Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module-38
Magnetostatic Fields-II: Calculation of magnetic fields
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Hello and welcome to NPTEL MOOK on applied electromagnetic for engineers we will continue the discussion of calculating magnitude static fields for a few more cases because this is quite important the next example that we are going to consider is the field of a coaxial cable.

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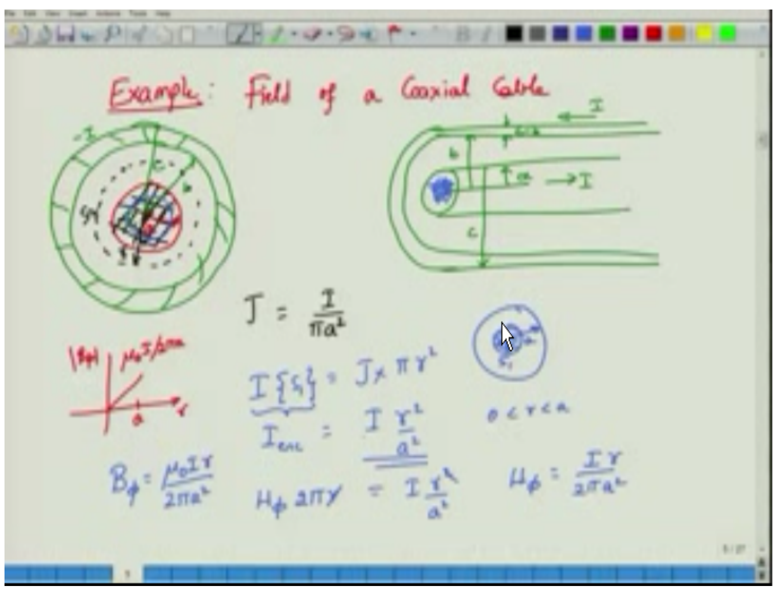

Now a coaxial cable is a very important transmission line as we have seen earlier and this has a structure that looks like this so there is a central conductor of some radius a and surrounded by this central conductor the central conductor actually carries a current of I whereas the return current is carried by the outer conductor and the outer conductor has a certain thickness of C-B because I am assuming that the outer conductor is again given by two conducting circles one having radius B and the other having radius of C.

So the thickness of the outer conductor is C - B we will assume that this current is uniformly distributed in the entire cross section so what we mean by that is that the current density J will be equal to I which is the current being carried divided by πa^2 for the inner conductor because this is the inner conductor having a radius of a and therefore an area of πa^2 the current density is always given by the current divided by the area.

So this is $I/\pi a^2$ okay now my goal is to determine the magnetic field for all values of our first I will consider the case where this value of R okay will be within the inner conductor that is I am going to consider this particular loop or this one in the longitudinal cross section which is at a radial distance of small r okay now notice that because this R is within inner conductor the limit of R is that this R can be 0 to a that is it can go from 0 to a what is the amount of current being carried by this particular cross section.

So if you now look at just the inner conductor this radius is a but then I am considering this particular cross section which is given by this one so this is just the inner conductor I know that from the entire the area or the entire inner conductor cross section the current density will be $I/\pi a^2$ and the current that comes out will be $I/\pi a^2$ so you can again go back to this example this is the inner conductor the current coming out uniform and has a current density of $I/\pi a^2$ whereas I am interested only in the current through this particular radius R or the cross-section of radius R how much current do I actually get.

Well you can easily see that if this is the current density then the current that is coming out of this you know cross section let us say S1 okay I threw S1 will be equal to current density times whatever the area of the cross section S1 and that cross section area is πr^2 right therefore this would be given by Ir^2/a^2 as the total current that is coming out from the cross section s1 where R can go from 0 to e okay.

Now this is the right-hand side for our amperes law what about the left hand side of the ampere law well we have in the previous module seen that if I have a wire you know carrying a cross section or if this is a conductor then the magnetic field will be circulating this particular wire right so the magnetic field will be along the Ø direction only thing that you now have to understand is that you are considering within the inner conductor the radial distance R but on that radial distance R the value of H will be constant H will of course be along the Ø direction.

But the corresponding line integration will be equal to 2πr this will be equal to the current coming out from the cross section s1 so this would be Ir^2/a^2 clearly are on this side cancels out with R on the other side and HØ is given by $I/2\pi a^2 * r$ okay and for future reference we will also write BØ, BØ is given by $\mu_0 Ir/ 2\pi a^2$ and if I sketch this BØ how does BØ look as a function of R well at R equal to 0magnetic field will be 0.

Because there is no cross-section out there so there is no scope of having some current coming out so this is 0 but at R equal to a it reaches a maximum value of $\mu_0 Ir/ 2\pi a$ so it increases linearly and reaches a value of $\mu_0 Ir/ 2\pi a$ at R equal to a beyond that what happens we will have to calculate that now when I consider the radius R to be outside the inner conductor but within the outer conductor so this is my new value of R now I know that the total current enclosed by this cross section which we will call as cross section s2 will be equal to I itself correct because the entire inner conductor is now contributing to the current.

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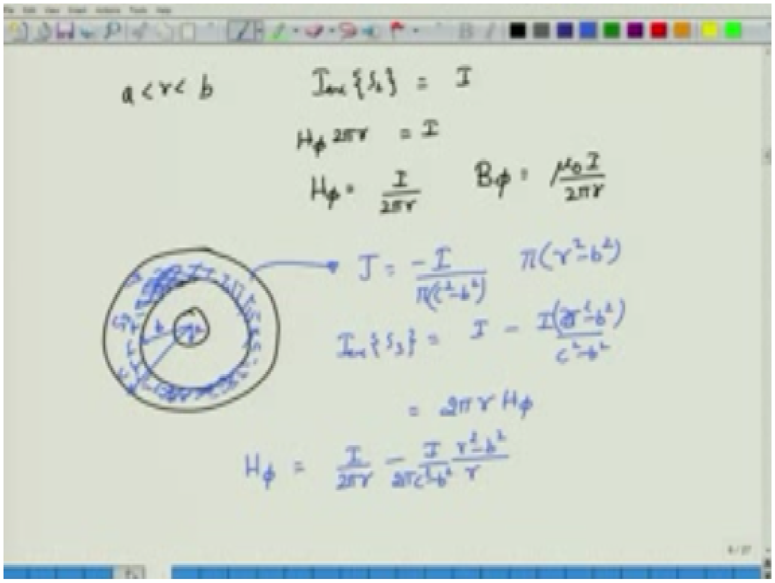

So when R is between A to B the total current enclosed of the cross section s2 will be equal to the current of the inner conductor I okay the left-hand side doesn't really change so you still have HØ 2πr that must be equal to I and HØ is given by I/2πr okay now you see that the magnetic field is actually inversely falling off with respect to R for future reference again BØ will be equal to μ_0I/2πr so at R=a you will have Bπ(μ0) I/2πa therefore there is a nice continuity out there.

So this was μ0 I/2πa and from there onwards it actually starts to fall off, when you go to B right, it falls off inversely I have not drawn it very nicely but when you go to B the value will be μ0 I/2πB okay, that is the value for Dπ. Now we are we have only one more cross-section to consider okay, and that cross-section happens to be within B but less than C, so this is the cross-section that I am going to consider or this is the radial distance that I am going to consider.

The corresponding cross section we will call this as s3, so this inner radius is A this is B and this outside conductor radius is C okay, now what is the current density. Now this one actually has two components one there is a current contribution from the inner conductor because now that you are on the outside there is a contribution of the current I and the total current enclosed will be equal to I.

However the current enclosed over this cross-section will have you know in addition to the inner current there will also be a contribution from the current being carried by the outer conductor, so part of the outer conductor current will also be present here okay, and what is that part of the outer current well first we calculate what is the outer conduction current density is J=-I/ - IπC^2-B^2

why is there a –π, I mean - sign to the current well because this is the return current that we are considering.

So on the inner conductor the current is I whereas on the outer conductor the current is -I therefore this is the current density that you are going to get, but what is the cross sectional area of this hatched thing that we have talked about right, so in this hatched cross section that is the cross sectional area is given by $\pi r^2 - b^2$ therefore the total current enclosed from the cross section s3 will be the inner current or the current contributed by the inner conductor minus whatever the current that is contributed actually plus but in this case I is minus therefore you get $Ir^2 - b^2/c^2 - b^2$ okay.

This is the current of course this current must be equal to then 2πr where again r is the radial distance of corresponding to the cross section s 3 that we are considering times HΦ so HΦ will be equal to I/2πr which is the contribution from the inner conductor $-Ir^2 - b^2$ so $I/c^2 - b^2$ well there is also 2π over there and then you have divided by r okay, it is a little complicated expression but if you now look at what happens to HΦ at c that is at r=c you will see that this would be $c^2 - b^2$ that would cancel out here which would again cancel out with I/2πr and you will actually get this value equal to 0.

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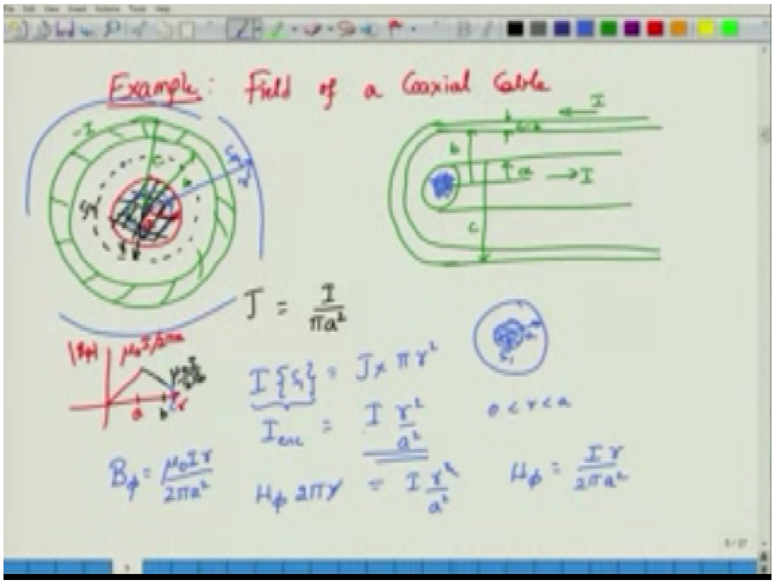

So HΦ will change in some manner which I not plotting but when you go to c the magnetic field will be equal to 0, because if you go to any region outside right so if you go to any region outside clearly the total current contributed will be equal to the plus current I by the inner conductor and the return current of the outer conductor which is -I therefore there is no current enclosed if you go to a cross-section s4 which is at a radial distance r greater than c, so for r greater than c no field.

In fact this is one of the reasons why there is much a nice thing about coaxial cable because all the fields are confined only within the structure itself there are no fields on the outside this is all true as long as you have a perfect coaxial conductor, but unfortunately in real world you do not have a perfect coaxial conductor therefore there will be some amount of magnetic field present outside some amount of electric field will also be present outside, in order to protect you know the cable from all these external ones you actually jacket this coaxial cable.

So a coaxial cable will have an inner conductor outer conductor and a jacket the medium in between will be filled by some dielectric or an insulating medium okay, this completes our calculation of magneto static fields using amperes law or biot-savart law, the previous problem can also be solved by biot-savart so it is a little more difficult compared to the amperes law so I would not suggest you do that one wherever possible take advantage of symmetry, wherever possible take advantage of amperes law. But even these amperes law and you know the biot-savart law kind of are very difficult to handle when the situation goes slightly difficult what do I mean by difficult scenario here is an example.

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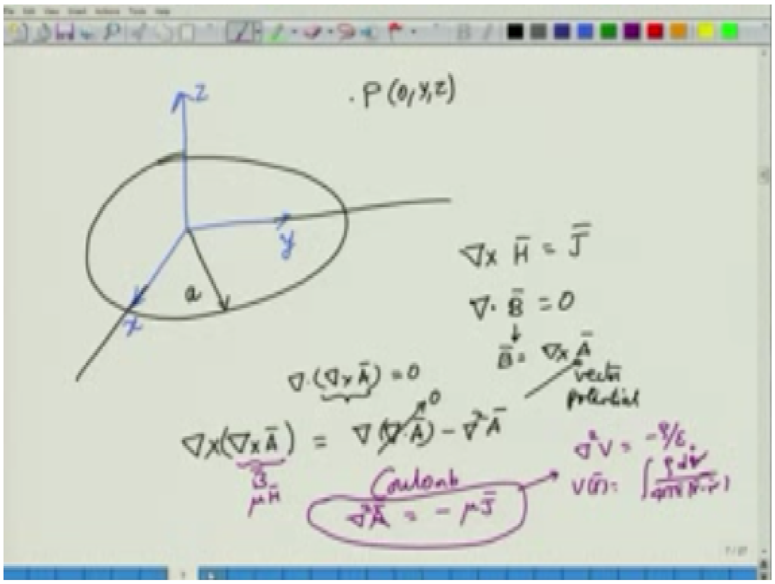

Suppose I consider you know current but not in the form of a wire, but in the form of a loop okay, and I put this current carrying loop in z=0 plane that is I actually consider the current in the z=0 plane of course this is the y-axis this is the x-axis you have to pardon the way I have drawn this one. Let us say the radius of this wire is a and I want the field to be calculated at this point P which will be 0,y,z why am I taking this a 0,y and z with a little bit of convincing yourself you can see that because of symmetry it is just sufficient for us to calculate the field at any constant value of x and what better value of constant of X then X equal to 0 to simplify our calculations a little bit.

So I want the field P here now amperes law is very difficult to apply in this case biot-savart law is even more difficult to apply in this case therefore we need some other means of calculating the magnetic field and it is here that we introduce another quantity called as a vector potential a, what is the vector potential we know ∇xH is given by J vector okay, but ∇.B is always equal to 0.

Now when ∇.B is 0 I can write this B as a curl of some other vector okay, and this some other vector a is called as the vector potential a or he magnetic vector potential a. Why is this true, because ∇.∇xa will always be equal to 0, therefore if I write the B field in terms of another field called a okay which is called as the vector potential then first I calculate the potential a which will be reasonably simple to calculate reasonably I am not saying that it will always be but reasonably simple from there we go back and infer what is the value of B, okay.

How do I relate all these things so of course since B is μH, H will be ∇xa/μ but you do not really need to worry about that, now consider what happens to ∇ x∇ x a right, that is I know what is ∇xa which is B and I take the curl of this B itself so I get ∇ x ∇x a and one of the vector identities is that this is ∇ of ∇. a -∇^2a and we are complete liberty to specify what is this ∇.a, we specify the simplest case of ∇.a=0 this incidentally is called as the Coulomb Gauge okay, Gauge being simple meter kind of a thing and ∇ x a is nothing but B, B is nothing but μ into H in general μ is constant we are considering in this particular case.

So what I have is ∇ x H I know which is J and this right hand side is -∇^2a therefore ∇^2a vector is equal to –μJ. If you look at this equation, this equation should remind you of poisons equations so you had ∇^2V=- ρ/ε only thing is that this V was a scalar in that particular case here this vector a is a vector of course okay. However, for each component of a you can simplify the equation to make it look like the poisons equation type and we know the solutions of this poison equation right, so V at any point R is given by integral of ρdv' which was the volume distribution divided by 4πε R- R' right, magnitude.

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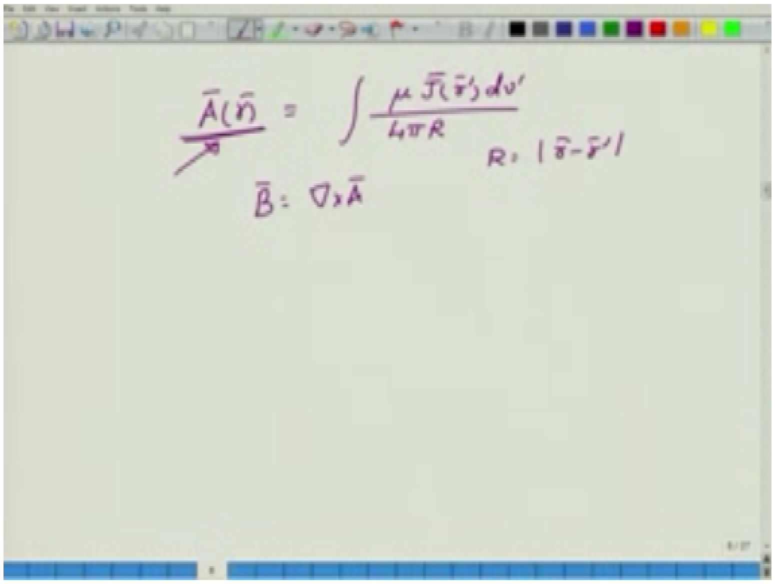

In a similar way you can write A at any point r as the integral over the volume distribution of the current so instead of 1/ε you will now have μ so μJ at the field point and the source point R' integrated over the volume integral divided by 4πr, where r is the magnitude of r-r' vector ,why is this vector potential and you know a easier method than biot-savart law well one important thing is that if J is varying in a particular direction the magnetic vector potential will also vary in the same direction or it will have the same direction as J vector whereas the B field will have a direction that should be perpendicular to this J as well as the point which connects from the source to the point where you are evaluating the field.

So the cross product is kind of eliminated now that that is eliminated of course in the form of the vector potential calculation then the cross that makes its reentry when you calculate B in terms of ∇ x a. However, if I know a which is reasonably simple to calculate than B okay, reasonably not extremely easy so if I calculate a you know which will have the same direction of J then follow it up by calculating the curl then this is an easier procedure okay, and we can also show that the calculation of vector potential in many cases especially in antennas is more straightforward okay, than calculating the B field. So whatever that we are developing will actually be very, very useful when we discuss antennas.

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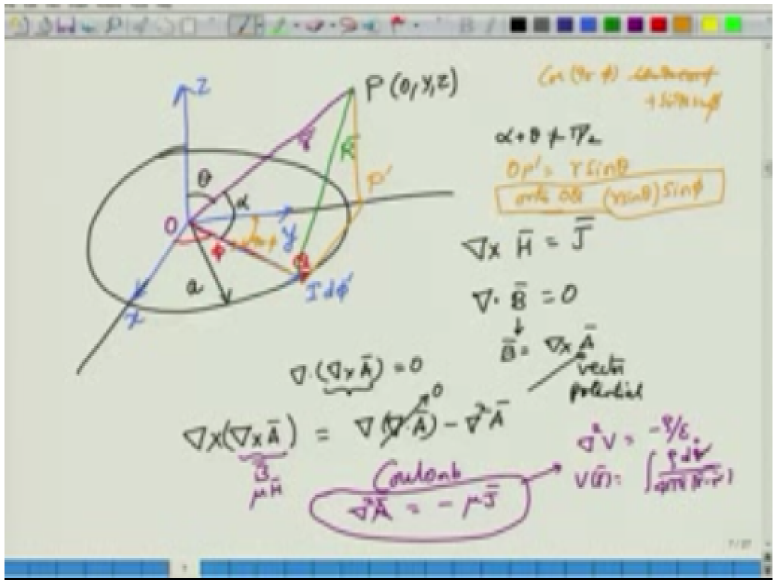

So let us go back to that loop that we were considering at this point I am going to calculate the vector potential correct, so let us say first I need to write down the position vector of this one of the point which we will call as O of the point P which is the line OP so let us label this position vector as R I know that this is in the spherical coordinates along the radial direction and we will now assume that there is a current distribution which I am looking at so this is the current distribution that I am looking at which will be along the Φ direction right.

So it would be IdΦ' where Φ is the angle that this is making so this is the angle Φ so from the x axis whatever the angle that this line OQ makes Q being the current element okay, so this would be 4Q but I am not interested in OQ or OP what I am interested is actually in this vector R okay, which is the vector from the source point to the field point okay. For our own sake we already know that this angle from the z-axis to the point P will always for the radial vector will be equal to θ and we will call the angle between OP and OQ by α.

And in general this α+θ will not be equal to π/2 what is this, whyi s it not equal to π/2 let us go back to this kind of a simple picture over here okay, so I have this picture this is my z axis okay so this is the z axis this is the loop this is the current that I am considering now imagine that I have no I am considering the current and I am evaluating the field at the tip of my index finger okay, what is the angle so this point you can consider this as the origin so if I had one more finger I could have pointed it like this and you know are pointed like this and then shown you that the angle made by this vector which is from the origin to this one is the angle θ okay, whereas this particular line is the capital R vector okay that angle between that one and the position vector okay.

So this is the position vector so the angle between these two so you can see this particular angle right so shown by my left hand thumb this angle is α. Unless I am actually on the y axis at which point you know this α will be in such a way that this angle α plus the angle θ will be equal to $\pi/2$ in general they are not okay, so in general α and this one are not in the same direction. However I can draw a projection okay I can project this P on to the y-axis and this amount of projection that you are going to get let us call this as some P' so OP' is certainly equal to R sinθ, where R is the magnitude of the vector OP okay.

So this OP' is r sinθ but if I know further project so this is the unit vector let us say along the OQ so if I further project OP' on to OQ line okay, that is the OQ line is this one so if I further project it on to this one the corresponding projection that I am going to obtain will be whatever the magnitude of OP' which would be r sinθ times this angle okay, this angle is now in the XZ plane given by 90-Φ therefore this would be cos of 90-Φ the magnitude of vector of OP' the magnitude of the unit vector OQ which will be equal to 1.

The angle between OQ and OP', OP' is on the y axis so this cos of 90-Φ cos of this angle times this one will be equal to r sin θ sorry, this is sinπ, why is it sinπ because cos of 90- Φ is what you are looking at so cos 90-Φ is cos 90, cosΦ +sin 90 sinΦ and we know that thisquantity is 0 so this is sinΦ okay, so this is actually important for us to note later on okay, we will come back to that expression.

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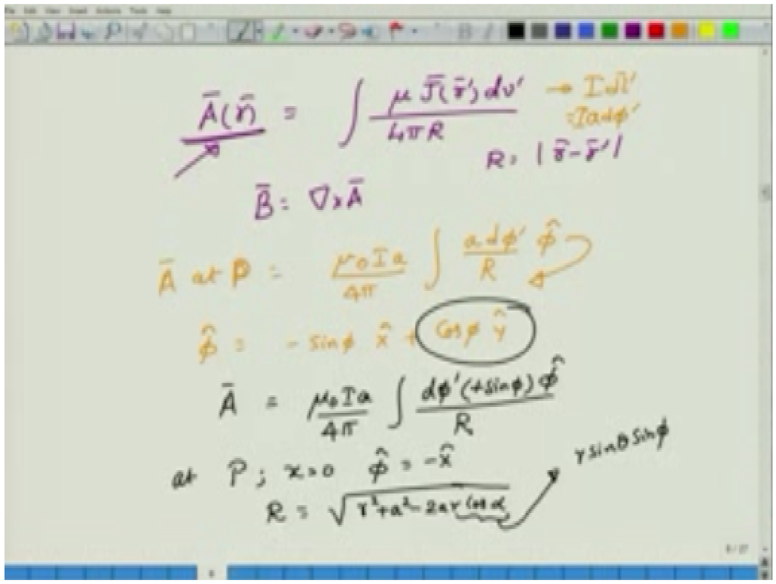

First write down what is A here, A at Point P is given by $\mu_0 Ia/4\pi$ these are the constants which I am pulling out of the integral so there is because I am assuming this one is μ_0 and the current is now not in terms of J but it is in terms of Idl' because I am considering the filamentary current there is no integration over the volume the integration is over only the line and then we have already seen that this Idl'=I =adΦ' where adΦ=dl so multiplying it by I, I get this one okay, so I can write this $\mu_0 Ia/4\pi$ and then I have on to this particular expression which is adΦ'/R where R is the distance that we have already talked about.

And what is the direction for this a the direction for A is actually the direction of the current which is along the Φ axis, but a unit vector Φ can be written in terms of x and y coordinates as Φ=-sinΦxˆ- sinΦ' because I to represent this one as not Φ but Φ' or we will simply represent this as Φ itself kind of simplify Φ's are I know this one so this would be - sin Φxˆ+cos Φyˆ okay, so I know that conversion from the vector Φ in the cylindrical coordinates or in the spherical coordinates onto the x and y coordinates okay.

Now here is an important point let us go back to this loop okay, there was one current element along this dΦ' located at Point Q if I consider symmetric point okay, about y this is not a nice symmetric point but if I consider a symmetric point about y there will be a current element going in this way correct, let us call this point as some Q' point and what is the direction of the current element along this Q' point that would be along the Φ direction, but if you now look at the two lines that we have or the two contributions of this Q and Q' current elements one will be in this direction the other one will be along this direction okay.

And what is the resultant of these two the resultant will be along this direction which happens to be along -X direction okay, so the resultant field is only along the – X direction and therefore we do not have to worry about the y component of the field, because the field will be along the $-\hat{x}$ direction okay, so I can go back and write this a as $\mu_0 I$ small a is a constant so I can pull that out divided by 4π I still have an integral of $d\Phi'$ okay, and $-\sin\Phi$ and this was along the $\hat{\Phi}$ direction divided by R okay.

But at point P, x=0 right, so when x=0 the corresponding vector Φ you know unit vector $\hat{\Phi}$ will be equal to $-\hat{x}$ therefore I can substitute for $\hat{x}=-\hat{\Phi}$ in this expression replace the minus sign with the plus sign, okay. So now I have the potential a written on this one from this triangle which is POQ where the angle between OP and OQ is α I can write down from the law of cosines you have to remember your geometry for this one I can write down the law from the law of cosines as $R=\sqrt{r^2+a^2-2a\,r\cos\alpha}$ okay, and r cos α is actually a projection of r onto the line right.

So this r cos α in fact should be equal to r sinθ, r cos α is the projection of this vector r or the vector OP onto the line OQ and this correction of OP onto the line OQ precisely what we calculated by first projecting OP onto OP' and then projecting OP' onto Q which gave us r sinθ sinΦ therefore this r cos α will be equal to r sinθ sinΦ okay, so you can substitute for that one and instead of cos α you can write this cos α sinθ sinΦ okay.

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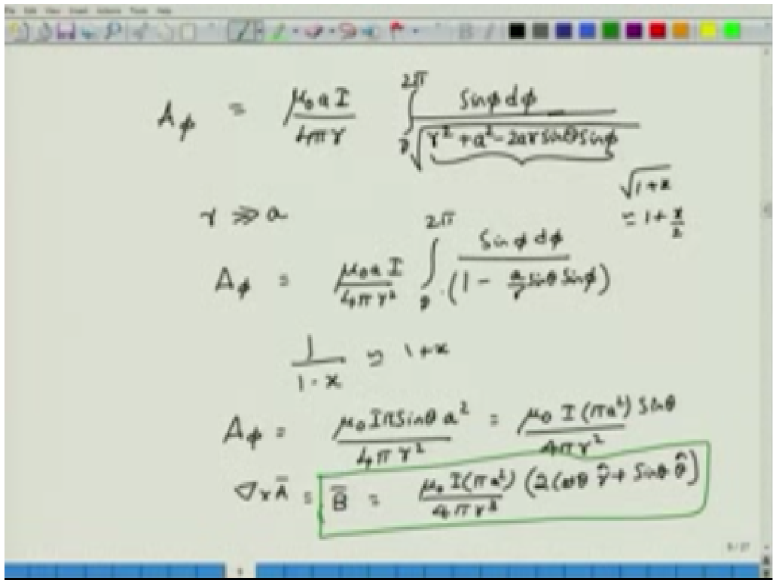

So what happens to A, with A will be along the Φ direction so we now know that a is along the Φ direction at the point P and this would be given by μ0aI is a constant divided by 4πr integral 0 to 2π sinΦdΦ remember this was in the numerator in the denominator I have $\sqrt{r^2+a^2-2ar \sin\theta \sin\Phi}$ okay, under root this in general is very difficult to evaluate but we make an assumption that r is much larger than either is we are at a very, very far away distance from the loop and in that limit.

I can simplify this expression I can remove a^2 and I can simplify this expression by removing r^2 outside the square root and I can rewrite this as $\mu 0aI/4\pi r^2$ okay, because r came out and integral 0t o 2π sinΦdΦ/1-a/rsinθ sinΦ 2a/r sinθ sinΦ under root but if I know that 1 plus square root of from binomial theorem √1+x is approximately 1+x/2 therefore if I remove the square root then this would be this 2 will go away because there will be a division by 2 there so it would be 1-a/r sinθ sinΦ I also know that 1/1-x is approximately 1+x okay, when x is very small and in this case t is small.

So I can write aΦ as μ0I okay, sinθ $a^2/4\pi r^2$ or there will be a π here okay so I or I can write this as μ0Iπ a^2 which is the area of the loop times $\sin\theta/4\pi r^2$ you can now apply ∇ x a to calculate B and if you do that you will see this can be written as $\mu 0I\pi a^2/4\pi r^3$ you have to actually carry out this I will leave it as an exercise for you, so times 2 cosθ r^+sinθ along θ^ so this expression for magnetic field is a very important expression or the expression for the vector potential is very important when we discuss loop antennas at the end of the course.

I know I have left a few steps as exercises for you this problem is slightly difficult but I wanted you to understand, I want you to understand this problem how to solve it because these results are very useful for our antenna analysis. And if you look at the magnetic vector potential B that you have calculated and you go back and calculate the electric field of a dipole in the far away region the fields will actually be identical except for the constant factor so the field configuration of an electric dipole which is just a short you know two charges separated +Q and -Q by a certain distance is exactly equal to a dipole which is a small loop okay, and at a very far away distance the field concentrations are equal to each other.

Therefore many loop antennas can be analyzed by the equivalent electric field analysis of the dipole analogy I mean of the dipoles with this we stop at this module, thank you very much.

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