Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 37
Magnetic fields – I: Biot – Savart Law

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Hello and welcome to NPTEL mook on applied Electromagnetic for engineers in this module we will discuss calculating magnet of static fields just as we know that charge distributions that do not vary with respect to time can generate electric fields similarly current distributions that do not vary with time can generate magnetic fields, if you go back to Maxwell's equations.

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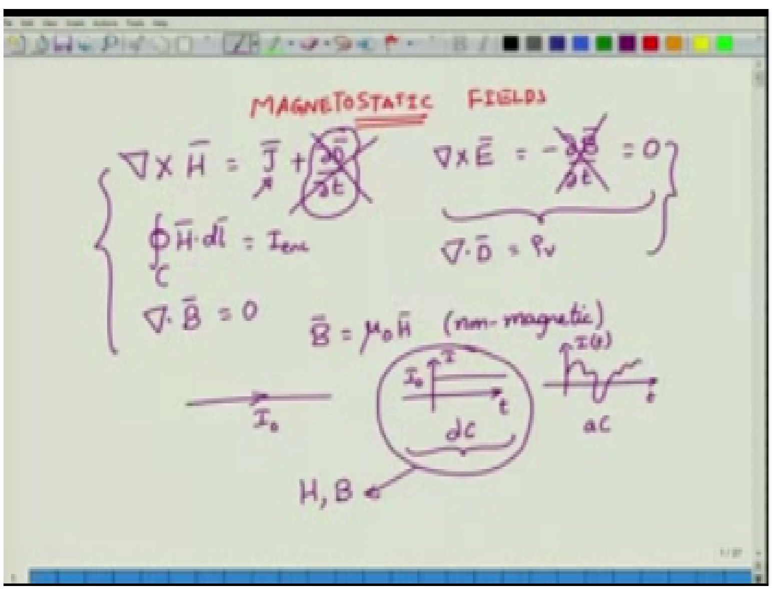

Then there are two equations which tell you how the magnetic field in space is related to the electric field for example you have the ampere Maxwell law that is curl of the magnetic field H is given by the conduction current or the current distribution J + the displacement current ∇d by ∂t similarly you have $\partial \times$ E which tells you how the electric field is changing with respect to space and time being related to the magnetic flux density vector B okay but we are going to consider a scenario where the current is assumed to be stationary that is to say we assume the current to be steady or DC.

In such scenario there is no time variation the current is constant with respect to time and there is no displacement current as a result of it similarly the magnetic field is not changing with respect to time and therefore ∂ x E will be equal to 0 in fact ∂ x E= 0 together with ∂ . d = ρv constitutes all the laws that we call as electrostatic release these laws are sufficient for us to tell what is happening or how to calculate the electrostatic fields because this displacement current is removed what we have for the curl of the magnetic field is only the conduction current J and this conduction current distribution if it is known will allow you to calculate the magnetic field H okay.

So this is the differential form of the Maxwell's equation or the ampere Maxwell know there is an integral version of which you should all be familiar with by now which tells you that integral of the magnetic field H over a closed curve must be equal to the total current that is enclosed together with this curl equation if we supplement the equation with ∂ . B = 0 and we considered a medium where B is always equal to some μ_0 x H this is the so-called non magnetic medium okay.

So if we specialize only to the nonmagnetic medium then B = μ_0 x H and the divergence of B is always equal to 0, so together these two laws or these two equations can be used to compute magneto static fields please note that if we consider a current in an extremely thin wire or a filamentary current as we would call it then the current is assumed to be not varying with time that is if you plot the value of the current carried by this particular wire then it would have some value whatever the value net state is I_0 the wire is carrying a current of I_0 this value will not change okay.

If you consider so that is why this is called as a direct current or a steady current on the other hand if the current work to change like this then this particular current which would be changing with respect to time would be called as the AC current okay sinusoidal currents are a very special type of AC, so in this module and the next few modules we are only going to look at the DC current is a slight misnomer it is direct current but that is okay the abbreviation of DC and AC are so widely y used that we call DC current and AC current although strictly speaking we should simply be calling it as DC and AC.

Anyway we consider that the current distribution is known and from this current distribution we want to find the magnetic field H and sometimes also find the magnetic flux density D.

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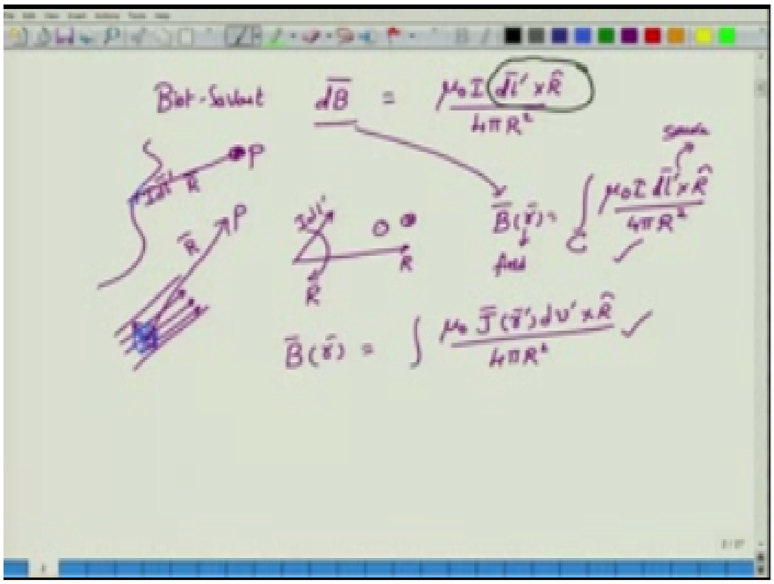

In addition to this ampere law there is another law which is called as Biot-Savart law okay which was discovered almost simultaneously as amperes law and this Biot-Savart law is also sometimes useful it tells you that the magnetic field because suppose if this is the wire that is carrying a current and we assume that this particular current has a certain value of current I and this DL` is this short segment of the wire which I have marked in the blue line so I do not know whether you will be able to see this but if you are able to see this then this blue wire tells you what is the short current element DL` and carrying a current of I the element will carry a total current of IdL I mean carries a current of I.

So this magnetic flux density DB at any point so this is the point at which you are trying to calculate the magnetic flux sorry magnetic flux density DB then this DB will be given by η 0 I am assuming that the space we are considering is free space or non-magnetic therefore μ is a constant μ_0 IdL` x R^ / 4 Π R^2/ R is the distance from the current element to the point P so this vector R is the vector from the current element to the point at which you are evaluating the field.

And the condition that you are choosing here is that this particular field DB that you calculate at the point P will have to be perpendicular to the current element as well as the vector R so in fact if you consider this current element and the vector R okay then the magnetic field will have to be perpendicular either it is coming out of this board or it is going into the board whether it is coming out or going into the board is decided by the right hand rule and the right hand rule tells you that if you curl your fingers starting from the current direction to the vector R then the magnetic field must follow your right hand rule.

So to show you that if this is the current carrying element then if I enclose write the current by my fingers then the thumb will be pointing along the direction of the current and the magnetic field we will be n circling so at a particular point for example if this is the current element and this is the point when the magnetic field must be perpendicular to the magnetic field you have to imagine as coming out and for the plane that will be formed by this current my thumb and this pointing index finger if you take that plane then the magnetic field will have to be perpendicular to it.

So I cannot show you this perpendicular thing but this is how the magnetic field should be looking like, so this is the right-hand rule which allows you to determine the direction of DB the magnitude of DB can be obtained by simply taking the magnitude of this expression okay the expression for Biot-Savart law that we have written is applicable only to what is called as filamentary current that is you have a wire which is almost having no cross-section therefore this is a thin wire that we considered but in general if you for example consider micro strip line on a printed circuit board then the width of the micro strip line cannot be neglected.

So the current will be flowing here no problem but then the width of the micro strip line cannot be neglected in this case it is more common to talk about the volumetric current density vector J okay, and the total current that would be contained in a particular this one will be given by integration of this volumetric current distribution J over the entire volume where this particular J is defined, so the corresponding expression for this DB or rather since we know that a small wire segment cannot exist the correct form for the Biot-Savart law although this is okay for the filamentary or an extremely small current element IdL`.

But the magnetic field that you need to calculate must actually be obtained by integrating this particular expression, so you will have to go over the entire curve through which the current is flowing and once you do that you obtain the full magnetic field at any point R, so if you remember this is what we called as the field point these primes are used to denote what is called as the source region or the source point, so as I was saying in the case of a micro strip line on a printed circuit board you do not have a thin wire approximation because there is a considerable distribution.

And then the proper way of specifying its volumetric current distribution J in that case B(R) needs to be obtained as a volume integral so this will be μ_0 J so that would be J at the source point which would be J(R)` and what is the volume element, so if you imagine that there is actually a small volume that we are going to consider here okay so this small volume shown in the blue line through which the J vectors are all coming out that corresponds to the current element now.

So you do not have the current element in the form of a line but you have it in the form of a volume times R^ / 4Π R^2 where R is again the distance R from the volume element where the source is located to the point where you are actually calculating the magnetic field, so this is the more general form of Biot-Savart law we will be using this form or we will be using the filamentary current form depending on the problem that we are going to calculate okay, so with all of this said let us now proceed to calculate magneto static fields for a few simple and important cases.

In some sense calculating magnetic field is slightly difficult okay then the corresponding electric field calculations why is it difficult the difficulty comes because you have across product over here okay because of this cross product you have your signs and you are the directions will be slightly non-intuitive okay therefore you have to pay particular attention to how you define this dL` that is the source distribution or the source line element sand calculating this particular unit vectors R from the source to the field point okay.

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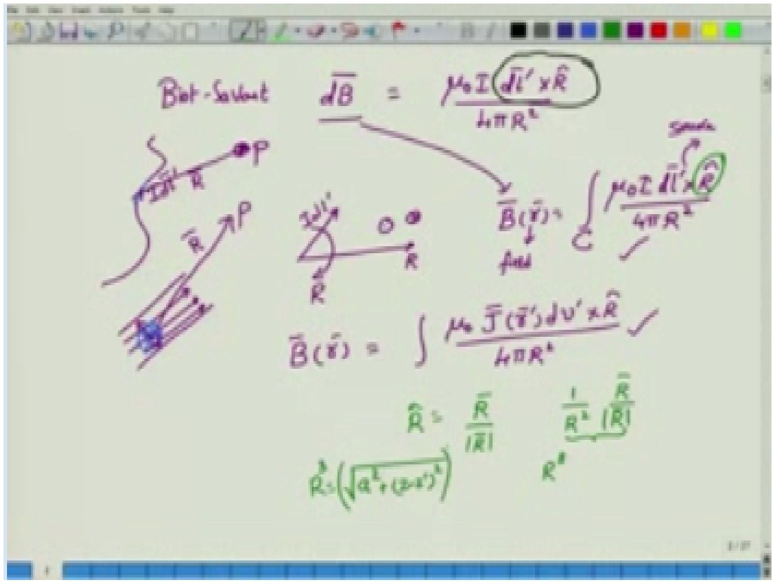

So we begin by a simple example of an infinite line charge okay so this is my XY & Z coordinate system let us assume that there is wire which is actually going all the way to ∞ okay for clarity I show that this wire has some cross-section but in what I am going to assume is that this cross-section is actually zero, so the wire extends from - ∞ to + ∞ we assume that the medium is some μ_o okay μ could be μ_0 or you can have a medium with a constant permeability so it will be μR x μ_0 either way we can represent this one by just giving you μ okay.

And then I will tell you in the medium which is there we use μ_o or if I give you a magnetic medium with some scalar magnetic permeability μR then you can replace μwith μR and μ_0 okay said now having said this let us now try to calculate the field, so to calculate the field let us say I take the field at this particular point okay which is shown by this red color dot okay there is a certain vector which is the position vector so at this point we will Mark has P, so the position vector that starts from the origin and goes all the way to the point P is the field position vector R okay.

However I am supposed to consider the current element at this particular point okay on the wire and let us consider the ∞ the element length to be dz ` and it is located at a height of z` from z= 0 plane okay, so my source point can actually be written as r` is given by z ` z^ okay that is I am assuming that dz` is actually very small whereas the position vector R which joins or which is the line that is joining the origin o to the point P is given by ar^ + zz^ what is this a and z well you know if I consider the projection of this line OP onto the z-axis okay and consider that this particular projection is constant and imagine that there is actually a cylinder sitting up here okay.

So this is a cylinder then the radius of this cylinder is taken to ba so a is just a point which I am considering at P which is projected onto the z-axis or the height above the z= 0 plane where I consider this point okay I am supposed to consider the radius of this one so this height OP projected onto the z axis is the z value that I am considering the radius which I consider is the radius of the cylinder which would be about a okay.

So the radius of the cylinder that we consider will be a and the height over which this one would be present on the z axis will be z so therefore the position vector R okay we have to define so we have to mark the position vector R over here, so let me try and mark it from this would be the position vector the green vector will be the position vector because that is what goes from the origin dz to the point P.

So we are assuming that we have a general point P where the radius of this cylinder you know you have to imagine that there is a cylinder over there with a radius of a and the projection of P onto the z axis will tell you what is the corresponding z coordinate okay, so we are assuming that this is the point where we are going to calculate the field, so we were considering this radial distance R from the source point of the wire to the point where I am calculating the field, so this R now is of course defined as r -r` vector correct.

So this is the position vector of the field and this is the position vector of the source and I know these two values in this cylindrical coordinate system so I can write this as a r + z -z` x z^ now we have all the tools that is necessary to apply Biot-Savart law and to calculate the magnetic field dB at this point which of course will be in the direction of ϕ as you will very soon see so this is μ_0I / 4Π which is the constraint so μ_0I / 4Π okay times what is the current element that we were considering I x dz` right.

So this is Idz` and the current element itself is located a on the z direction so this would be dz` z^ x ar^ +z- z^ times z so this vector is your capital R vector okay of course that is not so that is the vector that that is there divided by so this entire thing divided by r^2 is the magnitude of this particular R and that is given by $(a^2 +(z -z`)^2)^{3/2}$ 2 why have I put to the power of 3/ 2 because if I you know take this r^2 and then I want only the unit vector along r okay, so if you go back to this one this is the unit vector that I am trying to find out and the unit vector $\hat{R}$ will be equal to the original vector R divided by its magnitude.

And we have $1/R^2$ in the bio Biot-Savart law right multiplied by $\hat{R}$ so this $1/R^2 \times \hat{R}$ this $\hat{R}$ can be written as the complete vector R the complete vector R divided by the magnitude of the vector R and this when you multiply is you get R^3 since R itself is given by $(a^2 + z - z`)^2$ under root taking this R^3 you take this particular quantity on to the right hand side to the 3.

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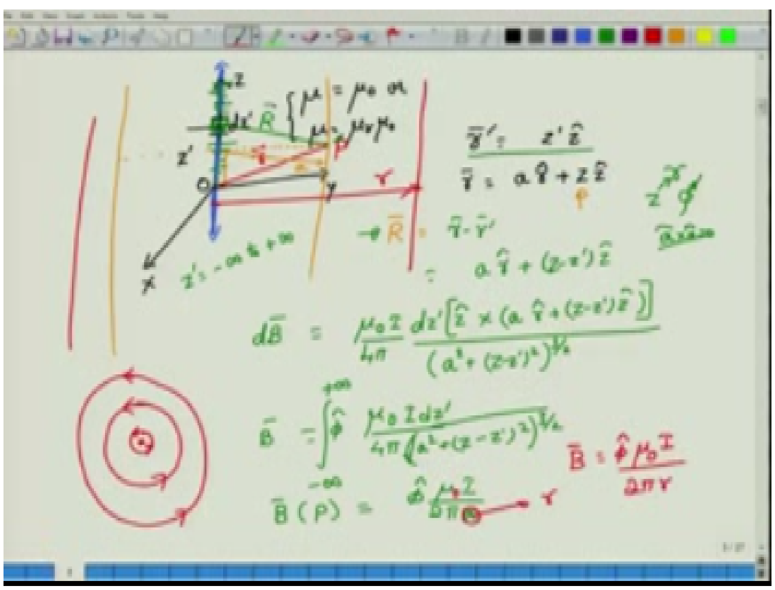

I mean to the power 3 and essentially obtain this equation, so I hope that you are you know you understand what is going on here the radius of this cylinder is considered to be a the cylinder does not exist your will simply imagine that there is a certain cylinder out there and you defined r` which is the source point and the source is actually along the z axis and located at a height of z` that is where you have r` in this way the field point R in the cylindrical coordinate system will have two components one is the radial component and the other one will be the z component correct the position vector of R is given by a r^ + z^ from which you define what is the vector R which joins the current element to the point where you are calculating the field and from there you play bio Biot-Savart law.

Now you can evaluate this cross-product here z^ x r okay because r ϕ and z form a triad of the vectors in the right handed circular system, so when I take z^ x r^ I will be obtaining ϕ^ that is I will obtain ϕ^ which would be the direction of the B field now okay so this z^ x r^ will be along the ϕ direction there is a constant μ0 I / 4 Π and z^ so there is Idz` z^ x some z -z ` times z^ will give you 0 because cross product of the same vector times same vector will be equal to 0 therefore the second term will actually drop out divided by $(a^2 + (z - z`)^2)^{3/2}$

So this is the field that you obtain at the point P okay Biot-Savart law and the contribution of this current element okay the reason this field db is written as db is not in the usual sense of an ∞ thing if this is the contribution db then you can actually repeat this experiment this db will actually be present in the entire region it is not in it is actually small but at the same time it is present everywhere that is weak field that is why it is a differential field or the ∞ field that we have written.

But at the same time this week ∞ field is also present everywhere okay now of course there is other no, the entire line has to contribute to the magnetic field at the point P therefore you need to sum up or integrate this expression in order to obtain the contribution of this element next segment next segment and so on all the way from when z` goes from -∞ to +∞ so you have to integrate this equation, so I will do that right over here by removing this d here and then trying to put the integration sign from -∞ to + ∞ with respect to z`

Now ordinarily this would be difficult thing to evaluate if the dependence was not on z but on other coordinate point r and ϕ because the vector ϕ is independent of z direction but it actually depends on the values of r and ϕ okay and because of this so if you are not convinced about this one you should go back to your coordinate system module and then you will be able to understand what you know this particular statement but in this case only in this case because the integration is carried out with respect to said you can remove this ϕ from consideration.

That is you can remove this ϕ outside the integral, so which means that the magnetic field will always be along ϕ in this particular case when you do that ϕ will come out of the integral and still you have this particular integration to perform since this is not a class or integration I will leave this integration as an exercise for you to take it up whenever you are free okay, so instead I will say the magnetic vector potential at the point P which is at a cylindrical radius of a is given by ϕ^ μI / 2 Π x a okay.

Now a is you know any distance I can consider any other cylinder for example if I want to find out what is happening to the magnetic field let us say this cylinder okay which is at a radius of r then I can do that so I just need to replace this a which was just a placeholder for this particular case by the more general radius vector R and I obtain the magnetic field to be ϕ^it is directed along the ϕ axis μ0 so because this is the medium that I am considering otherwise you have to replace μ0 by μR μ_0 / 2Πr.

So if you look at from the top you will see that if you fix the value of R then the magnetic field is circulating in this way and as R reduces the magnetic field starts to become increasing in the amplitude otherwise this magnetic field will be circulating about the current that would be present over here, so that is the reason why you know you have this current tool where the magnetic fields are circulating around a particular current carrying wire.

Now you might say that well we have used Biot-Savart law but is there a simpler way to obtain the same expression well there is a simpler way to obtain this expression now imagine that this goes all the way for up to + ∞ and you know below here up to -∞, I now want to find the field at this particular point okay which is at a radial distance of R from this particular current element, so from the current or the current wire I am at this distance R.

Now at this point right I am assuming that we are in the cylindrical coordinate system you will have an option of H being along the z axis or it would be along the ϕ axis or it would be along the radial direction that is it could be that this way or this way or it could be along the ϕ direction where it is circulating which of these components does you know do you really have can you have a component along this direction well you cannot have a component along this direction because the current element has to be in this way, so if there is a current element in this way only then there will be a magnetic field along the radial direction.

And since the current element is actually along the z axis there is absolutely no question of you having a magnetic field along the radial direction it cannot have at any no radial direction at all now can we have a component of H along the Z direction well for a component along the z direction there must be a source of the current element which would be present in this loop you know in the base xz = 0 or some z equal to constant group and there should be a current element in this way.

Because if there is a current element here so this current element if it is in this way then there will be a z directed field but there is no current element here similarly there as similarly was there was no current element in this way the only way that you have a current element is z and surrounding that z would mean you can now circle this particular current therefore there is only H ϕ component in out of the three components.

So that is the first thing that you will remember I mean that is the first thing that you will realize so symmetry tells you that you cannot have a component along Z you know along the radial direction you can have a component along z direction well not symmetry but it is the source distribution that is telling you but symmetry will now help us what is meant by symmetry now we have said that the magnetic field can only be circulating this current element and it would be along the ϕ direction.

Okay can this magnetic field be dependent on the ϕ value that is can the magnetic field H depend at say 45^0 from the x axis will it be different at 90^0 it will be different at 180^0 fortunately no why because if you go around if you imagine that this is the wire and then if you go around this particular wire at a constant radius okay ϕ is changing from 0 to 2Π the current distribution looks exactly the same for the current distribution to look different then if you are viewing this at zero angle if you are viewing this at 90^0 or 180^0 the value of the current should have been different.

But since this is an infinitely long wire carrying the same current I at all points there is no dependence on ϕ so symmetry tells you that H ϕ does not depend on ϕ can I have H ϕ dependent on z axis for example if I am there on a constant radius R and I move up or I move down but as I move up or as I move down I see the same current distribution over there the current is constant and hence there is no dependence on z as well.

Can I now have H ϕ at least as a function of R yes I should have because if I am closer then you can imagine that the viewing angle that you are seeing will be this much okay, so you imagine that you are at this no you have very tall building and you are at some particular distance and then you look at the building, so you will see certain height of the building as you move away from it right as you move away radially the building height will also keep changing or rather the more and more current elements start to appear.

So assuming that the building is actually going all the way from $-\infty$ to $+\infty$ as you keep moving away your viewing angle actually starts to increase more and more current elements start to appear and they will you know contribute to magnetic field but the point is as you keep moving away you can distinguish whether you know how close you are here and how far you are there and the field at these two points will have to be different in fact of course the field goes down as a linear value of R as we will show that one.

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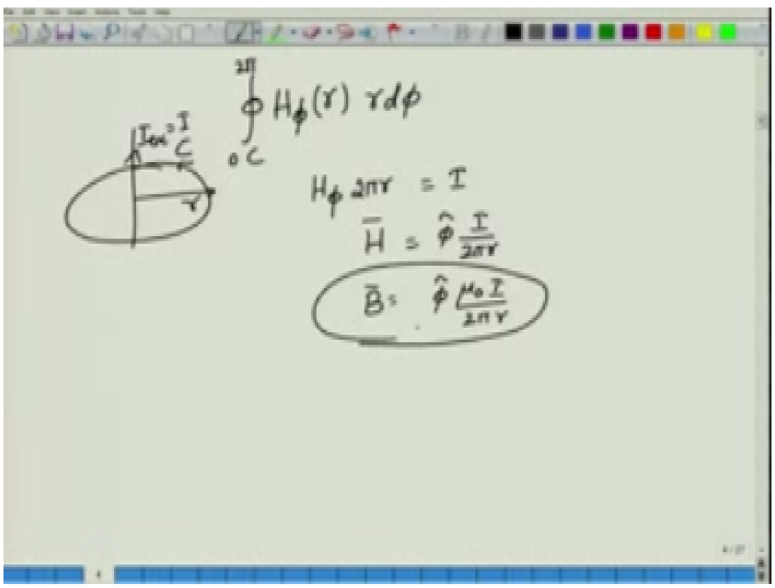

So what we have learnt from this exercise is that you can only have the ϕ component and this ϕ component can only be a function of r therefore if I have the current element and I am at a particular radial distance r okay which is a constant then amperes law tells you that H ϕ okay along this particular radial constant rd ϕ which would be the loop that you are going to consider so this would be the loop that you are going to consider closed loop meaning your ϕ goes from 0 to 2Π then because H is along ϕ I do not know what is the value of H but this H is not a function of ϕ and it is constant for a given value of R and therefore I can write this as H ϕ 2Πr.

Now this must be equal to the total current enclosed which is equal to I therefore this is equal to I and H = ϕ ^ I / 2Πr or B = ϕ ^ μ_0I / 2Πr just as what we obtained in the previous case so we will show you more examples of magneto static field calculation in the next module I hope you understand the previous calculations very thoroughly thank you very much.

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