformly and because of this you would expect electric fields to be generated in the region in between okay and that electric field will also b uniform. So this would be the electric field that you would be seeing or rather the D field that you would see and these D fields will be all in the direction downwards okay, so which we will denote / the unit vector e over there.

So what are the relationships that I am looking at as I said the D field is following the charge okay the charge deposited on the top plate will be Q(t) the charge deposited on the bottom plate will be - Q(t) since I know the area of the top and the bottom plates there will be a surface charge distribution ρ_S which is given / Q(t) / A and we know that the surface charge distribution okay must be = the D field because of the boundary conditions of the boundary condition tells you that the D field must have the surface area ρs.

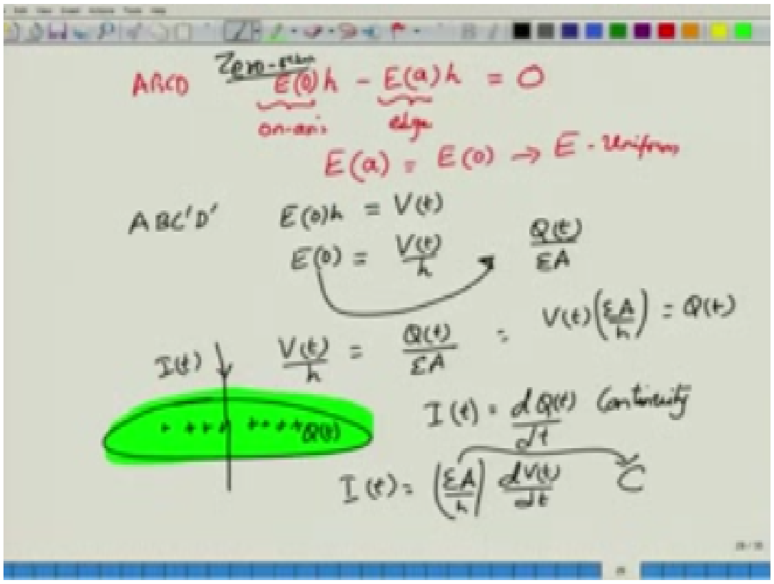
Where is the boundary that I am looking at well you know I am actually looking at particular surface, you can imagine that I actually have a surface over here okay and coming out of that surface will be the D field which is all perpendicular to this surface okay and that would be = whatever the charge distribution that you are going to see the charge distribution will be surface charge distribution given / Q(t) / A and that is exactly = the charge D(t) or C.

What would be the electric field E of T? Well electric field will be D(t) / ε, so this would be = Q(t) / ε A. Now let us try to relate this electric field E to this V (t) in order to do that one I consider you know one particular path, this would be one path I go all the way up to the edge and then come back I also consider an extension of this path I am assuming that, this extended path which I am denoting / this you know cross marks that I am different color pen.

It is not very far away from the edge okay, so this length that I am considering is actually very small so essentially, it is just outside the capacitor. Let us also label these points as some A, B C and D let us label this as C' and D'. Now if you imagine you know going through this path ABC and D right so through this path if you now travel and evaluate what is the ∫ of the electric field right, so I know that ∫ of the electric field ∫ .dl will be = whatever the total EMF that would be surrounding okay.

This is a very important thing this EMF is coming essentially because of the static almost static electric field there are no magnetic fields inside the region as of now.

(Refer Slide Time: 08:39)



So if I evaluate this ∫ for the paths ABCD what I get is E of zero which would be the electric field on axes, so this is the axis at ρ = 0 ρ being the radial distance from the centre point that I am considering, center line that I am considering h because that is the height - the side ones do not contribute to any unless. Therefore you now have E of A into H, so this one is the on-axis field and this is the edge field okay.

We do not consider fringing effects as I said so this is the on - axis field this is the edge field the difference between these two on the path ABCD must be = zero, indicating that the electric field at any point if you consider must be exactly = the electric field on the axis itself. So we have a uniform electric field happening unfortunately if you now try ABC ‘ D ‘ of course not unfortunate tried ABC ‘ D ‘ the corresponding field E of zero , H will be = V of T because that is the potential in this picture as you go from C ‘ to D ‘ you see a potential of V (t).

That would be = the potential on the line a B, so on this loop this equation actually allows you to find out what is the on- axis field which would be the same field everywhere throughout and that would be given / V(t) / H but we have already calculated what is the on-axis field this is given / Q(t) / ε A correct. So I can put that expression over here and then write V of T / H = Q(t) / ε A adjusting this equation gives me V(t) into ε a / H = Q(t).

I am not done yet I know that I have a to conductor right to which there is incoming current I of T and there is a certain surface around that, so which is what we considered. So there is a closed surface in fact around this one there is a closed surface to which we are sending in the current I of T. What is happening as a result of sending current I of T? The charges inside are changing right so if the charge distribution or the charge is Q(t) then I know that I of T must be = dq(t) / DT this is from the continuity equation correct.

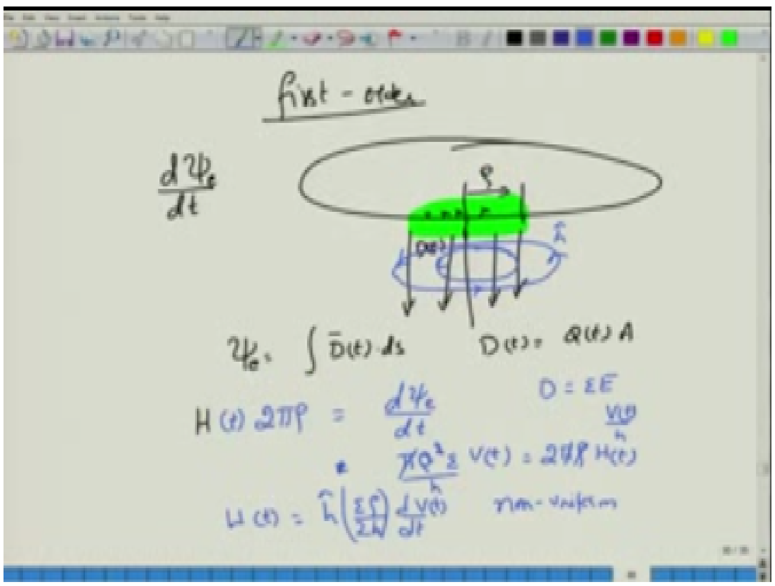
So from the continuity equation I know that I of T = dq(t) / DT I know what is Q(t) which is V (t) into some term in the bracket, so I can write this as I of T = ε A / H d V(t) / DT and if I identify this ε A / H as capacitance C then I have I of T = C dv (d) / dt. Since everything is fine so what was it I was talking about I mean I have shown you that there is an ideal capacitor but is this the complete picture I mean is this the correct picture to ask for.

Unfortunately we have neglected a major thing that is going on in the di electric region between the capacitors. What is that that we have neglected well we have said that there is a D field between the top plate and the bottom plate right, so the D field is supposed to be uniform but the D field is varying with respect to time, if the D field is varying with respect to time then D or ∇ D / ∇ T which represents the displacement current will not be constant or it will not be zero it will be varying with time.

And ∇ D / ∇ T / the connote continuity current continuity equation will take over the role of the conduction current whenever conduction current is zero of course in the region between the capacitor there is no conduction current but as a result of D(t) varying with time there is a displacement current. Now what does the displacement current do it will actually generate a magnetic field around you like this or around the current like this, so this is the direction of the current and these fingers tell you the direction of the magnetic field.

We will calculate some of this in the next how to calculate magnetic field in the next few modules but amperes law tells you that if there is a displacement current or if there is any form of a current there will be a magnetic field okay. So let us look at those fields.

(Refer Slide Time: 13:02)



I will call this as first order okay you will and we will call the previous one as the zeroth order you will soon see why we are calling these as different orders. So for the first order I need to find out what is the electric flux, that is being generated in the dielectric region between and I need to find out what is the displacement current which is that flux that I am going to find out and then D of E which are the electric flux that I have to differentiate this one with respect to time.

So again go back to the problem that we have this is the top conductor and now what I do is I consider a closed surface okay, so I consider a closed surface like this which actually has a radius of P okay where P simply tells me so this central line is the P = zero line and P = P will be the line at which this particular closed surface ends there are charges there is D(t) and surrounding this or coming out of this closed surface will be the electric flux density C which is ∫ of this D(t) times the surface ds okay.

And what kind of a surface we are considering we are considering a surface of what is the surface area of this one d is perpendicular to this fellow and D(t), we have already found out what is that right. So we know D(t) is given / Q(t) times a so if I apply amperes law which tells me that and there will be magnetic field surrounding this right, so that is the electric flux and there will be magnetic field surrounding this there will be different magnetic fields.

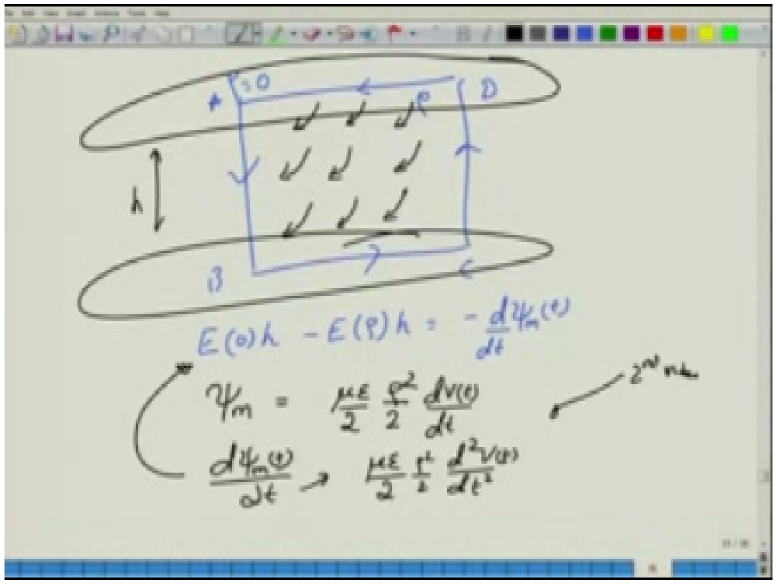
So I am considering the magnetic field at a radius ρ so that will be H of T because there is a time variation of their times 2 π ρ which is what your left hand side of amperes law will tell you. This must be = D φ e / DT okay we already know what is the area of this right so the area that I am looking at is $\pi \rho^2$ ρ being the radius of this closed surface 4 π P Square and times whatever the D field that I have right.

So what is the D field that we have we know what is the electric field from the previous example or from the previous this one we know where the electric field E is given / V (t) / H okay and therefore the D field D is given / ε times E and E is V (t) / h therefore I can write this as V of T so ε / h V(t) that must be = 2 π ρ into H of T okay which gives me the magnetic field H will be done o circulating this one so I will call the some H vector.

So it would be circulating and this row cancels out row cancels here φ cancels φ also cancels, so you get ε P / 2 H DV / DT. So clearly V (t) is not constant and DV of T / DT will also be not constant so you get H of T = or proportional to the first derivative of V (t) and it is very important to note that this h field is non-uniform which means that the H field actually will be 0 on the axis but it will keep increasing linearly at that edge where ρ is = A.

This is not the end (t) he story because if I have a magnetic field in this way there will be a magnetic flux also in the same direction and what is the magnetic flux do? This magnetic flux is also now changing, so this changing magnetic flux will introduce a change in the EMF along the loop that you are going to consider. So there will be a change in the EMF because of the change in the magnetic field who tells you that faraday tells you that.

(Refer Slide Time: 16:53)



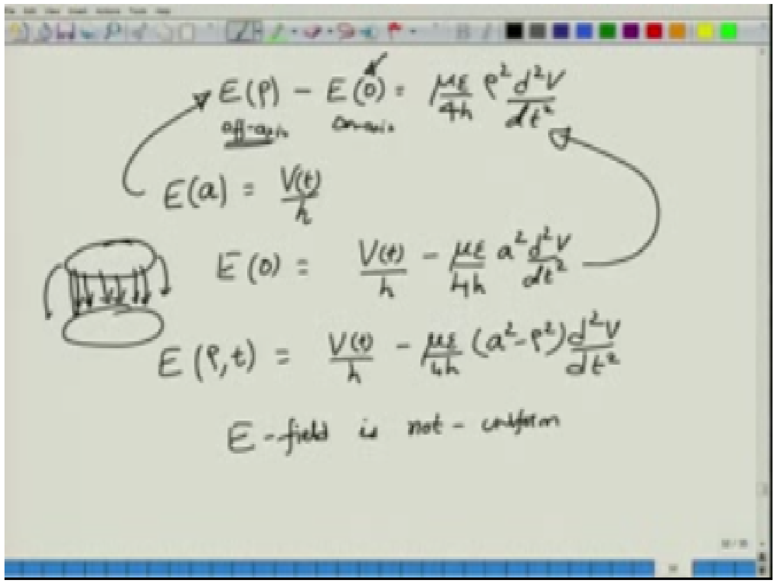
The Faraday law tells you that if i go back to the path you know ABCD path please refer to the previous this one to find this particular path inside so instead taking this CD path at the edge I will consider the path to be at some ρ = ρ itself and then write down what is the electric field change I will have a of 0 h - e (ρ) which is the electric field at this particular point P or on the line P along c and d H will be = - D / DT of magnetic flux varies.

The magnetic flux coming out well the magnetic flux is coming out because this is how the magnetic fields are coming out right. So the magnetic fields are curling and therefore they are coming out of this particular loop, so imagine I mean go back to this picture this is the top plate this is the bottom plate this is what you are actually trying to evaluate, so this is the height H okay. So the magnetic field is circulating and coming out and that will introduce a certain flux so you have to find the flux over this surface area ABCD.

I will leave this as an exercise for you to show that the magnetic flux is given / μ ε / 2 okay ρ^2 / 2 dv (t) / dt okay this is essentially integrating the non uniform magnetic field over this loop ABCD you are going to get the magnetic field of course what you want is d φ m of t / dt if you want this one you just have to differentiate the potential a second time so you get d square V / dt^2.

So clearly this would be what is called as the second order effect so this flux change or the EMF change should be equated to that expression.

(Refer Slide Time: 18:35)



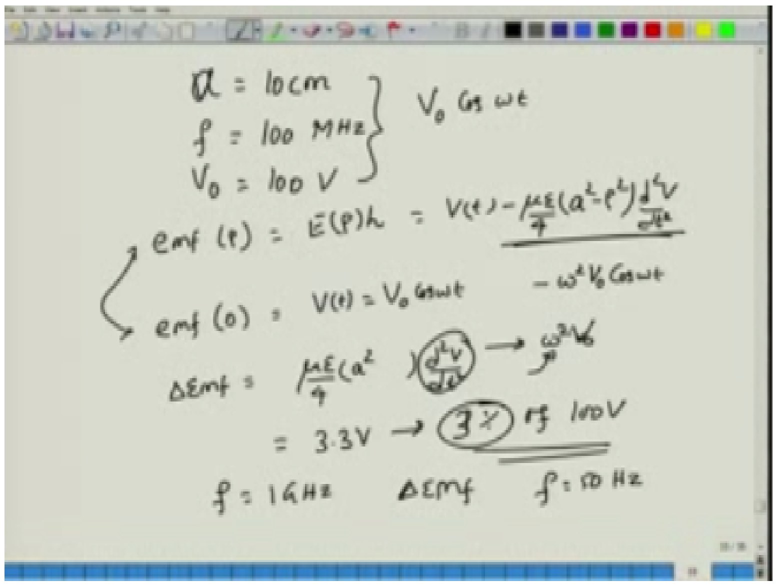
Which we have written so you can rearrange the equation and write E of ρ - E of 0 to be = μ ε / 4 H okay ρ^2, $D^2 V / DT^2$ this is a very important thing what it tells you is that the electric field is not uniform anymore the on axis field and the field at off axis or you know act away from the axis is actually different. How large is the difference in order to find out I first need to know the on-axis field now here is an important caution.

The on-axis field will not be the same as that we have calculated earlier okay because the earlier calculation assumed that electric field was uniform in the path ABC ‘ D ‘ here it is not uniform so in order to find out that I need to apply the Faraday's law applied the EMF condition only at the boundary where on the edge, if I take on the edge of a will be = V (t) / H so I can substitute for ρ = a in this expression and then find out what is the on-axis field and find this as V (t) / H which is no good thing that you expect this.

But you do not expect this collection which is μ ε / 4 h a^2 $d^2 V$ / dt^2this allows you to go back and then write the field at any radius ρ and 5T to be = V (t) / H which is what you would ignore expect in the uniform case but there is a correction - μ ε / 4 H $a^2 - \rho^2$ $d^2 V$ / dt square this implies that V field is not uniform this is a very important thing because when you look at the sketches of a typical capacitor okay in most undergraduate studies on circuit communication.

If you circuits if you look at the electric field is assumed to be uniform except of course there is a fringing but we have not even agreed the fringing. So you see that the electric field is taken to be uniform capacitance is very nicely defined but the electric field is not uniform the electric field changes as you move away from the axis okay to give you some numbers as to how this will change.

(Refer Slide Time: 21:03)



Assume that the plate you know radius a is about 10 centimeter and we are looking at a frequency of about 100 megahertz which is very small frequency okay is = 100megahertz we will also assume that v0 is about 100 volt okay this is a fairly large volt but if you now calculate what would be the EMF at any no distance inside the capacitor at a distance of P this would be = G of P into H okay.

So you can find this one / substituting for P or you know what is the equation for a G of P that would be the of t - μ ε / 4 that H has gone on this side $a^2 - P^2$ into $d^2 V / dt^2$ if I assume that this is a sinusoidal signal V 0 cos ω T then $d^2 V / DT^2$ will be $- \omega^2$ V0 cos ω T right because that's the second derivative of this one and what is the EMF on the axis EMF on the axis at P = 0 will be = V (t) itself which is V 0 cos ω T.

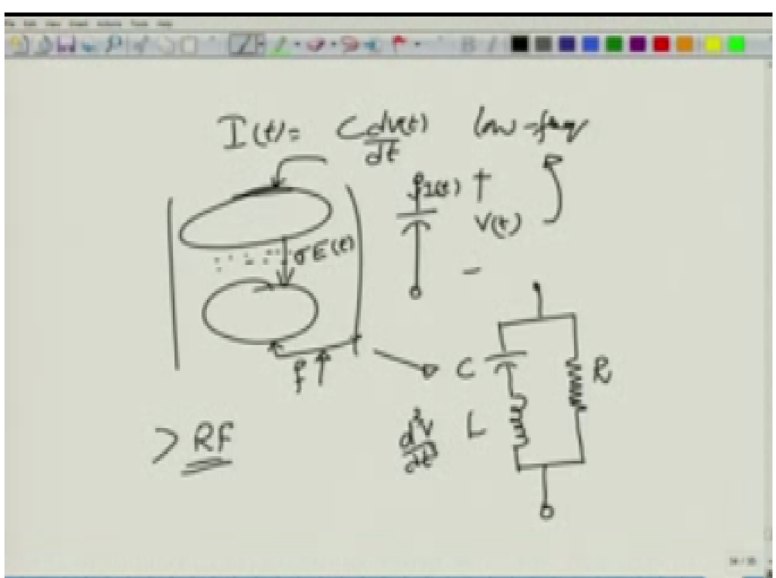
So if you look at the change in this potential okay the change we will represent this as ∇ta EMF this change is given / the term that is actually giving you the chain, so which is the correction term μ ε / for a^2 - P^2 D^2 V / DT square sorry this should be P = zero here because that is where we are looking at the change at the edge to the on axis change will be this particular quantity, so if you evaluate this will turn out to be approximately 3.3 volt which is about 3% of 100 volts.

It is not something to be laughed at because I asked you to do the same thing at F = one gigahertz you will be surprised to see what the change in the EMF is. Why would you be surprised? because if you keep all the other parameters same then the change in the EMF is actually going as a function of this d square V / dt^2 is nothing but ω^2 V zero right so there is maximum change I am talking about not the actual change in this prep time maximum change you see is that is proportional to the frequency square.

Now frequency if you increase / tenfold this change will happen about a hundredfold okay and that will be a quite a large change in fact many of the I mean inside the entire theory that we have derived will no longer be correct, when you are operating this ferrate 1gigahertz for this particular system okay. So it is actually something that you have to keep in mind that all the definition that we have seen of a capacitor is strictly valid at very low frequencies.

Because if you redo the calculation for the EMF at F = 50Hertz you see that the change will be very small it will not be 3 percent it would be very small because you are pulling down the frequency / a very large factor, so to sum up the capacitor that you see on the equation.

(Refer Slide Time: 24:04)



That you see I of T = C of dv / dt is really a low frequency approximation and the corresponding circuit that would look like this for the physical picture of large capacitor okay would be = a capacitor you know in the circuit theoretical approximation where current I of T is flowing and the potential V (t) is maintained is a very simple picture. As the frequency our active picture at low frequency as the frequency increases to the gigahertz range then what happens is you can show later on / a simple extension of the exercise that we are going do.

That there will be an inductive component added to this and this inductive component comes because there are leads which are taking the current into and outside of this capacitor okay and further if you assume that the dielectric inside is not an ideal dielectric but it actually allows you as mall conduction current of σ Y of T then you will also have to add an extra resistor to this would be the correct picture for this scenario okay. So there will be a capacitor there will be an inductive component I have not shown you how the inductive component to be calculated but this would be proportional to d^3/dt.

That is it would be proportional to the third order effect and there would be a resistor which would directly be proportional to how much is the imperfection in the dielectric, so this is a picture that is usually used a thigh frequencies or at RF radiofrequencies and about sort millimeter wave frequencies or at other places. The capacitor that we think is a very ideal element will no longer be an ideal element just relating IC and V but it will also have to be moved as capacitor inductor and a resistor okay.

So I hope you have come away with a very important fact that physical scenario is different from what is known in the mathematical approximation mathematically an ideal capacitor can exist because that is just a relationship between I and we you postulate that relationship say that this is the relationship that exists that can easily be done. But if you postulate that you are neglecting lot of other things that is happening in a real physical capacitor and that recognition that you can't just get away / writing I of T = C DV / DT.

And you have to include a term corresponding to an inductor and a resistor because the physical system is not the same as the mathematical system is the essence of this story okay. So I hope you have been entertained with this and I will show you how this other equivalent circuit came in the exercise and the problem sheets and from the next module onwards we will look at how to calculate, the corresponding inductance of the structures that we saw earlier so for example coaxial cable capacitor we have calculated we will now calculate inductance and so on with this thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production