Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 04
Terminating T-lines: Reflections and Transmission coefficients

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Hello and welcome to the NPTEL mooke on applied electromagnetics for engineers. In this module we will look at terminated transmission lines introducing you to the concepts of reflection and transmission coefficients. Now before doing that let us actually set up the problem that we are about to investigate. So far in the previous modules we considered that the transmission lines actually had no end there was no load on the transmission line right, there was no source, there was no load that we considered.

So magically voltages were appearing on the transmission line both going along positive or negative Z-axis. Now suppose I consider a semi infinite line that is I consider that the source is located far, far away from the transmission line, but I have a load situated here, here in my language means Z=0 okay. So this is a coordinate system that is typically used when dealing with transmission lines that loads are located at Z=0 and sources are located at some Z=minus some value okay.

So far for us Z=-α of the source initially, but we will relax that assumption later on okay. So what we are actually looking at is what happens to these waves okay when you terminate the transmission line with a load okay. Let us look at that one.

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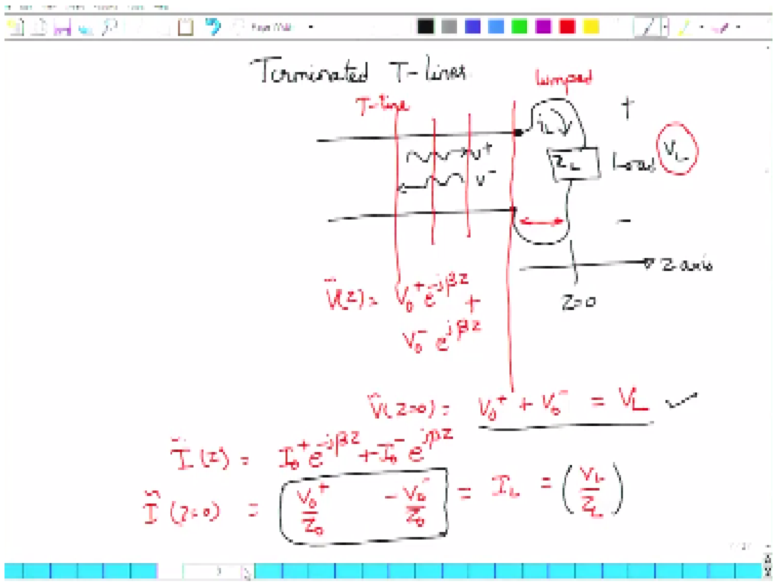

Look at the way I have drawn the transmission line the transmission line is indicated by these two lines which I have drawn okay I have also drawn two small sinusoidal kind of a thing to indicate that we are considering a frequency domain behavior and I have a way which is going in the positive Z-axis as well as in the negative Z-axis I also locate this node at Z=0 so this is the load that we have kept it could be a T, antenna or it could be a television set it could be anything that you are considering as a load in a high-speed digital system this could be a no address bus on one hand and the module that the address bus is actually addressing to.

So it could be any kind of a thing and this load could also be frequency dependent meaning that there could be some inductance or capacitance in this particular load, so that the impedance is what you would want to consider not just a load resistance. So you actually are considering a slightly complicated case of load impedance okay. The load impedance of course is the termination condition which tells you that there will be some current flowing through the load which we can label as I_L and there will be certain voltage across this load which we can label as V_L okay.

Now on the transmission line at any Z that I can take off at any point on the transmission line there will be both positive and negative going voltages right. So the total voltage here in terms of the phaser that I would have is given by $V_0^+e^{-j\beta z}+V_0^-e^{j\beta z}$ so the V_0^+ wave or the positive is a travelling wave will be going along the plus Z direction and this $V_0^-e^{j\beta z}$ will be going along minus Z direction okay, that would be the same case if I draw the plane here at this plane also it would be the same situation, this plane is all for the same situation it would exactly be the same situation at Z=0.

But what will happen to this V at Z=0 this would be equal to $V_0^+ + V_0^-$ that is the total voltage right that would be equal to $V_0^+ + V_0^-$ simply because Z=0 here, but at this voltage I do know that the total voltage is actually the load voltage V_L by the way I have used a very you know kind of a irregular shape of wires to connect this Z_L load right. The reason that I have done is to show that anything beyond this region where the load is connected the spatial extent of this load can be neglected.

In other words this region what we are considering is the lump region this is the transmission line region okay if this length of this region that I have considered or the shape of this region that I have considered has no bearing whatsoever for the relationship between voltages and currents. So to comeback at the load the lower voltage is V_L, and the load current is I_L, and the load voltage here appears from the left if you approach the load will be given by $V_0^+ + V_0^-$ which would then be equal to V_L okay.

It is fairly simple, because the voltages if they are different then there has to be some other element which is actually taking up that extra voltage and unfortunately that cannot happen here you can also confirm this one by applying a simple KV_L around the loop near the node if you apply KV_L you will see that from the left the voltage will be $V_0^+ + V_0^-$ and on the right hand side it would be V_L and those 2 voltages of the same.

Similarly the currents must also be the same in one of the exercises you must have derived what is the current phaser, and the current phaser is given by $I_0^+e^{-j\beta z}+I_0^-e^{j\beta z}$ you must also have derived I_0^+ and I_0^- to be V_0^+/z_0 and $–V_0^-/z_0$ with these phase factors going to one here because we are considering at Z=0 so the current phaser at Z=0 which will give you the total current on the transmission line as the current approaches the load will be given by $V_0^+/z_0 – V_0^-/z_0$ which will be equal to the load current I_L okay.

The load current can also be written as V_L/Z_L because I_L must be equal to V_L/Z_L. Now you have two equations okay one equation telling you that $V_0^+ + V_0^- = V_L$ and you have another equation which tells you that V_0^+/z_0 x V_0^-/z_0 must be equal to V_L/Z_L.

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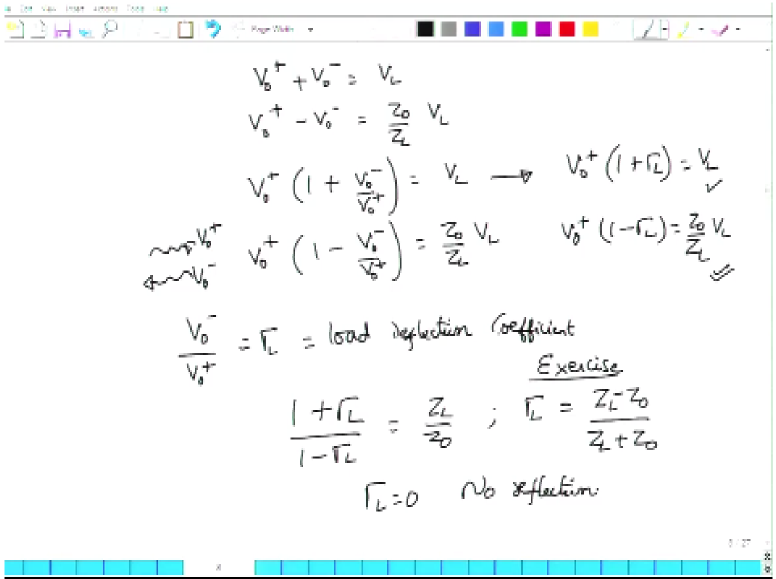

From these two you can actually reduce one of the constants you can show that or you can write this one as $V_0^+ + V_0^- = V_L$, $V_0^+ + V_0^- = Z_0/Z_L \times V_L$, I take V_0^+ as a common factor out and what I get here is $1+ V_0^- / V_0^+ = V_L$. In the next equation if I take this fellow common I get $1- V_0^- / V_0^+$ this would be equal to $Z_0/Z_L \times V_L$ okay. I can now divide this equation by this equation and all before doing that let me denote the ratio of the amplitudes V_0 - to V_0 + remember V_0 - is the wave which is getting reflected or wave which is actually travelling along the – Z direction V_0 + is the wave which is travelling along the + Z direction the ratio of this voltage is V0 - and V_0 + which represents the incident and the reflected voltages incident.

Voltage is V_0 + the reflected voltage which originated from the load and travels along the transmission line in the direction opposite to the incident voltage is called as the reflection mode reflected voltage whenever the load V_L will not be equal to the characteristic impedance of the transmission line Z_0 there will always be voltage reflection okay otherwise your KVL KCL would not be satisfied at the load okay so the reflected voltages are created necessarily as the mismatch between the load and the characteristic impedance.

As we will also see later on ok so this equation can be simplified to write V_0 + 1 + γL is equal to V_L okay where γL is the load reflection coefficient so this amount of reflected amplitude to the incident amplitude if called as the load reflection coefficient okay and this load reflection coefficient at the load point on a reflection coefficient at the load is denoted by γL and that is what I have used in this expression okay in this expression I have used that one I can simplify the second equation by getting this as $V0 + 1\ 1 - \gamma L$ this would be equal to $Z0$ / Z L into VL now I divide these two equations okay.

When I divide these two equations from the V_0 + will go away V_L will also go away from the equations and what you get is $1 + \gamma L$ by $1 - \gamma L$ is equal to $Z_L / Z0$ all right so when I divide the second equation from the first equation right I can rearrange this equation to solve for γL I will leave this as a simple exercise to you so let me also mention that this is an exercise that you can easily do ok invert the relationship to show that the load reflection coefficient γL is given by Z_L - Z_0 by Z_L + Z_0 okay please remember reflection coefficient tells you the ratio of the reflected wave to the incident wave if γL is equal to 0 the voltage that is getting reflected.

Will have an amplitude of 0which means that there is no reflection so γL equal to 0 indicates no reflection what is the nature of this γL well this just said that Z_L maybe complex correct.

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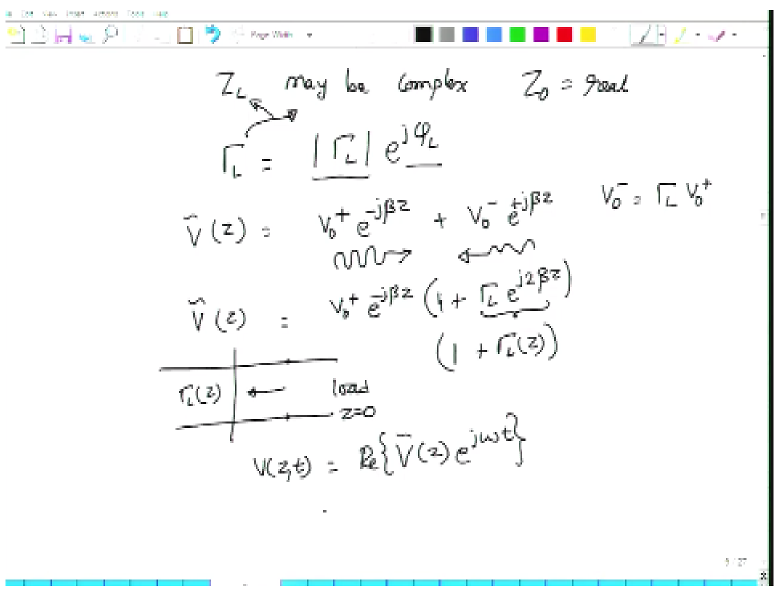

Because there could be some inductance and capacitance in the load if Z_L is purely a resistor then the reflection coefficient will always be real assuming that 0 is real we will assume that Z_0 is always real so in that case the reflection coefficient will be purely real otherwise it will in general be complex any complex number can be represented by its magnitude as well as its angle okay which we can write in terms of the magnitude and the phase angle we can write γL which may be complex because Z_0 Z_L is complex can be written in terms of its magnitude and a certain phase angle ψL.

Okay now once I know that this is the load deflection coefficient I can write down my total B of Z as V_0 +E $^{-j\beta z}$ which is really the wave which is or gating along the plus Z direction + V_0 – $^{ej\beta z}$ but I saw you pass +jβz but V_0 - can be written as γL into V0 + correct because the ratio of reflected to incident is this reflection coefficient voltage and this wave is propagating along - Z direction I can simplify and write this as V_0 + $^{ej\beta z}$ which I can remove outside no take as a common factor then I get 1 + γL $^{ej\beta z}$ β into Z further.

I can denote this γ L into $^{ej\beta z}$ as γL offset which will tell me how the reflection coefficient is very linked at different points on the transmission line so this is at the load point where Z is equal to 0 and at different points the value of γL will be changing to this γL if you add by 1 and multiply by V0 + e – $^{j\beta z}$ we you will get the voltage phase or the total voltage on the transmission line now how do I obtain the voltages function of Z and T from the phaser okay in order to do that one you simply multiply the phaser by $e^{j\omega t}$ factor remember this is the factor that you have dropped earlier to go from Z and T notation to the phase of notation.

Now you insert this $e^{j\omega t}$ and then take the real part of it okay so this is how you go from one domain to the other domain okay.

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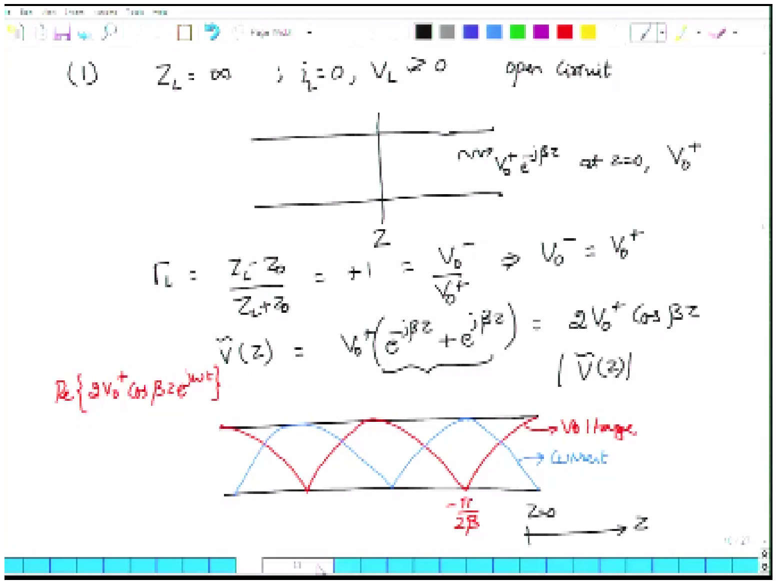

Now let us look at several different cases for γn let us consider the case where Z_L is equal to infinity when will I have Z_L equal to infinity when current is equal to 0 the load current is equal to 0 and the load voltage is some number greater than 0 right that is to say that I have no current flowing through that load terminals and the voltage is some value right this is clearly the case of an open circuit termination so an open circuit termination is simply the case where the transmission line ends are just left as it is.

What should be the total voltage at any point Z along the transmission line well first find out what is the load reflection coefficient what is the incident voltage incident voltage is V_0 + C – $^{j\beta z}$ at Z equal to 0 the incident voltage will have an amplitude of V_0 + but clearly this is the case where no current is flowing right so what should happen to the voltage that is incident entire voltages to reflect back you can see that by going to the reflection coefficient expression the load reflection coefficient expression γL is given by Z_L - Z_0 by Z_L + Z_0 this clearly gives you +1 okay γL of + 1 means full reflection.

and γ_L also turns out to be a real number here and remember what is γ_L this is nothing but $v0^-/vo^+$ which implies that the reflected voltage amplitude is exactly equal to the incident voltage amplitude, okay. Once that happens you can write down the total voltage phase at any point on the transmission line as $v_0^+e^{-j\beta z}$ I can take v_0^+ as a common factor out and then write this as $e^{+j\beta z}$.

I know that this is $e^{-jx}+e^{jx}$ kind of a expression so this will give me $2v_0$+cosβz okay. Now let us actually try and plot this $2v_0$+cosβz the magnitude of this one let us plot the magnitude of this voltage phase okay, so this is the point where that is equal to 0, z of course is increasing along this right-hand side, okay. How would this $2v_0+\cos\beta z$ magnitude look initially it would be maximum here with a value of $2v_0+$ and then it would go down to 0 at $-\pi/2\beta$ why would it go to $-\pi/2\beta$ because $\cos\beta$ into some value of z must be equal to $\pi/2$ for this fellow to go to 0.

And remember that z is negative as you go along the transmission line therefore this would be 0 at $-\pi/2\beta$ thereafter again it would be maximum it would be minimum acute maximum and it could go on like this what would happen to the current well you know that current has to start at 0 here because there is no current flowing in the open circuit region but then the current will reach its maximum at the same time where the voltage is reaching its minimum and then it would be out of phase with respect to the voltage by about 90˚.

So this is the current waveform and this is the voltage waveform that you would see on a terminated transmission line when the termination happens to be a open circuit, okay. Now this is the voltage phase what would be the voltage actual voltage as a function of z and t well let us look at that one all I need to do is to take this phasor and multiply this phasor by $e^{j\omega t}$ and then take the real part of it right in order to obtain the voltage as a function of z and t and when I do that what do I get.

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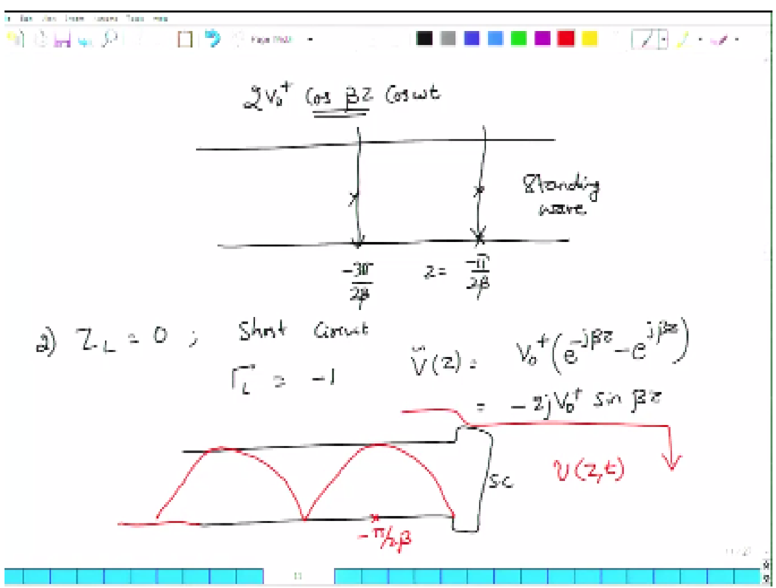

I get $2v_0+\cos\beta z$ right times $\cos\omega t$ okay, if you now go back to the transmission line and πck a point where $\cos\beta z=0$ right that is you know the case where we had considered at $-\pi/2\beta$ if you pick $z=-\pi/2\beta$ point you would see that no matter what time is changing the voltage at this point will always be at 0 right. Similarly the voltage will always be at 0 at after a π value here when this will be at $-3\pi/2\beta$ so these locations where the voltage is 0 is not changing they are always present no matter what time is changing okay.

We will contrast this case with a traveling wave in a very short way this wave in particular wave in this wave in a situation where this maxima or the minima is not changing with respect to time and hence kind of nothing is moving is called as a standing wave okay this is called as a standing wave, alright. Let us consider the second case I consider $Z_L=0$ which happens to be the short-circuit condition in the short-circuit case I know that the voltage would actually drop down to 0and the current would actually be maximum there okay.

But what would be my load reflection coefficient here γ_L will be equal to -1 right, because γ_L happens to be -1 in this case the total voltage phasor we have said that I get will be of the form $v0+e^{-j\beta z}-e^{j\beta Z}$ or simplifying by utilizing the relationship of trigonometric and complex exponential this would be $-2jv_0+\sin\beta Z$ and again if I plot the magnitude of this I see that the voltage would have dropped to four this is a short circuit right so I have to indicate that this is a short circuit I have indicated that here and when I look at what would happen to the magnitude of the voltage here it would be 0 and then it would reach to its maximum at a later time and then again go down to 0.

There would I get a maximum I would get a maximum at $-\pi/2\beta$ this number – $\pi/2\beta$ is coming up again this is the location of the first maximum when I terminate the transmission line with a short circuit previously this was the location of the first minima when I had terminated the transmission line with the open circuit I will leave this as an exercise for you to find out what is the voltage $v(z,t)$ okay from the phasor you should be able to find out what is $v(z,t)$ and also locate and find out whether you get our standing wave or you will get a traveling wave as a result of this what are the travelling goes.

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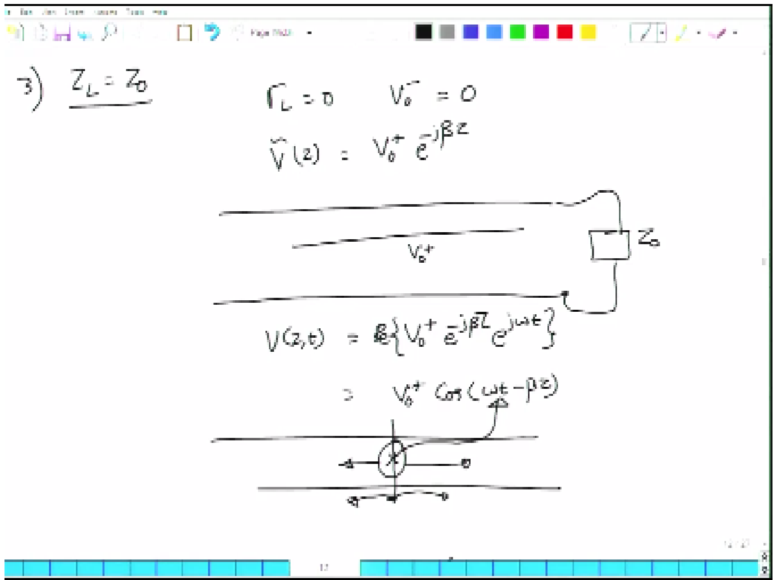

Let me go back to the situation where I consider Z_L=Z_0 okay, this is a very special case of termination where the load is terminated in the characteristic impedance of the transmission line when I do that what will happen to γ_L well γ_L is Z_L-Z_0 since Z_L=Z_0 β will be equal to 0 since γ_L=0 this is the third case where consistent so γ_L=0 simply implies v_0^- =0 which means there is absolutely no reflected wave and the voltage phasor is given by $v_0^+e^{-j\beta Z.}$
If I plot the magnitude what would happen this is something that is interesting so now again I am going to terminate this one by Z_0 right as I terminated here and if you look at what is happening to the magnitude the magnitude will be a constant v_0^+ that is kind of surprising that is surprising only because you go back to what is v(z,t).

For this case voltage and function of both certain t will be given by v_0^+ $e^{-j\beta Z.}e^{j\omega t}$ and then take the real part of it what you get is v_0^+ cosωt- βZ where is the minima located here or the maxima located here the maxima or minima that is located actually varies with respect to time correct this is a clear case where if I consider that at sometime t=0 the minimize located here at the next time instant the minima would have shifted either to the right or to the left it does not matter where it is but the point is it has shifted.

So this minima or the maxima will be moving along the transmission line and therefore they do not represent the stationary points along the transmission line they are actually moving along the transmission line and hence this is the case of a travelling wave, okay. Now before I consider other cases let me consider another situation okay.

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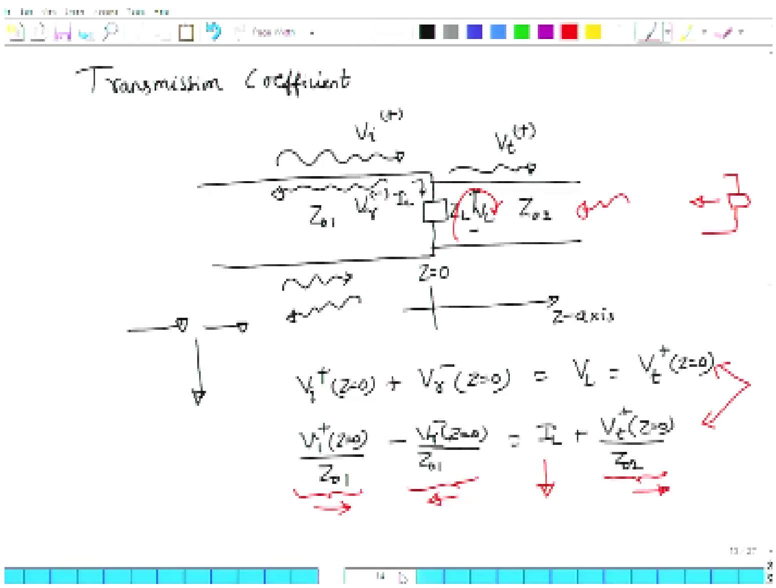

Wherein just describing everything in terms of the reflection coefficient will not be helpful I need to consider what is called as the transmission coefficient, okay. The transmission coefficient is best to consider what is called as the transmission coefficient okay the transmission coefficient is best obtained by considering this following scenario, so I have one transmission line over here I have some z0 1 as the characteristic you of this transmission line on this line I have both positive going voltage as well as negative going voltage I terminate this transmission line with the load ZL okay through which some current I L can be flowing okay.

And then to the right of the load I consider a second transmission line I connect a second transmission line of characteristic impedance z0 to so there is now a voltage what happens physically is that a voltage would arrive from the source hit the load and if ZL is different from zero one there will be reflection okay. So there will be a reflected voltage however there will be some voltage drop across the load and the remaining voltage would actually be propagated along the second transmission line okay.

You can also think of a current doing in the same way so I have a current incident current reflected some current passing through the load and the remaining current that is being transmitted so I have incident the I reflected voltage V R and transmitted voltage VT please remember we I will be plus V R will be minus VT will again be plus all this happening is happening at the load where I have kept the load at Z =0 and my z axis is increasing in this way.

Now I am interested in not only knowing how much the voltage is getting reflected but also how much the voltage is actually transmitted in order to do that I have to write couple of equations those are kind of very easy to write them from the left if I approach if I approach the transmission line Z equal to zero from the left then the total voltage will be VI + Z =0 + Vr- z= 0 correct this is the incident plus the reflected voltage on the transmission line at Z = 0, this should be equal to VL but I know from KVL that whatever the voltage that will be there at VL must also be the voltage of the transmitted voltage right on the second line that on the transmitted voltage in the second line at Z =0 okay from this KVL this is very clear.

So at bt because if you approach this load from the right hand side that voltage will be beauty plus X Z = 0 and that would be equal to the voltage on the load clearly there is no reflection here because there are node that we have kept you skip that infinity and it would take almost an infinite time to come back okay. What about the current waveforms well there will be current the current will be the total current corresponding to the incident which would be VI + Z = 0 / Z 0 1 - V R - / Z 01 at Z = 0 this is the incident current on this side there will be some current through the load and the remaining current will be transmitted right.

So this must be equal to the current through the load which is il +VT + Z = 0 / Z 02 okay clearly this is the current which is incident this is the current which is reflected this is the current through the load and this is the current that is transmitted okay. Now you can simplify these two equations I will not simplify them because this is mostly algebra and we do not really gain much of a physical intuition of there but if you simplify these 2 equations.

And then find out what is the ratio of V R - that is the reflected voltage at z equal to 0 – the ratio of the I + z = 0 this would be the reflection coefficient for the first transmission line or since there are only two transmission line it is kind of understood that this is reflection coefficient at the first transmission length therefore I will remove this subscript 1 I call this as γ L and this γ L is now given by Z L parallel - Z 0 1 by Z L parallel plus Z 0 1 in case you are wondering what is that L parallel you would actually see that once if you solve the two equations previously that I mentioned this is given by Z of the load ZL and the second transmission line Z 0- ZL + 0 - this is actually ZL parallel with Z 0 – okay.

Z 0 - is the characteristic impedance of the second transmission line and z0 1 of course the transmission line first transmission line characteristic impedance this is the overall reflection coefficient now, what is interesting to observe here is that the reflection coefficient does not only depend on the load ZL but also depends on the second transmission line characteristic impedance that we connect okay. If I am interested on what is the amount of the voltage that is actually transmitted onto the second transmission line I have to find out what is VT + at Z = 0 - what is the incident voltage that I have sent and this ratio of the transmitted voltage at the load to the incident voltage at the load is called as a transmission coefficient this is denoted by there is not a standard number or a symbol to denote this one I picked out okay.

Some people pick different symbol to denote this one so in my language tau represents the transmission coefficient and this value is given by 2 Z L parallel divided by ZL parallel plus Z 0 1 okay, you can show this again by some algebra that you would obtain by solving the previous equation so this is the transmission coefficient okay and we will later see what would happen to this transmission coefficient why would it be important when we consider the power that is carried away by these voltages to briefly tell you what it means there will be some incident power carried okay.

There would be some reflected power that is carried by the reflected voltage this is the incident power there will be some power that is dissipated in the load and there would be some power that would actually be carried away by the second transmission line in case I have the second transmission and if not then this would be that took 3 powers and the total power would actually be constant. So we will stop here and continue the solution of the transmission line in the next module thank you very much.

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