Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course title
Applied Electromagnetic for Engineers

Module- 40
From Maxwell's equations to uniform plane waves

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Hello and welcome to NPTEL move on applied electromagnetic for engineers in this module we will begin the study of plane waves okay, the concept of a wave is not new to you we have already seen an example of wave when we discuss transmission line that both voltage current on transmission lines are not some scalar component either components are which do not exhibiters any patient dependent but rather they are component which exhibit special dependent.

That is if you imagine that this is the transmission line and then you exist this transmission line from the source at some particular point then if you imagine there is a oscilloscope here and other oscilloscope on the other on the transmission connected then you will see that the wave on the transmission line will be appearing at the second transmission line so as you see on the transmission line you have 2 oscilloscope the wave or the time demand wave for that rm that appear at 1 oscilloscope will appear at the second oscilloscope as well however the will be a small delay between them.

So if you imagine that the transmission line extended all the way towards infinity and imagine that there are about infinity number of oscilloscope kept then when this peek arrives at the 1^{st} oscilloscope then the next peek will appear at the slightly different time the next peek will appear on the 3^{rd} oscilloscope will slightly appear different time and so on the peak travel and the entire wave plans travel along the transmission line.

We wrote this in the form of by written instead of V/T which usually write in a circuit approximation for the time warring voltage we have to include 2rd co-ordination over which the voltage or the current would depend and we conceded arbitral that coordinate would be relabeled as Z access. So we had a transmission line which extending all the way from Z=0 to infinity and at the any point Z,T the voltage or the current that if you measure or find out that voltage or current would be the function of both Z and T right.

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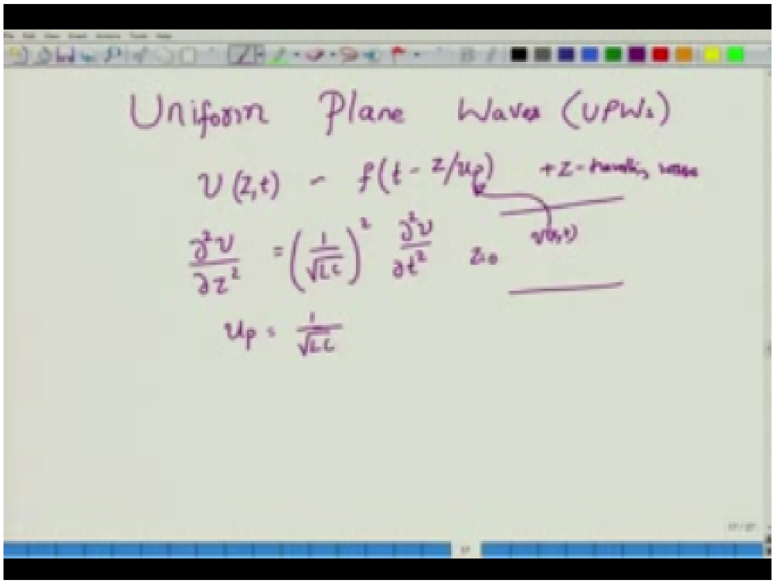

We opined this dependence of V/Z,T and that equation den square / den Z square which tells you how the voltage is changing along the transmission line with respective Z CORDINTE that is equal to $(1/\sqrt{c^z})^z$ denV^z/denT^z right so by solving this parcel different equation by the method of separable variable we are actually arrived at any solution which will be the form of some kind of function that could be ZT-Z/UP right, where we identify UP as the face velocity and UP was equal to 1/sq root of LC and important point was that when you had a transmission line which extended also here from z = 0 z= infinity the voltage of any point of time on this transmission line was not component it was just depend on time.

But actually exhibiting the wave like behavior because the voltage was in the form of some function team – Z/UP of course in this case we had used minus because we said that it was plus said travelling wave, so in simpler terms a wav is phenomenal were by energy is transported from one point to another point okay, sometimes those are called as traveling waves.

Sometimes it energy transportation may not be the idea but the voltage or current or any other quantity that would vary both as a function of space as well as time okay, in a prescribed manner in which the argument of that variation can be reduced as T-Z/UP or T+ZUP for the single dimensional case that we are considering those functions are said to be exbited they have like behavior, you might have seen an example like behavior you take a stone and then drop into the pond or drop in a small bowl of water small stone of course and then you would see those repels right.

So if you imagine that there are the small paper of boats that you can make and you place those paper boats then what will happen there will be a way which could be passing through the paper boat I like this and once that waves appears and gives it a bump the paper boat will just move up and then come down.

So this paper boat moving up and down and response to the corresponding water that is changing through the space is essentially mentioning in the wave phenomenal or you can take you can imagine that you have a rope which your tied to pool and then you swing that rope okay, when you do that you can see that all part of the waves also move ups an down in a certain way I mean they need not be periodic but you might want to create a periodic way by you know repeatedly doing it.

In generally if you just take the rope and then just you may not do just kind of action in just moving up and down then it will the kind of that motion that will be carried out further by the wire itself o the rope itself so this the wave like behavior okay, and we are going to considered very special kinds of wave which are called as plane waves and we will also consider those waves to be uniformed okay.

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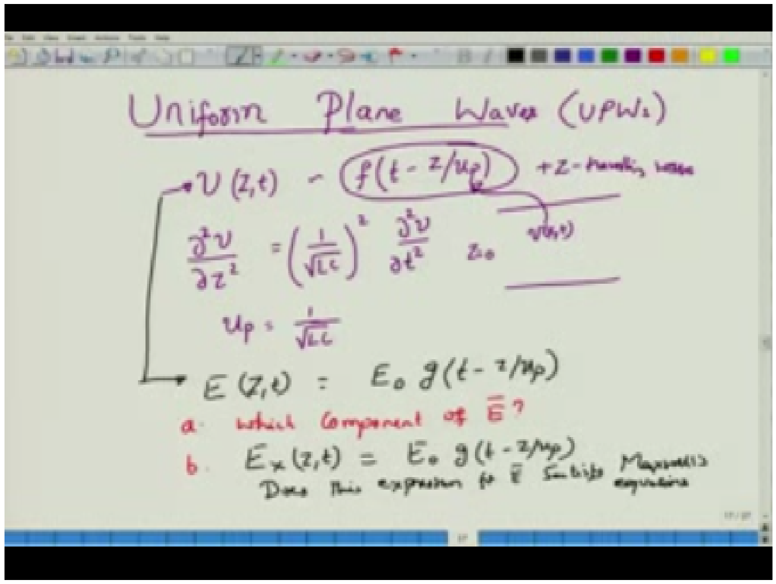

We assume that the waves are going to be proper getting along the Z axis okay so all are functions are supposed to be in the form of T-Z/UP okay were UP will be identified by some other names there will not be 1/√2^ because what we want to consider is how can electric and magnetic field exhibit a wave like behavior okay, we are not interested in the voltage changing n the function of Z and T we are interested in the electromagnetic variable being a function of function OF Z and T.

And the relevant electromagnetic variables that we will be considering will be the electric field E okay, so we want to write an equation that is similar to V of ZT we want to write an equations for the electric field and say E (Z,T) will be in sum E0 okay that is an E0 is the constant and some function g with the argument of t-z/up we have no idea what up is we have to find out that value of up it wont of course be equal to 1/√c but this is what we want now immediately we will have to raise two questions.

If we think about a little bit you will have two questions which component of the electric field are we talking about, right which component of electric field did I mention I mean did I assume when I wrote the electric field that way, why is this important unlike a voltage which is a scalar variable electric field is a vector. So I should be talking about the component of the electric field which exhibits this Z and T behavior okay, so I will have to specify whether you know I am considering the ex components ey component ez component in the carnation coordinate systems or I am considering the er, e5 and ez component circular cylindrical coordinate system or in the spherical polar co ordinate system I will be talking about e∩ and e5 so clearly I have to specify the component because electric field is a vector.

So that's the first question that you should have the second question that you should have is if I assume that I have specified a components let us for now assume that I have actually specified the x component of the electric field as a function of both z and time if I say that this is equal to some constant E0 g of t – z/up I would have answered the first question but does this equation right or this expression for the electric field satisfy maths well equations remember there are other maths well equations that needs to be satisfied right, so when I right down the electric field on the specified component of electric field that is Ex will this form will be sufficient or will this expression will be correct that it will also satisfy the other maths well equations right.

So there are two primary difficulties when we talk about waves one is that the quantity that we are going to specify will be a vector it would not be a scalar okay, that's the first problem that you have unlike on a transmission line were we consider a voltage or a current and both voltage as well as current okay, were scalar quantities here we are talking about electric field E and possibly the magnetic field H okay.

And these quantities are inherently vector in nature so even when they are vector in nature and you manage to specify one component acting like a wave does it mean that the entire electric field will act like a wave does it mean that the magnetic field H will act like a wave if it does act like a wave is it independent of the electric field if not the combine E and H field that we specify will lead satisfy the other maths well equations.

So these are the two primary question that your looking to answer okay, it tells out that the moment that I specified the electric field component Ex here I cannot specify the magnetic field independently why cannot I specify the magnetic field independently because maths well equations constrain that if Ex has certain expression then there will be a some relationship to the agnatic field which of course will be given by either faradize law or will be given by ampere maths well , so we have to apply this laws to see that the expression that we right down are consistence with each other so it is nobly important that we have Ex exhibiting a wave like behavior but there has to be a corresponding relation for the magnetic field.

Such that maths well equations are not violated so what could the magnetic field be well I would not derive it right now this is why we have the this modules and other modules but you can see that whenever we specify Ex to be a sum.

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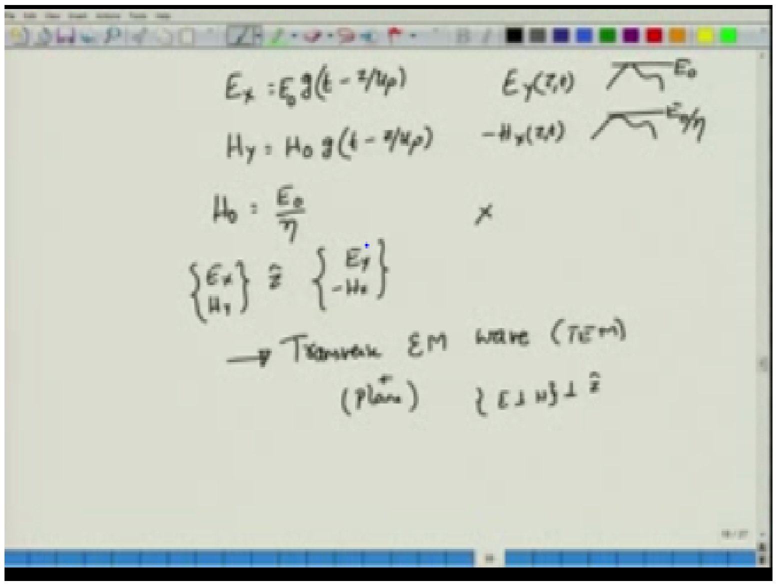

Function g (t- z/up) that terms out that the magnetic field so with an amplitude of E0 let us say in terms of that it will decide further that the magnetic field Hy must be non zero and this magnetic field Hy must be related to electric field by its constant H0. So we can show and we will show that H0= E0/η and η is what we call as free space impedance okay, we will talk about all these later.

But Hy will have the same functional dependence on Z and time that is very important thing both Ex will be as a function of G(t-z/up) whereas Hy will also be the same function same arguments and everything it will also have the same face velocity as it is the Ex component okay. If instead I started out by specifying Ey as a function of z and time then you can show that the corresponding component which will be non zero in this case will be –Hx as a function of z and time, so if Ey where varying in this manner – Hx would also vary in the same manner.

But the amplitude of this one compare to the amplitude of Ey will be reduced by a factor of 1/η okay, so this particular group of or the set of electric field that we have return Ex and hy as well as Ey and – Hx also satisfy very interesting condition over here if you know you know imagine that you have a right hand rule that where you go from X to Y that is you go from Ex to Ey then if you take the cross product of that.

That cross product will be in the direction of Z and what is Z, Z is the direction in which these waves are assumed to be propagate let me just show you the previous slide so that you understand the concept between waves on the transmission line and waves in a free space that we are talking about okay, so we are going to talk about a wave in a free space and by free space we mean space were the values of new explants sigma are all going to be constant so such spaces are usually called as uniform loss less isotropic media and we are going to consider only uniform loss less isotropic media.

Unless we consider different type of media as I will talk about, okay so for the waves in space there is a direct analogy with voltage waves on a transmission line expect that voltages on the transmission line or current from the transmission line are always scalar quantities waves are vector qualities, but if I have only Ex component and the corresponding Hy component E acts like a voltage and H act like a current.

so I have a kind of one to one correspondence to the voltage and current on a transmission line so these waves exhibit the property that when you take a cross product that is you take the electric field E and then go from Ex to Hy if you go then that direction the result in direction will point to the direction of the propagation oaky, similarly Ey cross minus Hx will also point to the direction of the propagation and because Ex is perpendicular to Hy Ey is perpendicular to minus Hx.

And both this quantities are perpendicular to Z axis we have a very special type of a wave called as transfers electromagnetic wave so what we have just said uniform plain waves are also sometimes called as transfer electromagnetic waves okay, of course these are transfer electromagnetic plus plain waves. I should point out what a plain waves okay I will point out what a plan waves in shortly.

But consider that when we group Ex and Hy with the wave propagating along Z axis if I group Ey and – Hx with the wave propagating along Z axis because Ex is perpendicular to Hy, Hy is perpendicular to Z axis these waves are called as transfers electromagnetic waves Transfers meaning perpendicular so E is perpendicular to H both of these quantities are further perpendicular to the direction of propagation okay. In short form we called this as TEM Transfers Electro Magnetic wave so 10 waves we would call them now having kind of given.

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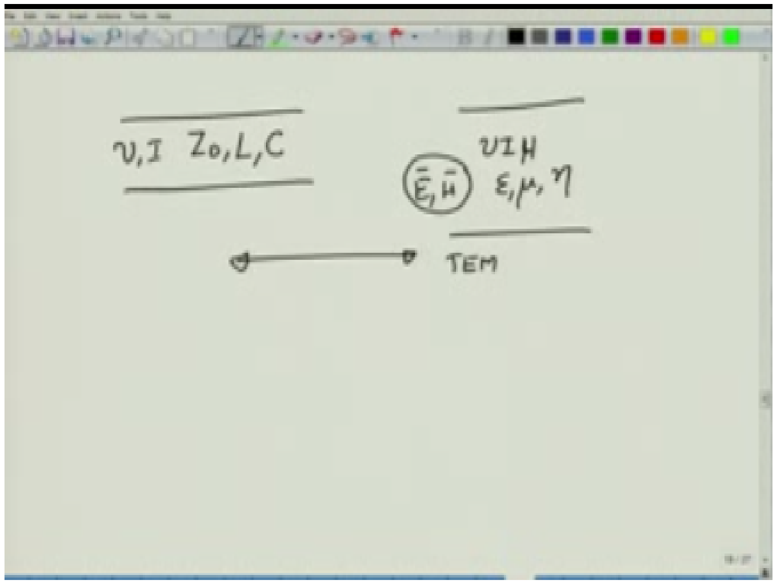

You the analogy between the transmission line you know this is the transmission line with some characteristics impedance Z not primary constant of the loss less transmission line of M and C there is a corresponding equalance with free space or the uniform isotropic loss less material so you have a uniform isotopic homogenous material homogenous means the property do not change with space okay, so were you have epsilon mu okay, we will also have an impedance η just as V and I are the primary voltage and current variable from the transmission line you will have electric field and the magnetic fields okay.

The difference that lies between these two is that you will have to specify a particular component of E and corresponding component of H such that you are not really considering the vaster nature okay for the tem cases both this quantities or both this saturation are exact they are exactly nano lockers okay. Whatever that works for a transmission line kind of work for the same thing for the electromagnetic transfer waves propagating in free space okay.

Having kind of informally told you what a wave is and what the characteristic now we will put little bit mathematics into will bring mathematics into picture to prove all the things that we said you know specify EX once we specify EX what should be the component for HY can I specify that the wave consist of both EX and EY well in that case does it mean it will have both HX.

And –HY or can I consider that the direction of electric field or the magnetic field of this particular wave or the direction of the wave itself will not along ZX it can be along –ZX can it be XZ plane well all these question we will try to answer them. And to start answering that we have to drive an analog equations for the voltage and the current.

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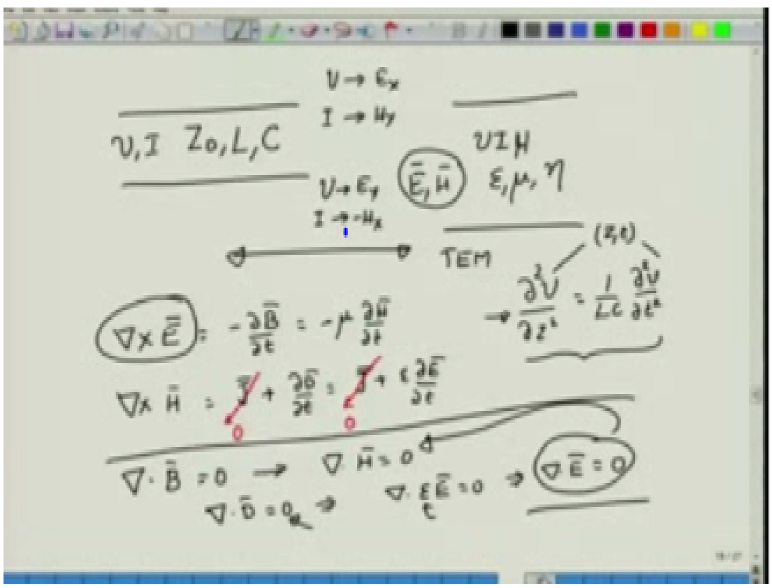

Remember we had derived an equation for the voltage we are said the r^2 v/ r^2z = 1/lc this voltages are function of both Z and time, so what is the corresponding expression for the electric field right we have made an analogy we have said that V correspond to EX and I correspond to HY correct so in the previous case we have made this analogy or we can say that V correspond to EY and I therefore must correspond to –HX.

So I can straightly substitute this EX into the from V I can substitute EX and obtain an equation for EX okay but I don’t want to do that one I want to show you from the Maxwell equation how can I arrive an equation that look like this so how does Maxwell equation tell you predict that if you have a time varying electric field and this are varying with space as well then they will form a there.

So Maxwell equation in fact predicted that based on these equation there has to be waves and this fact that electromagnetic electric field and magnetic field combine together to form an electromagnetic wave was confirmed by heard ok all right. So we begin where we begin at the Maxwell equation so we have dell cross E = -dell B/T this is coming from faradize but I'm not interest in B I'm going to assume that I'm dealing with a non magnetic media therefore I can replace by scalar permeability B I can write it as scalar permeability mu types the magnetic field H so B – dell B/T can replaced and rewritten as – mu dell H/T okay.

I have also have dell cross H = J+dell B/T again I don't want in the term of D I want in everything in electric field E so I can rewrite J+ dell E/T and I will and I will assume that the J = 0 when can I assume that J=0 when the medium the wave is propagating or when the wave I travelling is does not contain any wire or conductor placed in that.

So the medium is completely dielectric medium or insulating medium and because of that the conduction current density J cannot exist and therefore J=0 so we are in the case where there is no currents in that particular region what are the other Maxwell equations well we have dell.B=0 but really this is telling you that dell.H=0 because B=mu$^{\times}$H and then you have dell.D=0 these does not tell me that dell.E=0 all that it tells me that dell.timesE=0 but luckily for us we are considering to be a consent therefore I can pull these out and therefore I also option dell.E=0 okay.

Look at these you have dell cross E and then you have dell.E, dell.E=0 right so in the same since that dell.H=0 or dell.B=0 I forgot to mention to you that there is no charges okay there is no current and there are no chargers so in that case electric field do not start or some positive charge and negative charge because there is no positive charge and negative so electric field lines have to then formal loop okay.

Just as dell.B=0 right indicate that there are no magnetic mono poles or there are no magnetic charges similarly in the free spaces where there is no charges dell.E will also be = 0 which means electric field will be actually be in the form of loop cutting that loop will be the magnetic field through it that will be also formal loop because dell.H=0 however carrel magnetic you know carrel of H when you take through that particular loop the right hand side dell cross H will be non zero and it would be = to the displacement current that is coming through it.

And then when you consider the electric field loop the EMF around that loop is not 0 because there will be a magnetic field time varying magnetic field changing and hence giving you a induced voltage so this happens no electric field change there will be a displacement current displacement current changes magnetic field induce as a voltage or the EMF it induce as a EMF for the electric field which gets modified as a results of it and sawn and saw for the actually form of a interlinked loops okay which propagate through the space inside that the a picture that people are familiar.

So electric field is moving and the magnetic field is interlocking and also moving so we have seen that maximum equations in the region where are the no charges no current reduce to simple equations where dealt out H=0 and dell.E=0 now what is the implication of that.

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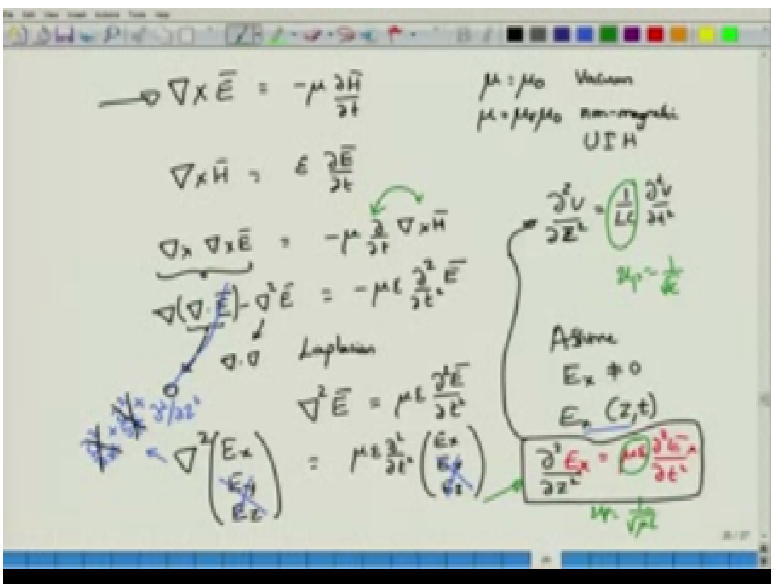

We have seen that curl of electric field E is –mu dellH/T again let me tell you mu=mu. For free space or vacuum and mu=muR mu not for the medium non magnetic but addition to non magnetic it is uniform isotropic and homogenous media okay. So it is uniform, isotropic and homogenous media. Similarly delxh will be equal to $\varepsilon \nabla e / \nabla t$ okay now what I do is I take the curl of the first equation the equation which I have shown by the arrow mark I take the curl of that equation what do I get I get ∇ x ∇ x e = - μ ∇/ ∇t ∇ x h here I have inter changed $\nabla/\nabla t$ operation with the ∇ x operation I have to interchange the differentiation and curl operation.

So when I do that and invoke vector identity to simplify the left hand side I get ∇ of ∇. e - ∇^2 e okay so I have $\nabla$of ∇ .e - ∇^2 e where this ∇^2 is actually ∇. ∇ and it is called as a Laplacian okay. So this is the Laplasian and of course we have seen in poisons equation and even when we talk of vector potential the right hand side can be simplified further by noting that ∇ x h is ε ∇e / ∇ t ε being a constant I can pull this out so I obtain - μ ε ∇/ ∇t ∇ x h is ∇ e / ∇ t therefore this becomes ∇^2 / ∇t^2 electric field where almost there remember our goal is to show something like this so ∇^2 v / ∇z^2 = 1/ Lc ∇^2 v/ ∇t^2 so our goal is to arrive at to this particular type of an equation we are almost there we know ∇.e = 0 why is ∇.e = 0?

Because there are no free charges and hence Gaussian's law tells you the ∇. E = 0 and –sine on the left hand side here will cancel with the –sine on the right hand side over here and I do not worry about that and I obtain a simplified equation which tells you this is the second order partial differential equation which tells you that ∇^2 e = μ ε ∇^2e/ ∇t^2 okay, so we have obtained this equation let us see what kind of an equation is this ∇^2 is a scalar kind of an operator okay, but it is operating on a vector E, vector E will have Ex, Ey and Ez in the Cartesian co-ordinate system this then must be equal to $\mu\varepsilon\partial^2/\partial t^2$ which itself is an operator this is also operating on the vector Ex, Ey and Ez okay.

I will assume that we have only Ex component or sometimes you want you can have an Ey component okay, but for now we will consider only Ex equal to non-zero we consider all the other components to be 0 that is all the other electric field components to be 0, okay. If I further assume that this Ex is going to be a function only of z and t so I am talking about a one dimensional wave in which the wave is propagating along the z axis in the Cartesian co-ordinates system.

So for that and further assuming that electric field so if this is the z co-ordinates or this is the way in which the wave is propagating this would be the electric field my thumb is the Ex direction okay, and correspondingly you have a Hy component so Ex and Hy together will show you the direction in which the way is propagating. So Ex which I have must only change along the transmission line and of course change with respect to time it should not change if I move up and down not the transmission line I am sorry, at any particular point.

So if I assume that the electric field so if I move up and down then as I move up and down in terms of x or y up and down or sideways in terms of x and y my electric field component should not be dependent on those components okay, so at all points I have a electric field which is a function only of z and t and I assume that there is only a Ex component okay, when I do that I do not have to worry about this Ey, Ez components out there.

And I obtain a simplified expression which tells you that δ^2ex obviously please note here this is the scalar okay x is a scalar the full electric field is a vector on even considered as so for b that I have to multiply that have to rewrite a scalar by multiplying the scalar with it is vector along the I mean I have to and further Δ2 because I know the electric function further I crop this Δ as well because I know that the electric field as a function to and if you expand this Δ^2 dependents on y these two components are, so essentially what is time after replacing this operator.

So you observe this equation voltage transmission line in fact the same mathematics one we calling b the other we calling as dx most importantly the similarity between okay since it has to be form of and in this case okay must there be equal to $1/\sqrt{\delta}$ We have shown that the solutions of this box equation for the electric field component EX has the function of both Z and time must be of the same form as the voltage on the transmission line that is there will be of the form some function of T – z UP and UP in this case is the phase velocity decided entirely by the medium given by $1 / \sqrt{\mu\varepsilon}$.

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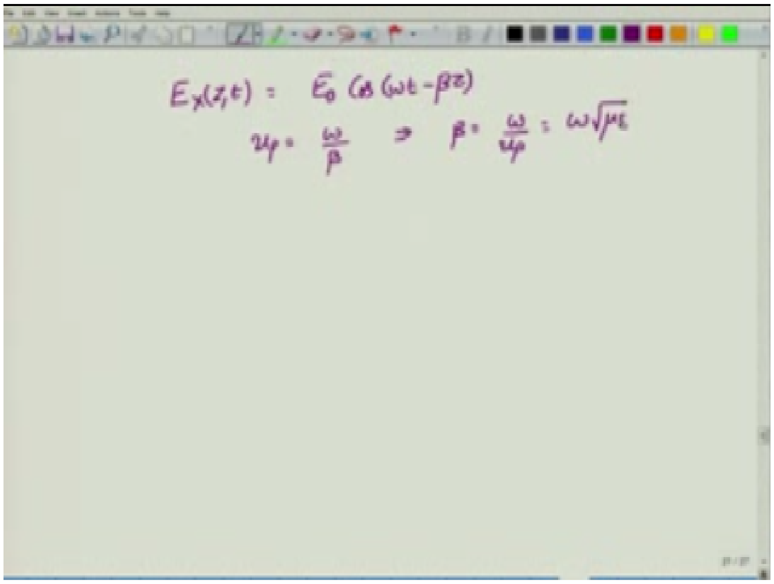

If I specialize to the condition of a sinusoidal one then I can write this EX as the function of Z and T as some constant E0 Cos of ΩT – ßZ where the phase velocity UP must be equal to Ω / ß this further implies that ß = Ω/ UP of course these are just the same relationship we are just putting them in any you know we are just using the same equations out there Ω is any way the frequency phase velocity UP is 1 / √ με and μUp therefore 1 / UP therefore = Ω / √ με with this we will stop here. And in the next module we will continue the discussion of uniform plane waves thank you very much.

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