Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL.)

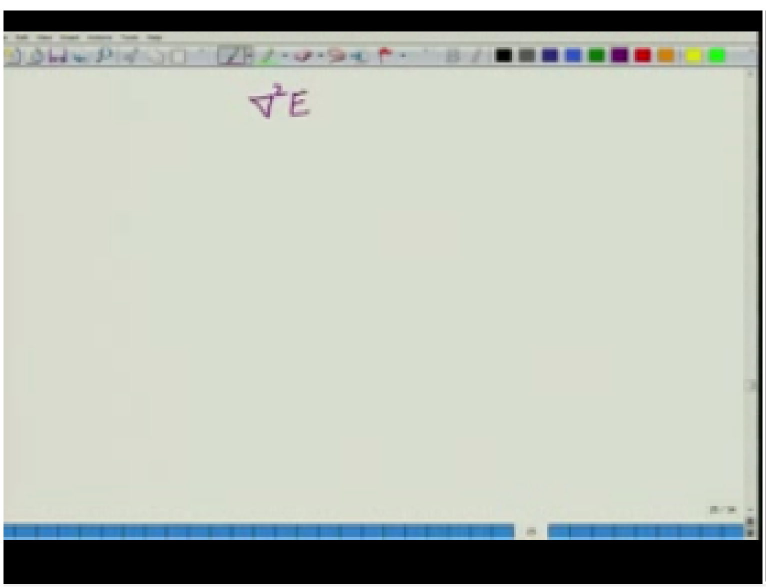
Course Title
Applied Electromagnetics for Engineers

Module – 42
Polarization of plane waves

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to input and move called applied electromagnetic for engineers. In this program we will continue the study of uniform playing waves. We have already seen that we have scanned propagate along the z axis can propagate any arbitrary direction. We have seen that the ways that we are transacting that transfer electromagnetic waves, but we have always assumed that they have either the x component or a y component. And the corresponding hy and the –hy component, but it does not have to be so. Because the real equation that we derived is a very linear equation

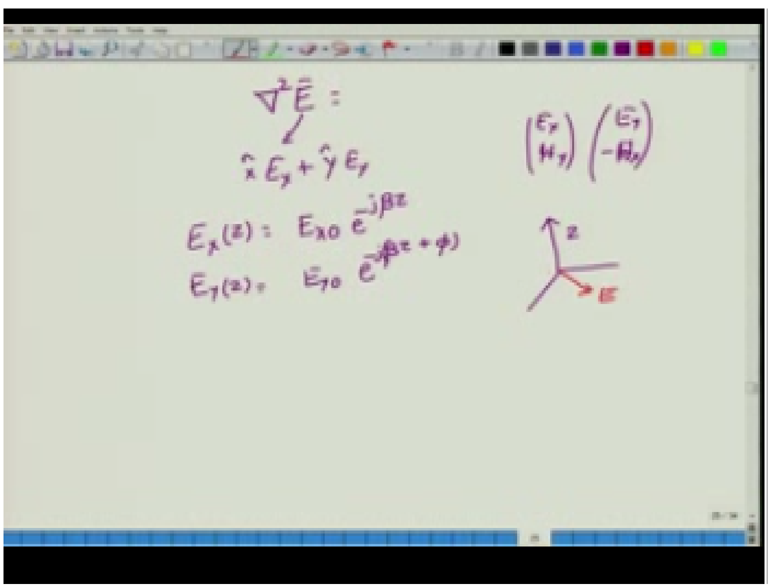
(Refer Slide Time: 00:51)



We can have electric field which has both x component as well as the y component. So we will have a super position principle applicable over here because the equation is linear, and which means that the electric field will have the x as well as the y component at the same time, okay. But before we get to that we will recall what is the assumed dependent of the x on z. We can write the dependence of x on ez by specifying the phaser notation, the phaser notation we have ex is 0 to the power -βz and Ey of z will be some altitude Ey_0 to $\beta^{-j\beta z}$.

Of course it is also possible for the Ex and Ey components to exhibit some kind of a face difference, so you can write this as $E^{-j\beta z+\phi}$, okay. Please remember the x and Hy are one group and Ey and –Hx are another group for the t index that are propagating alongside the axis. So along the z axis you can have everything which have both Ex and Ey components, so if this the wave direction, this is the z axis the electric field can be in a direction that will have both x as well as y.

(Refer Slide Time: 02:21)



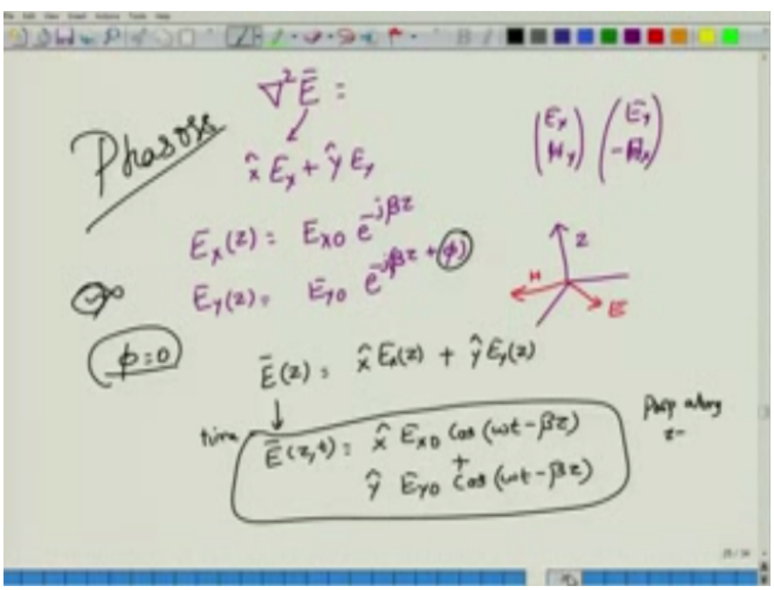
And the corresponding magnetic field will also have a component of Ey-hx, so E cross h must be along the z axis, so appropriately we will have to find another new field such that this E cross h would be perpendicular to e.

So the point is because of the linearity of the grade equation simultaneously both Ex component can exist as well as the Ey component can exist, okay. But as I said there can be a safe difference between the two components. Depending on the safe difference there will be a different type of behavior for the electric field of this vx.

So whenever we talk of a wave we normally specify its electric field, okay. Why do we do that? Because the magnetic field can be obtained easily from the electric field, we have already seen in the previous case how to obtain the magnetic field from the electric field so it is possible to always obtain the magnetic field either by applying Maxwell's equation or by the rule that we developed in the previous module.

Before we usually resort to only specifying the electric field, okay. And of course that electric field will be in the form of a very, which we have assumed to be a sinusoidal one and we are writing.

(Refer Slide Time: 03:30)



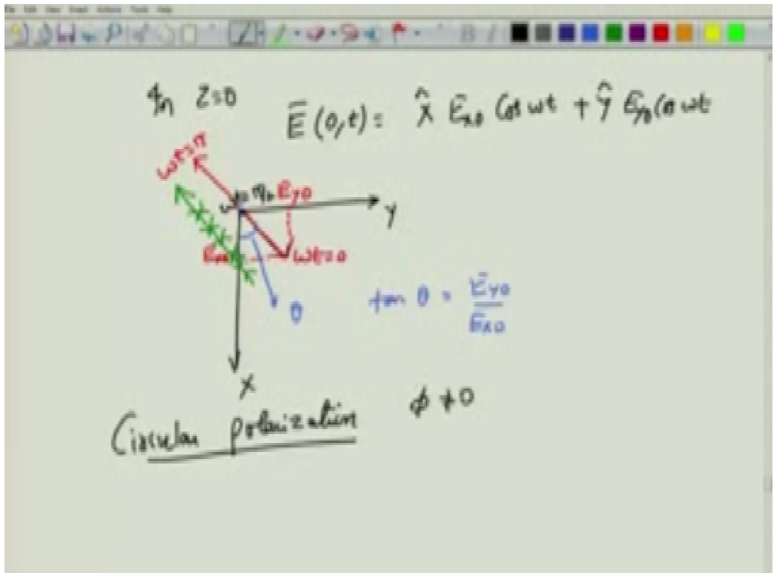
The corresponding phaser notation for those, okay. So those components all have the same frequency on either, so both Ex as well as Ey are of the same frequency omega and they also have the same propagation constant data except that they are like some phase difference between them, okay.

Suppose you consider the face difference $\phi=0$ then what will happen, the total electric said move which is a function of z and of course is a phaser but I am not writing the phaser till now everywhere, okay because it becomes a little cumbersome to keep carrying the tin, I am dropping this but please remember all that we are talking about are phasers only, okay.

At least up to some point where we will definitely introduce the time dependency. So yea that phaser will be = $\hat{x}\ E_x\ (z)$ which is another phaser + $\hat{y}\ E_x(z)$ which is another phaser. Now when I go to phaser to time, okay I can do that, so I will have the real electric field in terms of z and t given by $\hat{x}E_{x0}$ cos (wt-βz + ŷEy0 cos wt-βz. Why if that both components have cos wt-βz factor because phase is assumed to be Z.

Now this is the electric field of the wave propagating along the Z axis. So propagation along Z axis you can just write the corresponding magnetic field, but we are not interested in that this point of time, because whatever we talk of electric field is same thing for the magnetic field as well. The amplitudes Ex0 and Ey0 can be different okay. So it is not necessary that both components have the same amplitude. Now let us imagine that we pick a convenient plane okay, we pick a plane which will make this β(Z) term to be a constant.

(Refer Slide Time: 05:32)



So we pick that plane to be Z=0. So in the constant Z=0 plane okay, what will happen to that total electric field E at Z=0 as the function of time will be equal to $\hat{x}E_{x0}$ cos (ωt + ŷEy0 cos ωt okay. Let me draw two lines, I call these line as this line as X and call this line as Y. Why is it that I have written X down and Y here, because I am actually looking from the top. So this would be the Z-axis okay.

And if you look from the top right, you have an X-axis and a Y-axis okay, and the wave is propagating along the Z-axis which means if I look from the top the wave is actually coming towards me. And I have cut this plane at Z=0 that is just my reference, and I mean there is no reason or lying Y Z=0 has to be caused, and you can take another plane Z=Z1 where the only condition is that β(Z1) should be a constant okay.

So now that I have written my Z and Y axis on this plane and I have shown you that I am actually looking towards the wave travelling towards me, let us see what happens to the direction of the electric field at T=0 or at ωT=0. At ωT=0 what will be the direction of the electric field E, it will hang x̂Ex0 + ŷEy0 so the resultant will be depending on the ratio of Ey0 to Ex0, but it would definitely be along that direction.

So let us say this is the direction, let me make the different color to show you that. The projection of this electric field E at ωT=0 on the X-axis will give you Ex0 the projection of this one on the Y axis will give you Ey0 okay. The angle with which this vector lies initially at ωT=0 with respect to the X-axis is we denote this by θ okay, and this angle is given by tanθ=Ey0/Ex0 okay.

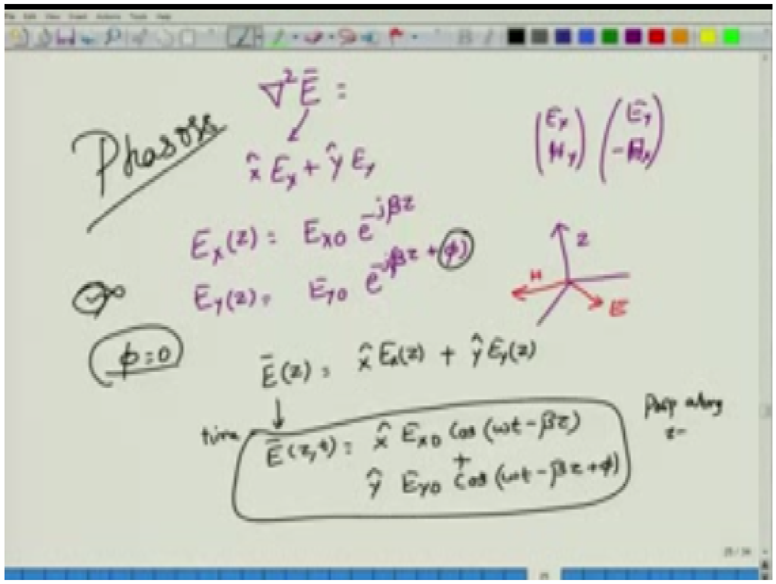
Of course, this tanθ will be equal to 1 and θ will be 45 when the amplitudes of both Ex and Ey components are same, if they are not equal then there would not be the same angle.

Now tanθ is so and so right. What happens if ωT=Л/2? Cos Л/2=0 so which means electric field at ωT=Л/2 will be at the origin, because then the amplitude is 0 what will happen at ωt =Π cos Π is -1 and you will get – Ex0 – Ey0 y^ right so this was a condition that we have at ωt =Π/2 but at ωt =Π the direction of the wave will be along – Ex and – Ey right which would be along this particular direction, so this is when ωt =Π what will happen at 3Π/2 cos 3 Π/2 is 0 so which means it would again be back at the origin.

So if you look at what is happening you can see that the wave is executing a motion such that it initially starts off with this point goes through the origin goes through Π point and then comes back so it initially goes in this way right and then comes back so essentially it is going along this particular line happily it goes along this particular line from one point to the other points so it is oscillating along a line okay, so that type of behavior is what we call as polarization of the wave we say that the wave is polarized along the line and a line polarization is also called as linear polarization.

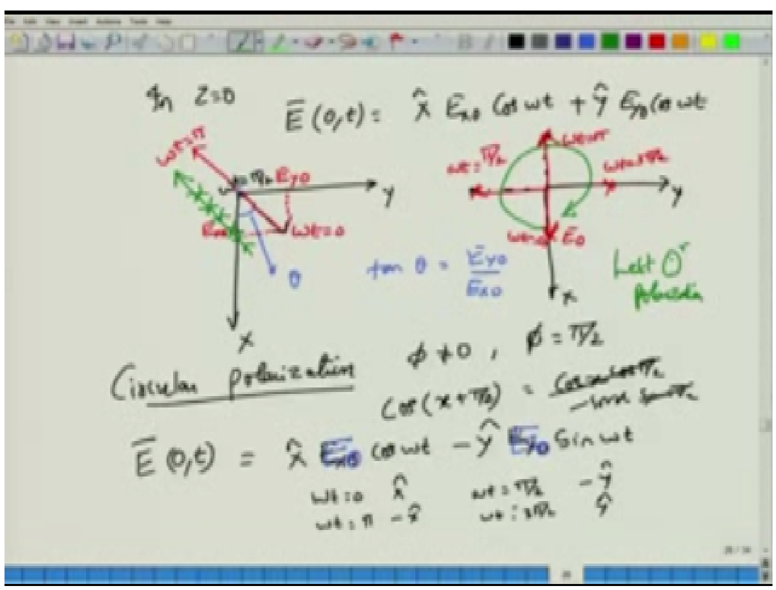
We also specify that angle of this line that makes with respect to the x axis so this is a θ linearly polarized wave okay where the angle the θ is given by E / 0 tan θ = E/0 Π Ex0 well it does not have to be that the wave are polarized along a line waves can be polarized in many in a more exotic state one of that is called as a circular polarization in the circular polarization case we require that electric field and magmatic field have some amount of phasor difference okay so when I consider ϕ to be non zero I have to modify this equation.

(Refer Slide Time: 10:07)



Right so the y component cannot be - β z but there as to be a phasor ϕ component so let me just write this as + ϕ okay does not matter I could have written as - ϕ but let me just write it as + ϕ now if I assume.

(Refer Slide Time: 10:23)



Now if I assume that ϕ non zero but $\phi = \Pi/2$ what will happen I have some $\cos(x + \Pi/2)$ what is this equal to $\cos x \cos \Pi/2 - \sin x \sin \Pi/2$ x so clearly this fellow is gone so you will get $- \sin x$ so this is what I have right so my electric field E at z = 0 plane will be = $\hat{x} E_{x0} \cos \omega t - \hat{y} E_{y0} \sin \omega t$ correct I will also assume that the amplitudes of these 2 are constants therefore this would be just E_0 and this would be just E_0 again so the amplitudes of x and y components are same.

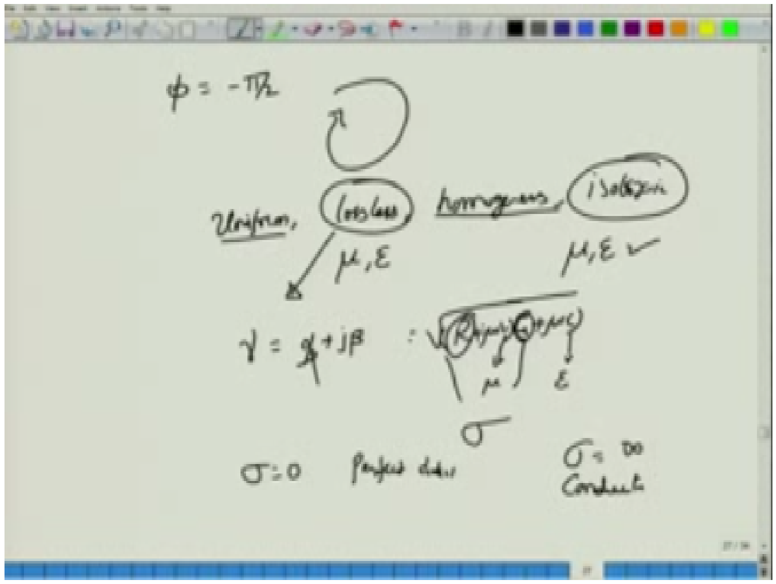
As before let me draw the lines over here call this as the x axis this I call as the y axis of course as no I am looking at the wave as it is approaching okay what will be the initial direction of the electric field at $\omega t = 0$ at $\omega t = 0$ the detection will be at along x axis at $\omega t = \Pi/2$ because cosine $\Pi/2$ is 0 the direction will be along $- \hat{y}$ then when $\omega t = \Pi$ the direction will be along – x direction and at $\omega t = 3\Pi/2$ the direction should be along should it be along + y direction because $\sin(3\Pi/2)$ is therefore this will be along +Y direction.

So let us plot this directions at $\omega t = 0$ and a long the x direction with an amplitude of E_0 right this is at $\Omega T = 0$ then what will happen at $\Omega T = \pi/2$ the direction is along – Y but – Y means along this direction right, so the direction for the way wave will be at this point this would be at $\Omega T = \pi/2$ at $\Omega T = \pi$ we will have the wave propagating or the wave electric field directed along – x which will happen along this right, so this is where I have the wave at so this is the wave so this is the wave here this is the wave later on this is wave at $\Omega T = \pi$ finally at $\Omega T = 3\pi/2$ we have the wave along the y axis.

So if you look at this it is have though the waves and it is of course the wave starts at easy I mean along the x axis and it continuous and make a circular motion in this way it is making a clockwise motion in this particular way okay, if I now imagine that if I curl my fingers along this so this is the plane and if I curly my fingers in this direction I see that this thumb will be pointing in the same direction as the wave propagation and remember I am actually looking from the wave from the top and it seems.

As though that the electric field likes where executing the motion first at if I curly my left hand direction I mean left hand then I see that as the wave is approaching towards me the electric filed is making this left handed motion okay, so such a wave is called or is said to have a left circular polarization okay so this is left circular polarization is when you look from the top to the wave and it makes the left handed motion okay, you can take these as an exercise.

(Refer Slide Time: 13:52)



Okay what would happen when ϕ = - π/2 and then show that the corresponding direction in which the wave electric field vector the tip of the direct field vector would rotate is in the other direction is in the direction of the right hand kind of a motion so you have a left circular polarization a right circular polarization we have the linear polarization all these polarizations are how the electric field the tip of the electric filed would behave with respect to time in constant plane okay one of the frequently you know ask question is that if I have a wave which know whether this circular motion.

Is it possible for us to implement some kind of an optical or a some kind of a converter such that I convert a circular polarization motion into a linear polarize motion or sometimes I have a linear I want to convert into circular all these things are possible they all require the they understand how the wave propagates in a un isotropic media which is the subject of what a future module but yes polarization converters are one of the most common elements that you would find in the EM wave community in the optic especially in the optics communities especially you would find.

Lot of experiments involved and use of polarization converters okay further these polarization vectors can be thought of a unit vectors themselves so you have a polarization vector if you have a circular polarization you can define a unit vector along the circular and a unit vector along the radial and in a right circular and combined them and the resultant will actually be a circular polarized light so you have a circular polarized wave expressed as a linear combination of left and right circular polarization just as we have in fact in the previous case.

We have shown in that left circular polarization is expressed as some of two linearly polarized wave one linear polarization for the long x the other one was along 1 okay so these two are what we call as a basis mutually of orthogonal basis okay so one allow you to express the other component in terms of that particular unit vectors so this completes our elementary discussion on polarization we will revisit polarization in the context of polarization conversion when we talk of propagation through an isotropic media, okay. We now move on to other topics.

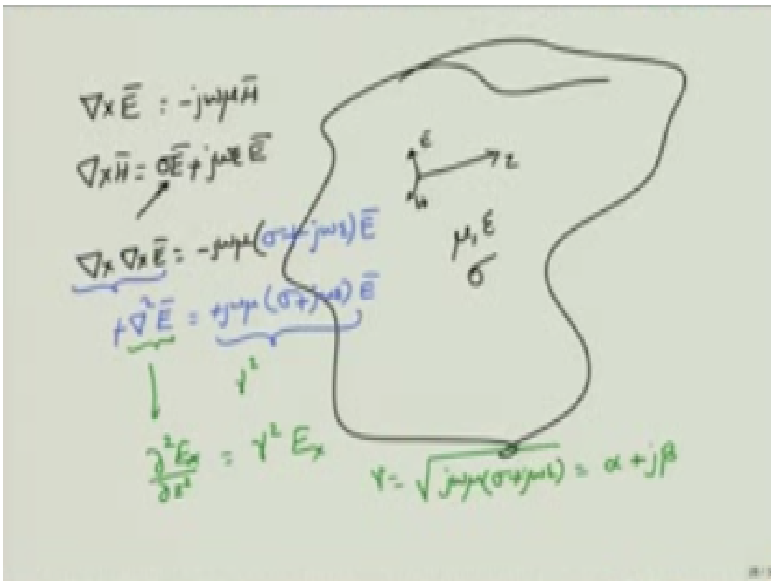
Well, so far we have considered a uniform loss less right, homogenous isotropic medium the meaning of each of this is that uniform loss, uniform and homogeneous means that the components μ and ε are independent they will have the same value at different points in space, isotropic means that if the wave is propagating along x direction the value of μ and ε is the same if it is propagating along the y direction also it is the same, if the wave is propagating along z direction μ and ε are the same.

Lossless media means that we do not have the wave amplitude does not attenuate as a propagates along the wave, so the if you look at the wave amplitude and z=0 and you look at the wave amplitude at z= say 5000km away the amplitude would remain the same, okay instantaneously they will be changing in the amplitude but if you wait for the time period then eventually it will go to the same amplitude as just the other point, so there is no reduction in the amplitude, there is no reduction in the power that is carried by the wave we will talk of power being carried by the wave slightly later on, okay.

So we now want to consider what happens when the wave propagates in a lossy medium of course I know what happens when the wave propagates in the lossy medium. Remember, this component γ that we derived in the case of a transmission line well γ was given as α+jβ and we have defined this one as √(R+jωl)(G+jωc) when the transmission line had resistive and conductive components, right.

So we wrote this one as the real part in the imaginary power and then we said that α is the attenuation coefficient pretty much the same thing happens, except that we have kind of identified l with μ and c with ε of the medium, what should be identified this R and G with it turns out that we have to identify it with the constant σ which is the conductivity of the medium, okay. If the conductivity of the medium is equal to 0 then the medium is a perfect dielectric medium, okay. If the conductivity of the medium is equal to infinity then this is an extremely good conductor or extremely prefect conductor or prefect conductor in some sense.

(Refer Slide Time: 18:40)



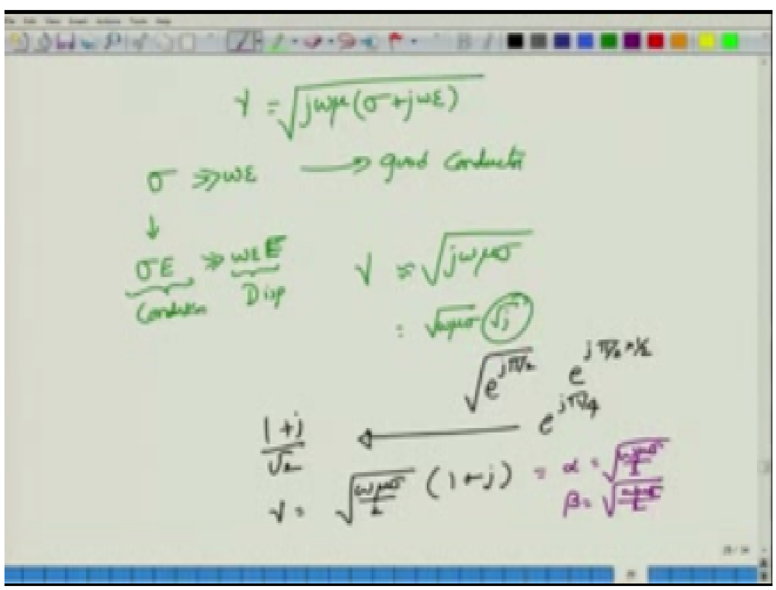
So if I consider a medium, so this is a medium that I am considering and the wave is propagating along the z axis with its electric field oriented along say x axis or y axis whatever that is and there will be a corresponding H field over there, if the medium in addition to be described by $\mu\varepsilon$ also has some amount of σ what will happen to the wave as it propagates well w go back to Maxwell equation to see if the wave equation that we derived will remain the same and it turns out that it would not be the same so you have ∇ x E = - j $\omega\mu$ h no problem but ∇ x h we had so far assumed to be equal to j ω ε x e.

And neglected the current density component but it cannot neglect that current density component anymore because of finite value of σ the current density component which is proportional to e and therefore given by σ e the point from a formulas we discussed this is equal to σ x e + j ω e.

So this term cannot be neglected and must be included oaky as before we take the curl of the electric field once again and substitute the right hand side of curl h okay so I obtain – j $\omega\mu$ x σ + j ω ε x electric field e because the left hand side can be simplified to ∇^2 e and the right hand side as become – j ω μ σ + j ω ε x e okay – sine and – sine can cancel with each other this complex number which is j ω μ x σ + j ω ε we call this as γ^2 we can reduce this ∇^2 e x some ∇^2 ex $/$ ∇z^2 assuming that the way this propagating along the z axis and z axis alone and a component of the electric field as only the ex component.

And the right hand side will be γ^2 x ex where this γ is given by $\sqrt{}$ j ω μ x σ + j ω ε okay and of course γ will be equal to α + j β you can derive the expressions for α you can derive the expressions for β if you really do that one here but one case is actually very interesting this case leads to what is called as the skin effect in the conductor and it is applicable when γ can be simplified.

(Refer Slide Time: 21:09)



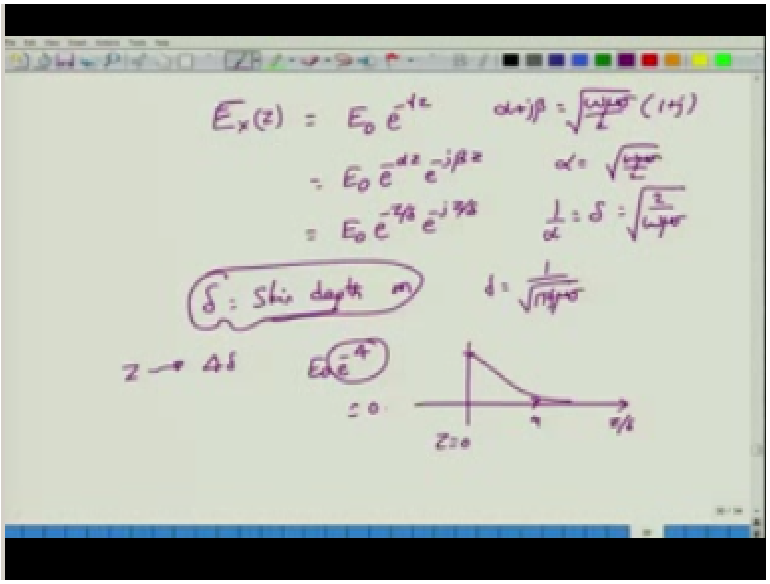
By taking an approximation which I will just show you remember recall γ is j ω μ σ + j ω ε right all under square root if I assume that σ is much larger than ω ε oaky if I assume that σ is much larger than ω ε which is equivalent of assuming that σ x e is much larger than ω ε x e okay in magnitude of course I am talking about so in magnitude of σ e is larger than magnitude of ω ε e and ω is time variation right time variation converted in the phasor form in terms of the frequency variation so this component is essentially the displacement current and this component is the conduction current.

So if the conduction current dominates the displacement current as in would happen when the medium is made out of a copper for example right, so you have a nice thick copper plate over which we are trying to or within through the medium that is trying to propagate in that case because of the conductive of the cooper being very high and for the typical operations of the frequency the value of the σ will be much larger then ω epsilon.

This indicates the conduction current of the current density in that medium will be mainly conduction current and extremely negligible displacement current. So for that case we call this as a good conductor, so we call the material and the medium as the good conductor, when σ is much larger than the ω epsilon. What will happen to γ then? γ will be approximately j ω μ σ under √ okay.

Now this can be written as √ ω μ σ Times √ j. what is √ j? One simple trick to find this one is to write this as $e^{j\pi/2}$ and then take a √. Luckily taking the √ means you multiply this one by 1/2 , so this becomes $e^{j\pi/4}$ is nothing but 1 + j/ √ 2. So γ will then be = ω μ σ / 2 under root 1+ j. indicating that this as α = ω μ σ/ 2 under root and beta also = √ of ω μ σ/ 2 okay. So both α and beta are = and they are = to ω μ / 2 okay.

(Refer Slide Time: 23:40)



What will happen to the electric field then? Well we know that Ex(z) you are considering the pharos of the electric field that we were considering let say it as some constant E_0 and it must be $e^{-\gamma \times z}$ this is the general solution that we are considering okay and we will consider that z > 0 is where the medium is present. Therefore we are putting the – to this one, so that we eventually want the wave to attain and not to amplify.

Substituting for the γ as α + j ß and knowing that this α is given by ω μ σ / 2 under root 1+ j for γ, I can write this as e_0 $e^{-\alpha z}$ $e^{-j\beta z}$ but e_0 or this particular α is given by the √ of ω μ σ / 2 and 1/ α if we define as Δ and Δ is given by 2/ ω μ σ under root and Δ can simplify further by writing ω as 2 π x f this is given by 1/ π f μ σ under root okay. so if I define 1 / α as Δ then I can rewrite this one as, this expression for electric field as e_0 $e^{-j/\Delta}$ okay and Δ is what we called as the skin depth okay.

It is measured in meters you can go back and show that this is actually measured in liters 4 δ so if z is such that the distance between travel stance of four δ then the amplitude would be e^{-4} and e^{-4} is such a small number that this is approximately zero so we say that as the wave begins sort some z=o let us say that the wave enters into the medium a good conductor.

And as it begins to propagate in terms of z about 4δ the way of electric filed would have dropped down almost to zero okay so it is δ which is the skin depth constitutes are you know is a very important parameter to describe a whether the medium is good conductor and if it is a good conductor how within to what distance inside the good conductor the wave can actually propagate because if δ is very large right when δ is large then you need to propagate further

Such that the wave actually drops to in amplitude but if δ is very small then mean that for very short distance right after the conductor the waves actually drops out most importantly this skin depth is not independent of frequency that is the very important factor to remember skin depth is inversely proportional to f so the same medium which might be you know having a very large value of δ for small value of frequency the corresponding value of δ will actually start to decrease as the frequency increases okay.

So if you had initially considered a conductor or a wire a solid conductor let us say the current through this conductor would be uniform at f=0 because at f=0 the current or the current density which is proportional to the electric field will be you know having a skin depth of infinity right. So which means that there is no attention as a way go through this conductor but very frequencies what you will see is that the current is kind of confined only along the edges within this particular material okay the higher frequencies the current is just confirmed to this thin strip okay.

And that width of this strip is given by the skin depth δ and this width keeps decreasing and decreasing when the frequency goes off to optical frequencies then this be equal to very small number provided you keep the same value of μ and sigma we will have more to say about skin depth we will in fact have a cause analysis of skin depth in the next module until then thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra

Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production