Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 41
Plane wave propagation in lossless dielectric media

by
Prof. Pradeeep Kumar K
Dept. of Electronic Engineering
Indian Institute of Technology Kanpur

Hello and welcome to NPTEL mook on pretty Electromagnetics for engineers in this module we will continue the study of uniform plane lives, in the previous module we have seen that the x component of the electric field and we assume only ex component for a moment that exhibits a dependence on Z and T where we have assumed it to be the direction of the propagation of a wave then it satisfies an equation that is very similar to the equation satisfied by the voltage wave on a transmission line okay.

That equation is the one-dimensional wave equation where one dimension refers to the fact that the voltage is propagating along the z axis and therefore it is a function only of z coordinate of course it is a function of time coordinate because, we want to allow for any general time variation while we were discussing in the last module we also said that we can assume in general or we can specify in a particular case in fact not generally in a particular case we can specify that the electric fields or the waves associated with the electric fields are you know sinusoidal type of a wave of frequency.

Ω propagating along the z axis with a propagation constant of ß and we are of course assuming that there are no losses in the medium and therefore the question of attenuation as of now has not arrived, okay so we have only looked at media which is uniform, isotropic, lossless, homogeneous media okay, so for this media waves are assumed to be or waves are lossless and they are propagating in as a sinusoidal waves along z and varying with respect to time with the frequency of F^ okay the corresponding expression for that.

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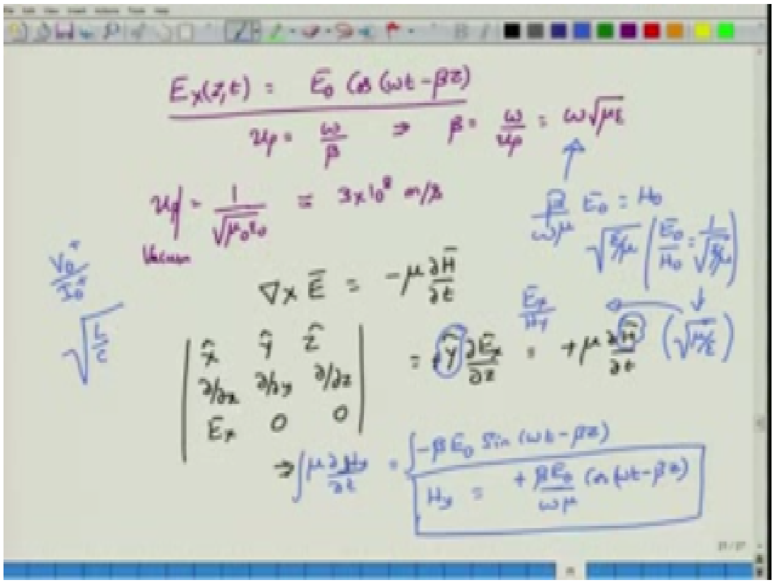

Is given by this ax of said t is along this expression where U_P the phase velocity is given by Ω / ß and we know that ß = Ω / Up I mean I am just inverting this relationship a little bit and I know that phase velocity UP for the free space media which contained or for the media which had the medium constants of μ and ε then this ß was equal to Ω√ of με into and the face velocity UP which is 1 by √ μ ε for the vacuum okay, so this for the case of a vacuum becomes1 by √ μ_0 ε_0 and if you plug in the values of μ_0 ε_0 into this expression this turns out to be 3x 10^8 m/ s approximately 360 x 10^8m/ s 3.

So in free space in vacuum a wave would propagate as a light wave the light is an electromagnetic wave because it does propagate at a same velocity as a light wave ,so this was infact the argument that Maxwell used Maxwell found the values of μ_0 ε_0 and then calculated the velocities factor or the velocity value and then sure that that was equal to the speed of light.

And once that took off then people started recognizing that it's you know light is an electromagnetic wave at a different frequency, just as the microwave signal is a light at a different frequency the frequency of a microwave is very low whereas frequency of light is very high, so it apart from that distinction in terms of the frequency they are all electromagnetic waves okay what about the H component we have shown that only ex I mean we have written only the ex component.

But I should be able to find the corresponding magnetic field content and I can find that by appealing to this equation ∇x E = - μ ∂H / ∂T and because we are considering the Cartesian coordinate system writing this ∂ x E is very easy it is easy as a pipe, so this is X^ Y^ and Z^ what component do we have well I need to first write this as ∂x / ∂/ ∂y and ∂/ ∂z what component of electric field we have only the X component EY and EEZ components are 0, so when you take the determinant of this one you will immediately see that there cannot be any x component this will be 0.

There cannot be any Z component as well because ∂/ ∂x time 0 is 0 - ∂Ex / ∂y and EX is a function of Y EX is a function only of Z component therefore what component of this one we will have we will have only the Y component ,okay and what would be the Y component given by ∂EX / ∂Z and this should be equal to - μ ∂H /∂T which therefore implies that H will be equal to so this is the Y component that I am looking at therefore there will be a - sign to this one so this minus sign and this minus sign cancels with respect to each other and clearly because electric field is only along the Y component the curl of electric field is along the Y direction.

The magnetic field must also be in the Y direction itself because that is equal to H or rather direct time derivative of H so I have μ ∂H, so instead of writing this ∂H in a vector form I would like to write this as ∂HY / ∂T this will be equal to ∂EX / ∂HZ but I know already what is EX is E0 Cos Ωt – ß , so if I differentiate this with respect to Z I will be pulling out – ß from this so I obtain – ß E0 and then I have this instead of Cos sine I will now have a Cos sine Ωt – ß said okay this will be equal to μ ∂HY/ ∂D but now integrating on both sides with respect to time what do I obtain this would be μ and integration with respect to time will take this away.

So I will have HY and to the right hand side we will have – ß E0 and what is the integration of sine with respect to time will give you this will give you – Cos right, so this will give you – Cos and there will be a division by Ω correct because it is Sine ΩT with respect to DT that you are going to do, so when you do that when this becomes Cos ΩT – ß into Z, so therefore HY can be written re written by putting this mu down here and writing this as ß / μ ε times E0 and if I call ß / μ ε okay x E0 as H0.

Which is the amplitude of the magnetic field what will be this ß by Ω μ ß as we have already seen from the above is given by Ω √ μ ε so dividing this one by Ω will cancel Ω on both sides, so I obtain √ of substituting this Ω √ μ ε for ß what I obtain as√ ε / μ okay but I already told you that E0 / or rather I told you that H0 is given by E0 / η okay in the previous model, so I can define η as the impedance okay I can define η as the impedance as the ratio of electric field amplitude to the magnetic field amplitude and that is equal to 1 by √ of ε / μ alternatively this is equal to √ μ/ ε.

Okay this is very similar to the transmission line characteristic impedance in a case of a transmission line characteristic impedance you had √ of N by C in this case you have √ μ/ ε characteristic impedance of a transmission line was the ratio of a positive travelling voltage to the positive travelling current in this case the characteristic impedance is the ratio of the electric field amplitude to the magnetic field amplitude, and the electric field will be along the X Direction magnetic field is along the Y direction, so that is the key difference that you have to see between the two.

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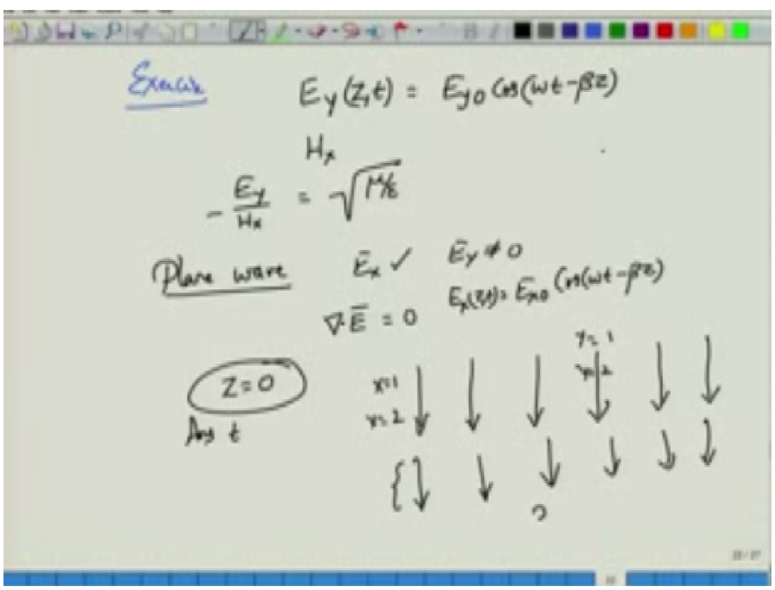

I will leave this as an exercise to you okay or you can try if I assume instead of the EX component if I assume E of E / Z ,t as some constant Ey_0 Cos Ω T - ß Z okay then you can show that it is the H_X component that will be important and the ratio of E / 2 – HX will be exactly equal to √ of μ/ε okay, so far so good why is this called a plane wave right if you look at the electric field E assume only that there is a E_X component E_Y component is assumed to be 0 this X component if you look at or if you try to find what is ∇.E right can you see what will happen to ∇.E this will be equal to 0.

Because E_X will be function only of Z and this one so if I consider the case where Z is equal to some constant and I can consider Z equal to 0 as a constant and if I plot at a given value of time if I plot the electric field component, so I know that X of Z and T is given by EX_0 or some a 0 constant or Cos of Ω T – ß X right, so this is what my X of Z and T is if I consider the Z equal to 0 plane which is a constant plane in the space that I am considering and I also freeze time T to a particular value, so let us say T is equal to 0 in this case okay.

And it can be anything so for any value of T if I fix the value of T and if I now look at what would be the direction of the electric field and what would be the strength of the electric field in this plane set equal to 0.

I see that the electric field will be directed along x axis this is the direction of the x axis that I am considering and the strength of this electric field will be completely independent along the x axis itself okay, so it does not matter where I am standing the strength of the field at X equal to 1 it will be the same at X equal to 2 it will be the same at Y equal to 1 it will be same y equal to 2it will be the same so it will essentially be completely independent and if you look at how these amplitudes are all you know present they will all be in the form of a plane okay.

So this is the plane wave that we are talking about now that is not the complete story this is what we called as the constant amplitude, so if I collect all the constant amplitudes okay at many different times these constant amplitudes will always form a plane okay so it is like a flat surface kind of a thing but that is not the full story, because we also need to talk of the phase friend what is the phase front it is a collection.

If I have to have a common face I need to maintain both Ω T - Ω T and ß said in such a way that Ω T – ßZ must be equal to some constant and whenever I consider that particular cone for example I consider the phase front 4 π by 2 in which case Ω T- ßS it must be equal to π by 2 so I for example fix my said and then I find at all different times where this condition is satisfied and when I look at that condition what will happen or when I consider that particular thing then the phase constant okay implies that the phase front will also be a form of a plane okay, so that is the reason why we call this as a uniform plane wave okay.

In contrast to that suppose I have a source here and then the wave is you know generated from this source and it is in the form of a cylinder, so it is expanding in the form of a cylinder or you can think of a wave coming from an explosion where it would be moving in the form of a spherical cloud so in this case although if we consider a constant I mean so if because of the constant amplitude on the constant phase those constant amplitudes will always be forming a surface which is spear and spear is not a plane.

So spherical waves as what we call them or cylindrical waves as we call them are distinguished by the way their amplitudes friends and the phase fronts are distributed okay, so that is the difference between a plane wave and non plane wave we will for now and in most of the cases in courses in most of the lectures in this course we will be considering only the plane waves okay.

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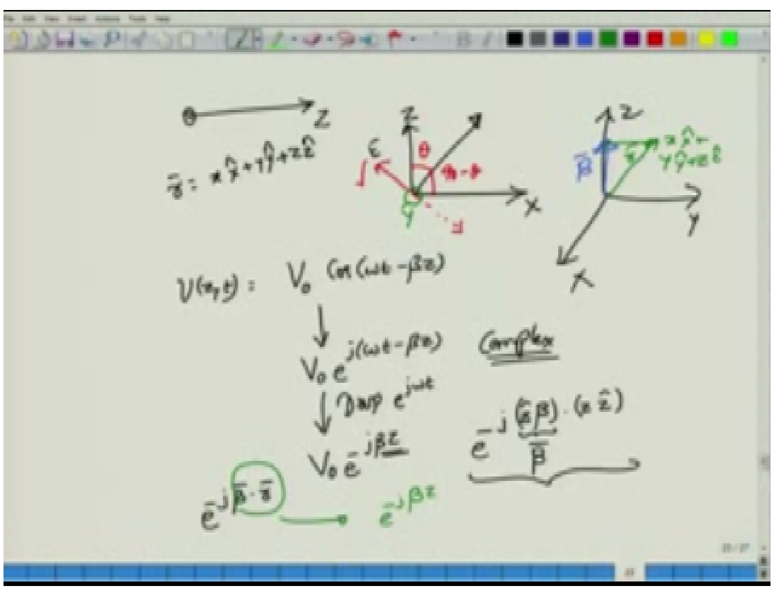

We have seen one type of a wave in which the direction of the wave was always along the z axis what if I want to specify a wave in an arbitrary direction okay, so I want to now specify a wave which is propagating along let us say this direction okay so I will consider this as X and Z and appropriately this is y so I do not know this should be - Y or so X cross Y well this is okay so we can consider this as the Y axis so let me say I want to talk about a wave which is propagating in the and z-plane and making an angle of θ with respect to z axis and an angle of 90- θ with respect to the x axis.

How do I describe this way can I still have a plane wave yes you will have a plane wave where the wave is propagating along this direction okay, so what should happen to my electric field and magnetic field well when the wave is propagating in the XZ plane I know that if I still want to claim a tem wave electric field must be perpendicular to the direction of propagation, and this electric field must be perpendicular to the direction of propagation again you have two choices right.

So you have one choice which is shown here or you can have a choice which would go like this if I fix electric field to be perpendicular and lying in the same plane then the magnetic field must be perpendicular to both of this and it should be in such a way that a cross edge must point in the direction of propagation, so if this is my electric field then the magnetic field must be perpendicular so let me raise this Y field so if I show that I know in the two dimensional case I am looking at if my electric field is lying in the plane right then the magnetic field must in this case lie along Y axis.

So that this e cross Y should be showing you the direction of the propagation which is along the XZ plane okay, so this is one solution for me or I can have an electric field perpendicular that is along the y axis so that electric field times H, H will now be in the in this particular loop I mean in this particular plane X and z plane and E Y cross that H must show you the direction of the propagation okay as demanded by the tem condition okay, so it is possible for us to specify waves propagating an arbitrary direction.

Before I can show you that I need to introduce you to the phase notation or the complex notation okay you know already this is V of Z T is given by V 0 Cos Ω T - ß Z we said that if you want to obtain the corresponding phaser you first convert this Cos Ω T - ß Z into $e^{j \Omega t - ß Z}$ okay times V0 this is the complex notation okay, this is not the phase or this is just the complex notation very electric field in this in the form of e to the power J I mean voltage is in the form of $e^{j \Omega t}$ to go to the phaser domain.

I need to drop this factor e $^{j \Omega t}$ so as to obtain V0 e $^{- j ß Z}$ I can rewrite this e $^{- j ß Z}$ as e $^{– jZ^}$ ß. Z, Z^ where Z is the length or the direction that is the coordinate value and Z^ is the unit vector along the z axis, this I can consider it to be a vector itself right I can consider this to be the propagation vector and this propagation vector in this case will be directed along z axis, so I can rewrite this exponential phase factor as e $^{– jß.r}$ where r is the position vector, so if this is the z axis from the origin the r vector is given by X, X^+ Y Y^ right, so let me go back to the three-dimensional case this is X this is Y.

And this is said the position vector at any point the position vector at any point along this one or at any point can be written as from the origin what is X X^ +, Y^+ Z Z^ if I consider the wave propagating only along the z axis then what will happen to the phase factor it would be e $^{– jß.r}$ this is the direction for ß this is the position vector r and ß . r will be the dot product between this propagation vector and any point or the position vector right, and that would be just the projection of this r on to the said axis okay.

And that is precisely what this ß dot R is telling you right if the wave is propagating along z axis then e $^{– jß.r}$ is a fancy way of saying e $^{– jßZ}$.

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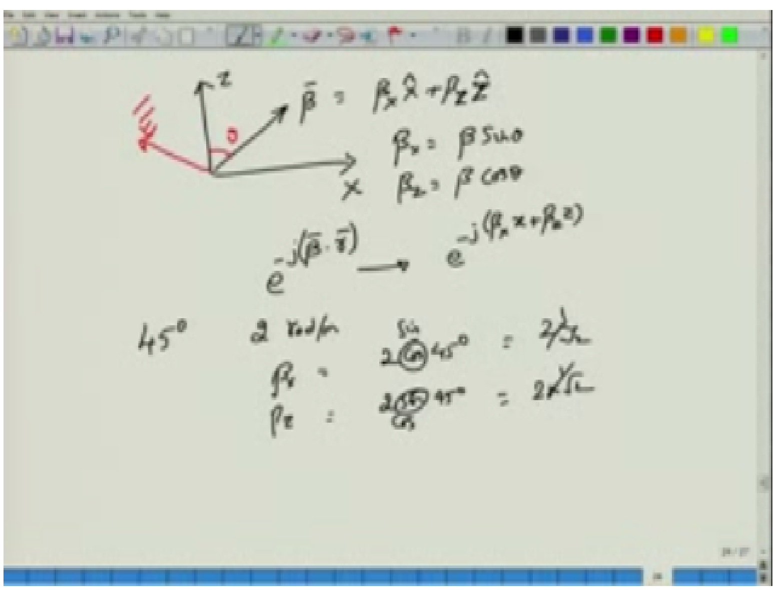

But the situation is different than I can propagation in an arbitrary direction so I said that you will have an X Z plane and I am considering propagation of the wave in the X and z plane correct and this is making an angle of some θ with respect to said if this is the propagation vector ß having the magnitude ß then this propagation vector ß can be rewritten as ß X, X^+ ß Z, Z^ correct and I can find out what is $ß_X$ $ß_X$ is nothing but propagation constant ß times Sine θ whereas ß Z is equal to ß times Cos θ okay.

Once I have this what will happen to the phase factor the phase factor becomes $e^{-jß.r}$ okay so ß is ß Sine theta ß Cos θ along the x and z components the position vector r is X, X^+ Y Y^ and Z Z^ and therefore this becomes $e^{-jß\,XX + ß\,Z}$ Z okay, so you see that the direction of propagation is along plane in which the propagation vector has two components ß X and ß Z and if I now assume that the electric field is perpendicular to this and lies in the same direction I can show that electric field also has two components one component will be along the z axis and the other component will be along the x axis.

Okay I can find out what those components are but right now that is not the point for example if I give you a wave propagating in the XZ plane at say 45^0 with respect to the z axis and having an magnitude of the propagation constant of 2 radians per meter what will be the component for ß X ß X will be 2 Cos 45^0 ß Y orbital said will be equal to 2 Sine 45, but ß X will be equal to 2 into root 2 this would also be 2 into √2 Cos 45 is 1 / √ 2 so this is 2 into 1 by okay, so you can find this so this is you can still have a tem condition but the wave is propagating as a tem wave but it is propagating in a arbitrary direction okay.

There is no reason why you should stop on with two-dimensional you know plane kind of a propagate in the sense that ß can have only ß X and ß said you can actually have ß propagating in any direction, so you can have ß X ß Y and ß set the condition for a tem gave us that electric field must be found which is perpendicular to this ß vector and the magnetic field must be found which is further perpendicular to that too before we close this module I would like to actually show you the relationship between E H and ß in a slightly different manner okay which will allow you to find out H given electric field and the propagation vector ß okay.

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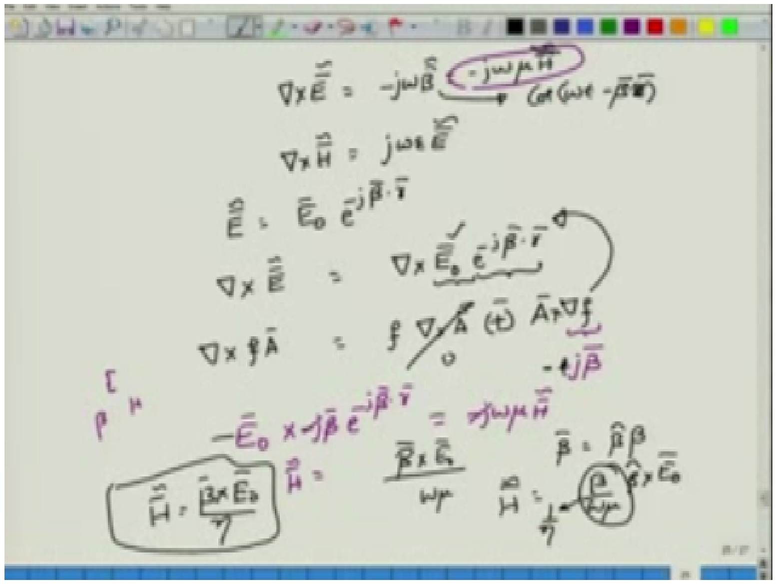

We start with ∇x E = - ∂ B / ∂ T now here because I am I was considering the phaser notation by dropping the time dependence and I will do the same thing for the magnetic field vector B so if I express everything in terms of a phaser or and this is how I express the phaser in the vector or I express the vector phaser I put a small tilde on top of it, so for the phaser there is no time dependence because that is precisely what the definition of a phaser is and instead ∂ / ∂ T becomes J times Ω where we will assume that this B field original B field is of form Cos Ω – T - ß into Z okay.

Or rather Cos Ω T – ß.r okay so ∂ / ∂ T will be replaced by – j Ω I can do the same thing for the magnetic field magnetic field will be equal to J Ω ε times electric field okay this J Ω B can be written as - J I mean J Ω B can be written as - J Ω μ into μ into H because B is equal to μ into H in the phaser notation as well okay now what is the assumed phaser for e this was some constant E_0 which is the vector constant E_0 telling you the direction of the electric field times e – $j^{ß.r}$ this is what we have assumed this E_0 is a constant vector right.

So if I consider ∇ E right this will be equal to ∇ x E0 e $^{-jß.r}$ so now I am looking at one scalar multiplying a vector and then I am taking the curl of this entire thing, so I am looking at a scalar F times the vector A so what is the curl of FA, so if I remember it correctly the corresponding identity for a vector will be F ∇ x A okay + A times gradient of F, so will that give me the correct answer or a cross gradient of F so this one will be a vector which is okay gradient of F will be a vector.

But then I A vector to multiply so in this case we have a vector identity which is given by this one so ∇ x FA = F∇x A + Ax∇F and identifying a with a zero constant vector and F with the phase factor e $^{-j ß.r}$ this fellow will be equal to 0 because curl of a constant will be equal to 0 and you can show that gradient of F simply pulls out this - Jß vector out of it okay, and of course it will still be e $^{-j ß.r}$ that you can write in the same way and a was easy E_0, so I can go back and substitute for a which is E_0 in this case cross -J ß this is a vector it will still be associated with this phase factor e $^{-Jß.r}$ or that let us write this should then be equal to from the first equation - J Ω μ H phaser okay, now minus J on both sides will cancel and H phaser can therefore be written as E0 x ß.

So I have e H ß so this should actually have been Δ gradient of F x A okay, so there is a - sign over here you can put so this would be a plus somewhere our gradient of F will be a minus so this has to be a minus sign somewhere and therefore ß should come on this side and E_0 should go on the other side okay, so I have E0 vector cross whatever that ß that I had right, so ß was the vector ß itself divided by Ω μ I am assuming that e $^{-jß.r}$ is the same for both sides so I am removing that e $^{-j ß .r}$ phaser on that.

So what I get is E_0 x ß / Ω μ so this is the phaser for the hedge without the phase factor that I can put in but I can write ß as a vector unit vector along ß multiplied by the magnitude of the vector ß itself substitute that into this expression to show you that H is equal to this is ß x V0, so this is to be ß 0 x E sorry ß x E0 okay and then you can substitute for ß as the vector unit vector ß hat times ß in this one and then show that the magnetic field H will be given by ß by Ω μ the vector ß ^ x E0 and this ß by Ω μ we have already shown it to be equal to 1 by η.

Therefore the phaser H can be computed without really worrying about that so you will have ß cross E0 divided by ß, so this is this equation okay allows you to calculate the magnetic field provided someone tells you the component or the vector propagation vector ß as well as tells you the direction of the electric field okay, I request you to verify this equation again by the help of the vector identities and try to solve this one for the previous example for the arbitrary directed electric field that we considered thank you very much.

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