Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

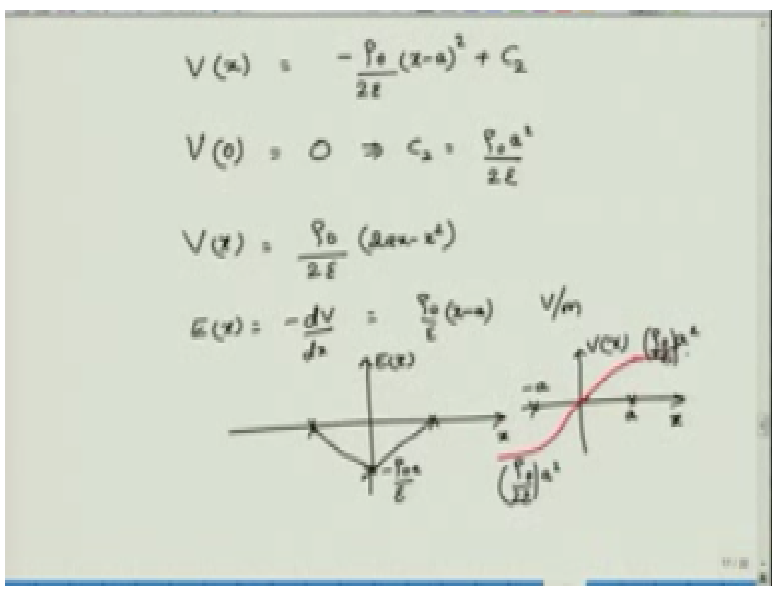
Course Title
Applied Electromagnetics for Engineers

Module 36
Electrostatic finite difference method for solving
Laplace equation

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to applied electronic for engineers, in this module were will complete the analytic solution of the poison equation in one dimension and then talk about the numerical methods for solving the encloses equation we will concentrate on one method today called as the final difference method. Now Poisson equation.

(Refer Slide Time: 00: 34)



Is applicable whenever there is certain charge density so the general expression for the Poisson equation is the α^{2} V= $-\partial/f$ r so the R is the charge density measured equally in the meter cube and the material we have we will apply this Poisson equation in order to the solve the potential and the field of the very important electronic device known as the P N junction.

If you remembers from you’re the solid states some you remember our solid circuit and the P BN junction is formed when the p junction and the N junction and the N junction is equal to the close to the each other off course the modern P junction or not really manufacture in this way in the way and the olden days the P N junction are grown separately and then brought in the contact what would happen is that the charges would esssiantely migrate right because of the diffusion there v would be the excess positive and the P side or the holes and the holes could be migrate actually the electron could be migrate and the but it bi does not matter.

And the living behind in the electron could be diffused the left hand said on the mobile positive charges now whenever you have the positive charges now whenever you have the negative charge and the positive charge and the negative charge is separated and the negative charge is separated and th3eere will be the electric field and in this case the electric field is directly in the n side in the p side now you cannot have the situation of the uncovered charges and the mobile charges and in the p junction and the bar so it will actually happen and in the small region at the centre of the junction let us consider for the not at the both P type region and the N type region are sufficiently.

Equally between the two substances and the width of the space charge and it is called in the width of the space charge region is A in the N region and it is the same region in the P region as well because the charge in the symptoms and in which numeric which is unknown it is do know the charges solution and that is the in fact that is thing we have to find out one of thing we have to find or may be in this case we do not remember and in this we have in the doping profile.

So we have to the charge distribution it is uniform and there will be the- 00 charges to the elusion because we have to tie and it carry actually negative charge so this is the situation that the negative charge we have now we have and this is our expression to solve we will consider the solution of the position solution only in the regi0on X in the end region that is the X is greater than the o the same result can be obtain in the applied to the solve for the potential and electric in the field region in the p region as well as the and we knew as the exercise.

And because this is the one dimension Poisson equation because of my simplification I do not have a partial differential equation I can just solve an ordinary equation which tell you that when you are in the N region off course when you are in the P region this will be actually will be replace by the row 0 okay because the charged density varies negative okay integrating the image once you can get the D V / D X it will be the some constant and in they we will also integrate in the once more and in the it will be the X^2 will be the some constant C 1 x .

And the constant C 1 X - row will be the X 2 and in the adding one more constraints over her what will you do in the boundary condition and then you tried to the remove the right het and the boundary condition you are going t0o apply well be the D V / D X actually gives me in the trick in the method and in the minus electric field is D V /D X and the X is = to A and this is the right and the electric must be 0.

Now X is = to A and the D V / D X and the able to A is and it is = to 0 so when I put the X = to this expression and the said that the D V /D X and the said that the D V / D X = to and I will get the C 1 as row 1 as in the times so I know to simplify my life to putting this row 0 F and this expression and we write this D V / D X as the row 0 and the X – a in the integrate v and then next Time integrated a and then in will small exercise to you .

Show what I am writing and then what I am, weighting is correct then I integrated what is this and we often and the 9os the and the and the X – a and there will be $x – a^2$ and there will be divided by s = +some there will be integrating constant okay again what is the potential at the centre of this is the one of the center and this will the and the charges are between the two so the X is 0 and the X is 0 at the potential actually to be the and it allows me to find the C 2 as the row 0 P^2/2 subtitling the total expression for the V of the X put can shown the V of the X .

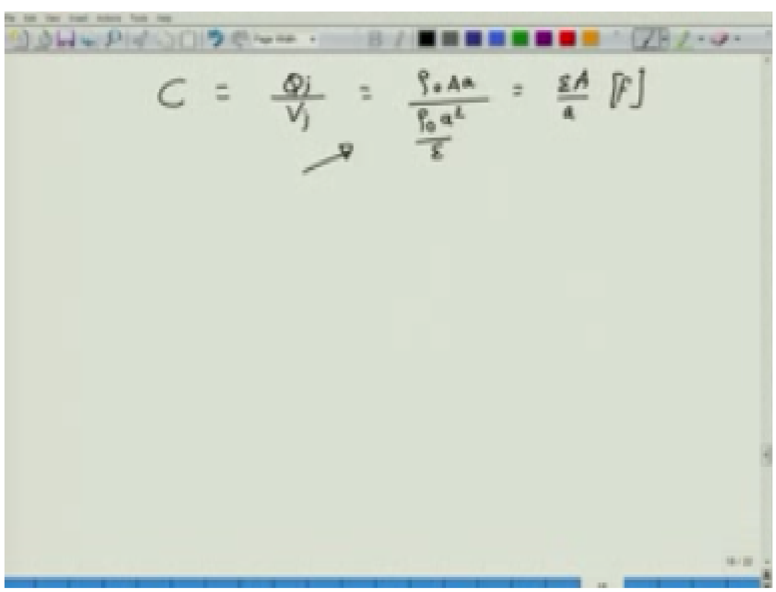
Shown in that that is given by the F silent and the 2 A X –X 2 and this is the potentially that we has in the what is the remember w electric field is – gradient is the but in thisiocase it is its the one dimension thing that is the one D V /D X and this is given in the row 0 and the f silent and the x and the X – a off course the electric field is measured in the volt per mete r and this is in and this is the electric field in the N layer in the p v layer of the row 0 / - row 0.

And this is the what is the kind look like so you have this is the X axis and this is the P if the X and the X is = to the 0 and this is center we have the maximum value of the electro field and the maximum value of the electric field it has the negative that will indicate the direct of the indication of the field it will be the opposite direction to the x okay this is the at the X = to the and that will be the 0like in the electric field on the p side region as well when you reply the row

0 and the – row zero and this will become X =+A and the X will come in the X some will again go to the zero.

So the electric field that you have in the triangular for t6he voltage that you have going in the saturated and this is the and we have derived that one and it will go someone like this okay so far of the junction is and this is the same value here and the row 0 you can obtain in the simplify the difference in the way of the A and that is the value you are going to get at the A and that is beyond that everything will be the constant potential term that is equal to the you can show that is equal to the row X 7 A^{2}.

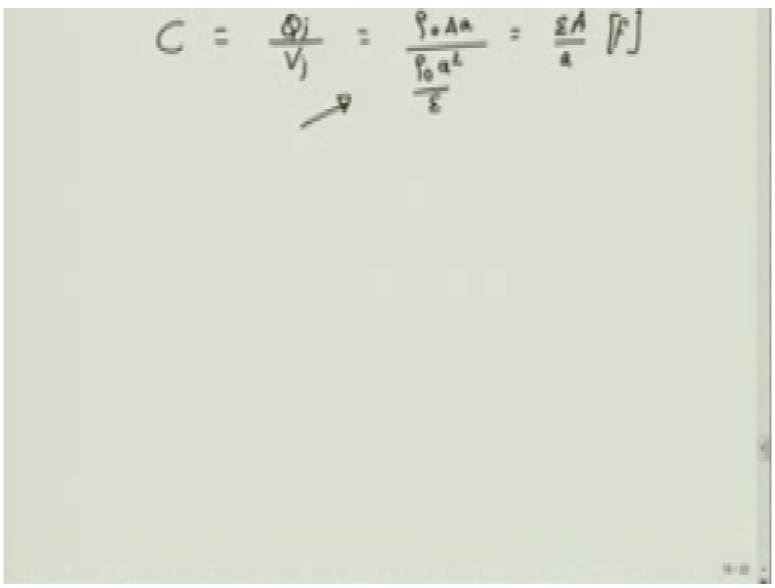
(Refer Slide Time: 07:57)



Now that we have as well as the electric field we can also calculate the capacitate of the junction and the capacitance will give the total charge that will be contain by the one region are the one particular size and that what is the potential that we are applied for that the potential difference and the potential difference that we have applied than potential difference we have applied looking at the between the 2 junction and the V of the X is positive.

Here and the OF X is negative here the total potential junction is a difference between the two and that is called as the between the two and that is called as the potentials difference and the junctions potential for the and this case the junction potential has to be and that is to be ant that is the because substitute that one here in to the reprensative form and the junction potential.

(Refer Slide Time: 08:40)



This is the two charged and that will be the amount of the charge density that have told and the is and the times and the area of the bar and that will not show you and in the expression in there uniform particular and the width of the and the area of the A is the volume towards the charged toward the space charged of the space charge stood and that is divided with the potential and the potential and that is and the potential and which was through and that is the and that what is the a silent and that is the small A and this is the parallel .

It will be the parallel like the capacitor so that were have a link obtain now here is an important thing in the case of the we have consider the plotting and that in the equation or The poison equation are in the quite small it will not enough examples we have consider and that if I consider and the list we have consider and the larger valve and that is the solve this equation analytically I still have very few cases of the were the analytic complete solution for the what we have and there line we have form expression can be obtain.

In the simple snaring   I consider the of the quakes cable with the circular cross section and the net of the rectangular and the electrical or the solution is the corresponding and this the structure we have thee ground lay of the people in the micros tic line we have the activity problem and that is the cases and this year we have the parallel courses and that we have the parallel idea of what the difficulty you go to the experience we have in the apply numerical methods to electrical method to solve the for the electromagnetic problems but our focused is to be introductory idea of how to approach the electrical of the electromagnetic for the numerical and the matters .

And towards the end I have pick the first numerical method which we have in the very simple numerical method called as finding difference method okay what is the finite difference method you going to apply the difference method to the difference v of the problem actually we have already solved that is the solve the problem of the Atlantic difference in the two dimension for the square trust problem.

The square trust problem is you know the imagine from the top view of this one you just imagine that there is a kind of the rectangular and the bucket keeps on the increasing so this is the square trust imagine of the small and that is the total and the image the rectangular box with the box is going to the infinity all the infinity the but then but then it know in the crows section in the actually that is the box of the certain gap over that is the over here and that that I have in the due in the potential is will be the zero are kept at the 0 moulds .

Okay we keep all this 0 volt and we have to sink me on going on the top so what we want so what we want the potential is the potential and the D of the X Y in the flux region so the earlier case we will simplify the problem seeing that the dimension along the dimension it is not really important for us it will be uniformed and in the form of the assumed in the assume that and what we want is the potential of the V of the X Y are in the electric field inside this particular region.

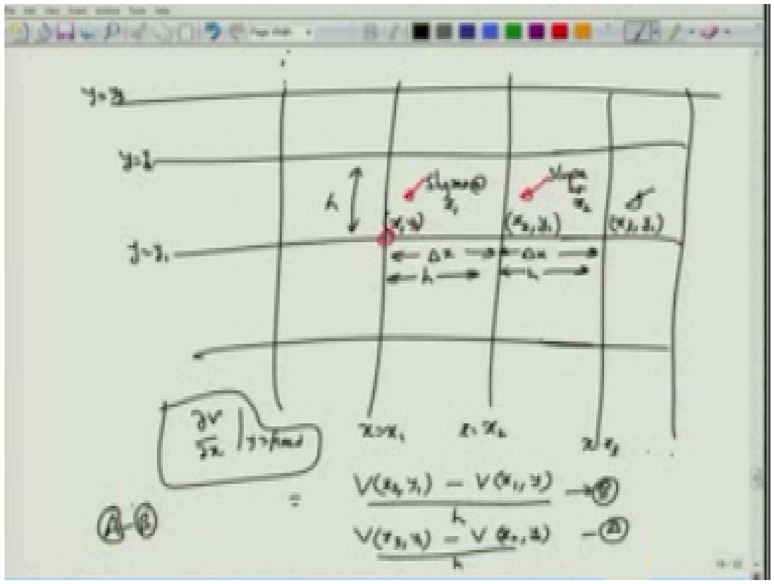
We already calmed and we have shown in the analysis solution for this problem but what we want to do is the actually in the numerical method and see however we do in the respective analytic method so the other advantage as I said about the numerical method is the it is not restricted to the only this problem for only if I consider this class there is absolutely no simple analytic solution for this problem but this problem can be passed easily handled as the corresponding square trophy so that is the advantage of numerical method that is the numerical method.

So you can easily generalize to other cross sections without the too much of the method the equation is the and the 0 subject to the boundary condition right there are certain boundary condition which we have given here subject with that we have to the solve this 2 and that will be in the certain and that is main and the if you never seen the equation and that is the before the partial derivative of the with the respect to the X okay it is actually at the some of the X =+ A for the given value of the Y .

So you have to find the X and the Y and we have to find X along at the certain in the X + the A now can the small step away from that at the same value of the Y and the – V of the X, y it is the potential at the X , Y and then divided by the Q X and the Q X is going to the 0 and that is the how the mathematician could define the path is the partially derivative and if you want to define and that is the second partially derivative and t6he Q 2 and that is the Q X 2 ant that will be the 0 but not of the potential value but the other but the derivative of the partially is the deems size.

So that equation can be written as d v / d x cos d v /d x so instead of the very function that is the Q V / Q X itself it has the function that evaluate that the Q V / Q X is that the two different points and the X + Q A in at the X / the whole thing by the X now you apply the one new definition for the one we have the X = above expression drawing the line okay.

(Refer Slide Time: 15:20)



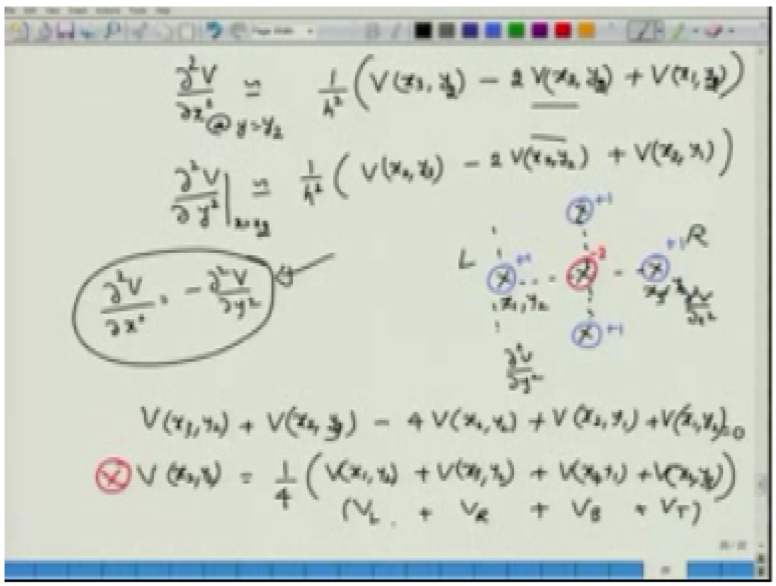
So this is the X axis so I am going to the some grid points as I would call then in the mark as the X , Y okay and this is the numerical X 1 and the X 2 y and this is called as the numeric called as the v f 3 y #b I will assume all these are equally space in the by the amount of the X and this is the of the H .

And that I am giving all the grid point that is the facing of their h even the top lines are the alternative of the space of the line X there is no restriction in that place buy it simplify in the introduction of the method and in the method in the practice there is no grid separation there are region in the higher axial you actually in the minor surface and the implement in the X and in the X 1 and we have to find the slope here and the slope here and then subtract the slope here and then divide one this by H so I how do that in the slope at the X 2.

So what is the slope at the X 1that is nothing but the X 1 and the X so in the along it I the actual mistake so this is actually the point X 2 Y and the X 3 Y 1 because this is the line where y = to y 1 and this is the line is and in the top and so on that is X 1 and that b is X 3 so that I the slope of the X 1 and X 2 and that is divided by of the X and in the computer because in the computer I cannot take the limit of the 0 right the all number are all the computer of the representative by the finite value and I cannot take and that is value of the 0.

On the computer this is what I am going to have so this is the slope at X 1 similarly the slope at the X2 will actually have V of the X 3 , Y 1 now similarly the slope at X 2 will actually have V of the X 3, Y 1 – X 2 , Y 1 will actually require the value of this potentially at this point and then it is divided by H okay so once you have done this thing right now what we have to do is to subtract the two right so this is A called this ca B okay what you need to obtain for the @ and the X 2 A – B okay when you do that one.

(Refer Slide Time: 18:38)



Okay when you that one you can see the $V X^2$ has been approximated so this is Y = to Y 2 has been approximated as $1 / H^2$ we hav3e the X 3, Y 1 –and this is case of the Y 1 okay we have there X of the 1 , Y 1 so this I what we are going to obtain and similarly you can shown what will be the Q^2 and the V / x s and the X 1 we will rewrite according to ands the equation accordingly okay.

So going back and the insulated in the point and the only there points we have and then w have looked at and there is the p out at the X 1 Y 1 and f there we have the X 2 Y 2 and X 3 so we will called as the X 1 Y 1 X 2 Y 1 and this is the X 3 y1 so only we look at the points which is writing the Q 2 and were v you can see that I am trying to evaluate them at the centre point I have the centre point I have to the give the wastage of the – 2 here and the wait age of 1 and the then I have to some of these values.

And all these values and then divide the entire thing / 1 / X 2 okay you can see that why I am seeing the L 2 in the and the y is the constant and the y 1 and the Y n1 and the X to the right and the left to the right and the added up here similarly we have the x 2 and this sis the particular point okay in the plain version of the instead and then it is d the X 1 and the plain of X the plain of the X and the X 2 and the be written as the 1 / h2 and the VF of the X and then it is the value of the top of the bottom they we have to applied in the wait age of the =1 and while the centre point have the wait age of -2.

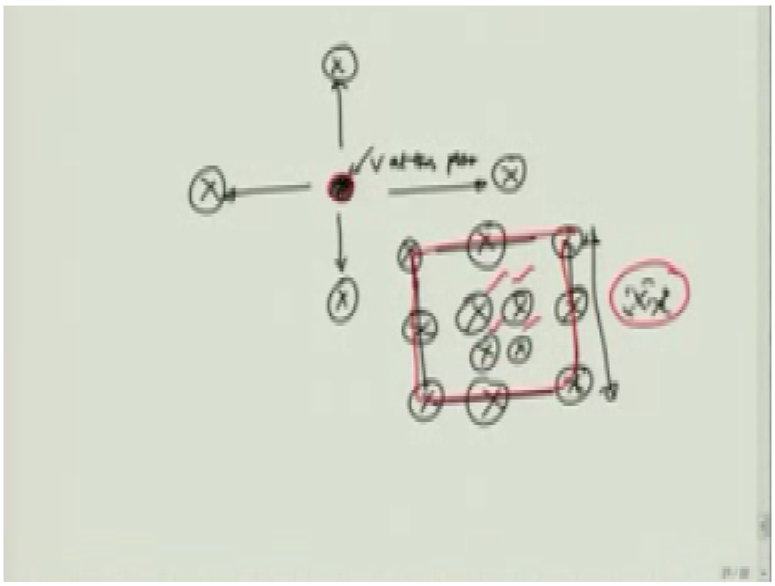
And the V of the X 2 and the X 3 and in the way of the X 2 and in the Y 2 okay then the + v of the X 2 , Y 1 okay now to the top and the grid of the one and what I want to show there is these are the points which are actually formation and this is the centre point okay which is in the wait age of the – 2 and in the + 1 it is the points that is the one the grid is the inform in the certain every where v and in the X and the y we look at the everywhere and then if and go horizontally then I obtain in the v Q 2 and if I go vertically then I obtain the Q 2 V / Q us okay so the horizontal and the vertical direction.

And that is the movement actually gives me that whatever the corresponding part that I am looking for the land that is the any point and the X and Y that is considering that is red colored centre point over here the derivative that will go horizontally must be then it will go vertically so then we actually add them up couple of the terms will added them up nicely okay in the couple in the X 2 and the Y c and then I can added up or the subtract the two expression over here so this is the one I am going here and this is the X 3 Y 3 and the X 2 and then it is applied in the terms of the Y 3 and that is the 4 times of the V of the X 3 and then 2.

So this is X 2 and then Y 2 so this is X 2 and the Y 1 so it could be – four times done over the times and this is the one of the formula lightly but the centre I am going and looking at the Y 1 so this fellow and this fellow will add up and all the other are adding in this way so this -2 and the -2 it becomes 4 okay then I have = v of the X and the Y 1 and the Y 1 has the +1 and the Y this sis = to 0 so you can rearrange write this is the centre of the red dot which I have consider at that point that is actually equal to ¼ V X 1 Y 2 and the V X 3 Y 3 V X 2 Y 1 = V X 2 Y 3 so this is the V of the X 1 and the rate of this point is this actually lies on the X 1 Y 2 and then I have so this is X 1Y 1 .

So this is actually X 2 Y 2 and that is the left of the centre point and the X3 and towards the right at the centre points of the write this as V at the left V at the right the potential at the V at the right so the potential at the bottom and the bottom of the top okay so what you see here is that / everything by the 4.

(Refer Slide Time: 24:26)



So what you see is that if there is the centre point okay and that is if I want to know the potential so here at the centre point at this particular point al have to look to is the point to the and the right point at the left point at the left point at the potential our the potential at the top and the potential and the bottom and the top and add all this the average of this 4 will be the potential of this particle centre point okay this is your you know the finite difference of method of the eposes equation .

This is coded on the computer as we will be discussed on the next module but for this module we have to understand that the potential at the centre will always be equal to the average at the point how do I deal with the edge condition for the edge condition for the edge on the certain some of the find out the potential here I do not have any right here and so I have any right component so that is the component at this side and this is actually missing.

So there we will extend the formula to the include the exterior and the write this terms and the other points and the solution or simply you recognize that you are never going to change the foundry condition on the boundary itself and then keep them fixed that is the fixed that we will evaluate the problem so this is the trough problem that I have that is the these are points that have the grid that we have the grid here so that we say that is the grid that I have only in the centre and then I have evaluating .

So then we have evaluating numerically I never update on the boundary so in the boundary I kept them fixed I trite al the update in the centre of the trough or the centre of the region okay this is the one method do the solving as I told you you can exterior fitnesous point and that point and then explain the solution but those are slightly complicated which I not be complicated alright the one we will do the numerical method and till that thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production