Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 29
Towards Maxwell's equations-Part 2

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Hello and welcome to NPTEL mook on applied electromagnetics for engineers. In the previous module we were discussing electricity relevant of history of electricity and magnetism. Now we will complete that history and then show you what Maxwell's equations look like. Remember we have already introduced due to the first electromagnetic quantity which is electric field, electric field surrounds any charge and these electricity would be varying at every position as well as it could be varying with respect to time.

If the phase are time in variant then we call them as static fields, if the fields are varying with time we call them either as time varying fields or dynamic fields. When parallel to this electric field is the notion of magnetic field, what does a magnetic field?

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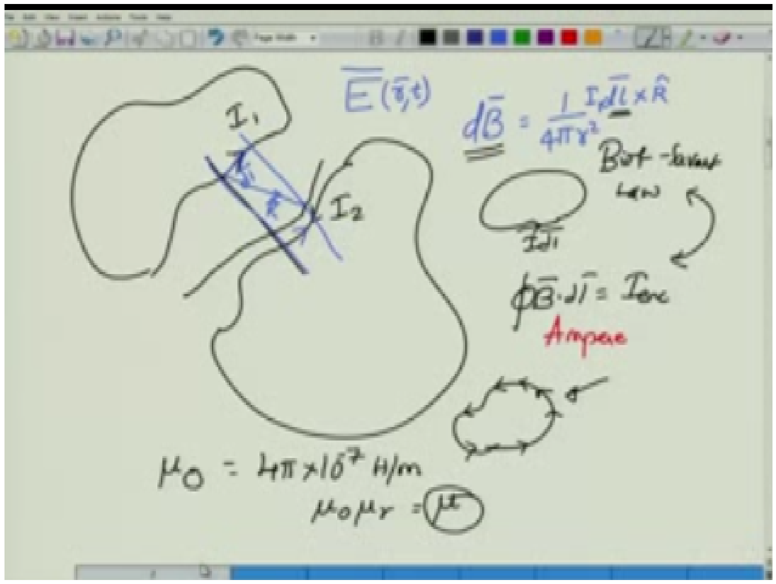

Let us assume that you have two current carrying wires, we introduce this one previously. And then there is one more current I2 now it is other circuit is carrying, so there is a complete circuit here, and then we have a complete circuit out there correct. Of course I have not shown you the sources of these currents okay, but imagine that there are these two sources on we seeking up there. Now it has been observed that if you focus your attention only to this small segment okay, the small segment that depending on whether the currents are going in the same way or the currents are going in the opposite way there will be either attraction or repulsion between these two pieces of wire okay.

And coupled with the fact that this current can change the magnetic compass deflection it can introduce deflection into the magnetic compass, people what they did, there are lot of people who work and this one the important ones are ampere and biotins of work, what they found was that each small piece of current carrying wire actually develops a magnetic field around it okay. So it is the magnetic field is developed by this current, and is this magnetic field which was deflecting the magnetic compass.

Because compass is known to deflects, so this is the magnetic compass, this magnetic compass would deflect in a slightly only when it is pertinent or when it is kept in a magnetic field. So if the surrounding magnetic field then this will be now pertinent little bit or it could be deflected like this. And back magnetic field was created by this current carrying wire and the amount of magnetic field that would be produced and magnetic field is a slightly different beast than, you know electric field.

In that the magnetic field if we call this B as a magnetic field here, sometimes B is also called as magnetic induction we do not worry about that, we call B as magnetic field. Sometimes you also call another quantity which I will introduce as magnetic field but the context usually will make clear what we mean by what so we electric field let us denote the electric field here first. So this electric field is a vector therefore it is E with a bar on top of it to denote that this is a vector and it is a function of both position vector as well as time and this magnetic field component I do not exactly remember the formula but it also has interestingly as square like behavior but then the direction of this magnetic field produced by the current element.

So if there is a current element of I okay or I_1 may be or looking at the effect of the current I_1 circuit there will be you know along this length there will be certain line element dL so there I1 dL x $r^{\wedge}$ okay, where $r^{\wedge}$ is the unit vector at which your actually trying to find out the magnetic field of influence so this is the magnetic field of influence, so at this point if you where to look at this r then the magnetic field will be a quantity which is actually perpendicular to both dL and r so the cross that we have written between dL and r tells you that this is a cross product and the magnetic field is actually given by the so-called right hand rule.

That is you curl your thumb you know or you curl your this is one in such a way that you try to enclose your you know magnetic field the direct of magnetic field when the direction of the current carrying wires is along my thumb so this is a the thumb circling this thumb will give you the magnetic field that is surrounding with particular current I mean current carrying weir. This is my current carrying wire and this is the magnetic field that would be present so you can see that form this equation as well.

In this equation you have dL which is along this direction and then r which is the unit vector along this direction perpendicular to both will be the direction of the magnetic field, there is a ;little bit of cheating going on here that is because currents are not line elements you cannot take a current P and then isolate this one and say okay this is my total current I because current always has to go into a complete loop okay but never the less we call this law as Biot-savart law and this is used in calculating morganatic fields closely associated with this one the close looped kind of a scenario.

Is what is called as amperes law and you would probably see amperes law in this particular manner I will explain to what this is but this is what you would actually see amperes so let us label this I as amperes law okay weather it is B here or another quantity I will explain to you in minute what does this mean this means that if instead of considering the current small element if I actually consider a path okay over a closed path okay and then evaluate the magnetic field around then this magnetic field.

You know over this particular path the complete path will actually give me the current enclosed by this conduction path, so these two laws was slightly different in the sense that one law allows you to determine the magnetic field the other law tells you that if you want to take this magnetic field and the integrate it around that closed path what you will see is the result the result of this integration is actually a total current that is enclosed okay.

Now these are all very nice and we can actually do this you know in certain scenario which is essentially called as vacuum or in this particular magnetic field case that there is only one magnetic I know the current carrying wire and then we have done this one, but if you want to try and do this experiments in a material medium you know remember we have already talked about material medium and tie electric permittivity, associated with magnetic field is also very similar phenomena okay.

You do this experiments in air and you do this experiment in a magnetic material such as you know right or something then the result would change slightly as before in the previous case we introduce what is called as permeability so we say that the better is the permeability then better is a magnetic property of that one the permeability of free space is denoted by μ_0 and this is given by definition this is given by $4\pi \times 10^{-7}$ / m okay, so this Hendry indicates that this is the concept introduced or link with inductance and therefore the permittivity just like permittivity was linked with capacitance, permeability is linked with inductance okay so this is what μ0 is and in a material you will have μ0 and μr x together as the overall permeability of some material okay, so what is this permeability that we are talking about the full discussion is slightly complicated I alone going into it.

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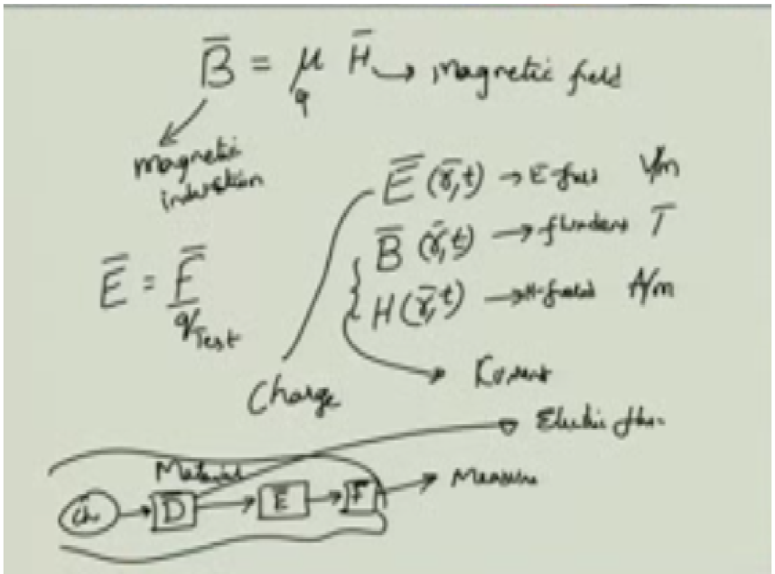

But then the field quantity be is related to another quantity called as H okay this H is what we sometimes call as magnetic field and B is sometimes what we call as magnetic induction or sometimes called as magnetic flux density okay there are this unfortunately there is not standardization of what we call as what B is what is called as magnetic field by most physics people okay whereas H is what we call as magnetic field, or magnetic field intensity by most electrical engineers and magnetic induction is a slightly older term for B.

And magnetic flux density is because it is similar in many ways of it is behavior to electric flux density which we will introduce very shortly okay so we have in a matter of few minutes we have seen 3 electromagnetic quantities one is the electric field quantity Ert and then there is a magnetic field quantity Brt and then there is another magnetic field quantity E, r, t.

And then there is a magnetic field quantity B, r, t and then there is a another magnetic field quantity or rather magnetic induction or flux density okay and then this H is called as the magnetic field okay or simply the H field this we will callas simply the E field know this does not introduce lot of controversy over here.

So simply call this as E field, B field and H field and you can be alright okay the way in which you measure E filed is volt/meter okay the way you measure this H field is ampere/meter and the way you measure this B field is the unit called as tesla, after the great scientist tesla that you might have heard it on, okay. And what are the loss that is I mean satisfy by this, the electric field at any point is given by the force/charge Q.

Of course in this equation we should remember that the charge should not be very large okay, the charge should be barley enough to you know correctly estimate the electric field because of a certain charge that is already present, so this is a test charge okay and the test charge magnitude should be very, very small. Numerically of course test charge if you set it equal to one or one coulomb because charge is measured in column.

Then E will be equal to F okay and the loss that are you know linking B, B is linked to the current E of course is linked to the force, force is further linked to the charge so you already seen that the source of E is charge the source of B and H because H is proportional to B atleast in a simpler magnetic phenomenon no hysteresis and all the other complications that we are bringing in at this point.

So in those materials B and H are simply related to each other by this permeability factor and the source for this B or for H as you can go back and check here is this current that is flowing in a conductive path and that current is what goes here. So this is the source for electric field is charge and the source for this magnetic field and the magnetic flux densities current. Now we said that you know you have this εr right which is the relative permittivity.

But in scenarios where things are in a complicated what we say is that we do not exactly obtain E what we say is that in a material so if I assume that this is a material okay just as B and H where related by certain quantity we say that charges produce not D but produce a D, Dis further related to E in a material E applies a force on the test charge and this force is what you are able to measure, okay.

And this vector field D okay all these are vector field quantity so this vector field Dis called as electric flux density okay electric flux density tells you how many flux lines are coming out on a closed surface or the charges emanating.

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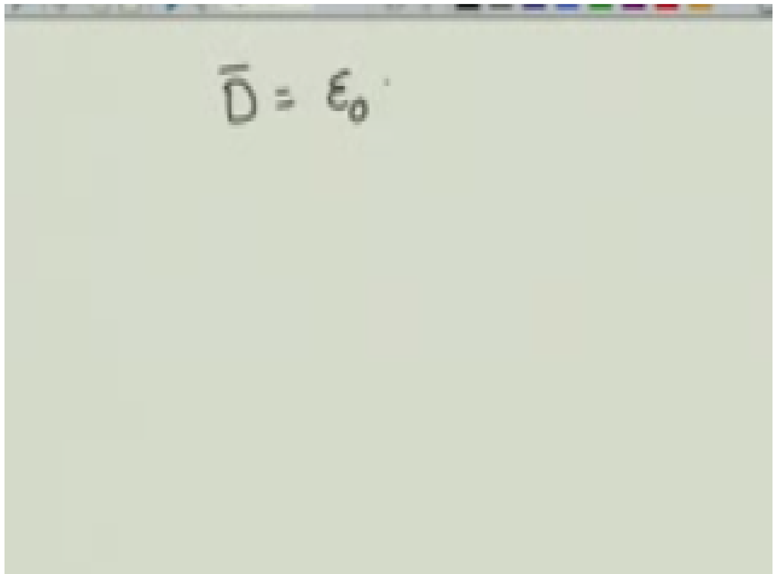

And this flux density if the material medium is very simple you know it just if the material medium happens to be vacuum then it would be $\varepsilon_0.\bar{E}$ okay, or if it happens to be a simple material which can be characterized by a relative permittivity ε_r then $\bar{D}$ will be equal to $\varepsilon_0\varepsilon_r.\bar{E}$.

Now you see the analogy between this and the magnetic materials, right so you have $\bar{B}=\mu_0\mu_r\bar{H}$ okay, and this $\bar{D}$ field is measured in C/m^2 okay, and the reason why we do this electric flux density is because if I take this charge okay, look at how many lines are crossing a particularly close surface.

So you can imagine that there is a close surface over here and then calculate how many lines of fields are coming out of this and repeat this experiment in another material say water or something okay, assuming that you are able to create an isolate electric charge positive charge over here, then the flux lines will be different okay, and this difference in the flux lines of air and this one is what we call as flux density $\bar{D}$ that denotes that material must be doing something else and our reasoning is that $\bar{D}$ is a mediating field.

So you have charges, you have producing them as $\bar{D}$ you know related to $\bar{E}$ by the material properties if it is water then ε_r will be different, if it is something else it could be different and there is also phenomenon of polarization which goes with $\bar{D}$ I will mention that one in a moment, and that $\bar{E}$ will further exact force on a test charge that it brought and that force is what you are able to measure.

Similarly in a magnetic phenomenon you have this currents or whatever the sources of the magnetic field are that produces $\bar{B}$, $\bar{B}$ is related to $\bar{H}$ okay, $\bar{H}$ produces further force or you know relates to the current and then you are measuring that particular or rather in this case it is actually $\bar{B}$ which measures you the force so it is actually material sorry, the current $\bar{H}$, $\bar{H}$ produces $\bar{B}$ or rather $\bar{H}$ is related to $\bar{B}$, $\bar{B}$ is related to the force okay.

This $\bar{E}$ and $\bar{B}$ fields are further related to each other by what is called as the Laurence equation it tells you that if I take a test charge then the force acting on this test charge okay, will be given by due to the electric component itself you know some q test into $\bar{E}$ this $\bar{E}$ must be coming from some charge that is situated over here plus if the test charge where to be moving with respect to some you know observer frame then the amount by which moves the velocity with respect to it moves and the corresponding magnetic field be there will be a force on that, so you can see that the direction of the force would be along the same direction as the charge but the direction of $\bar{F}$ because of the magnetic field will be actually be perpendicular to the charge motion, because charge motion constitutes current.

So far we have introduced all the electromagnetic quantities we have also looked at two loss, one is the Colum's law which gave us this one and then we have also looked at Ampere's law okay. we have looked at Ampere's law in the integral form of the equation over here. So we have this Ampere's in integral form Biot Savart law is use to obtain the magnetic field from the electrical this, so we do not any bother about that, at this point at least.

And then we say or rather we write on this Ampere's law in a slightly different fashion okay, rather Ampere's law is actually given I made a small mistake.

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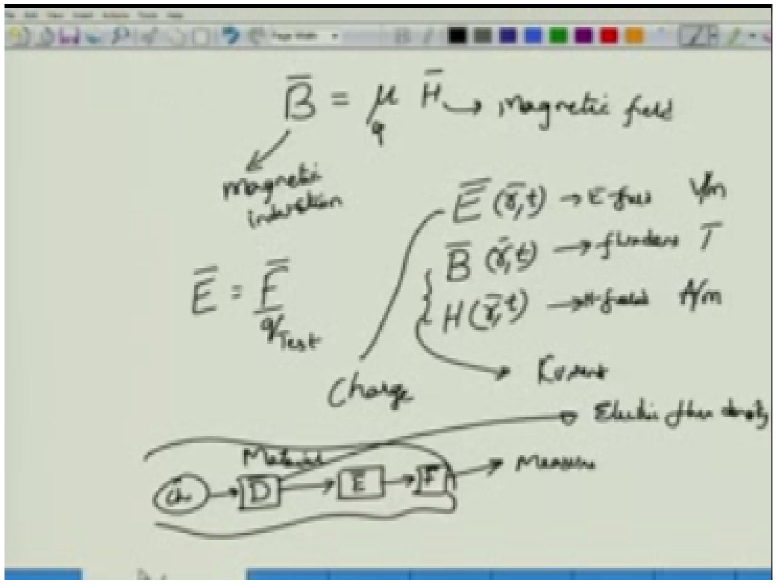

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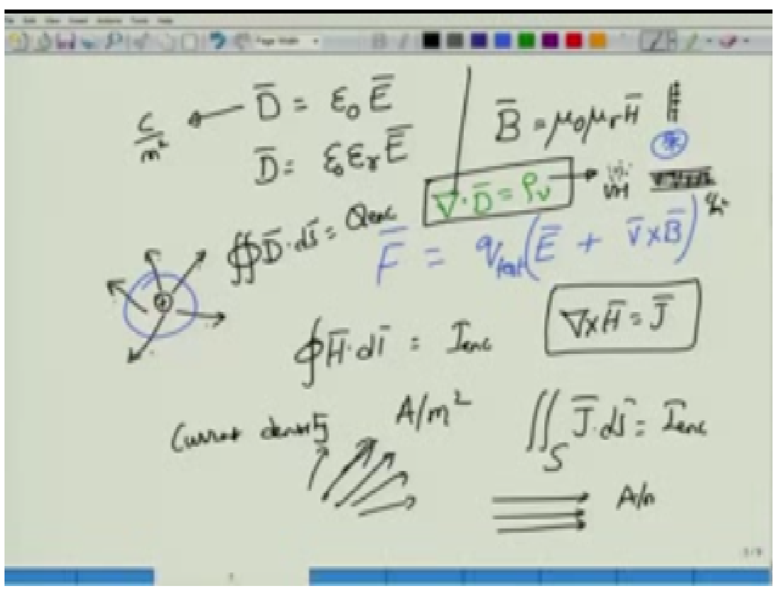

The Ampere's law is actually given by Ampere's law is not given in terms of b but rather given in terms of h over a closed loop. So h dot dl will be equal to the enclosed current or you need to multiply this one by μ0 if I work to express this one in terms of b I should multiply this enclosed current by $\mu 0$ okay. Any way so we have seen this law which is h dot dl = I enclose and the corresponding point form of this is called as curl of h = j what is this j?

J is what we call as current density okay, before explaining current density there is another law which I would like to talk about and that law happens to be that of counting this number of electric flex lines as these are called as counting the number of electric flex times and recognizing that if I perform a closed integral of this quantity d over the closed surface that will give me the total charge enclosed.

So you can see that you know things are slightly similar to the magnetic case that if I take the close surface instead of the line and then integrate this flux density, flex density tells me how many flux lines are coming out per meter square, so multiply that one by meter square in the sense that integrating will give me the total charge enclosed. So c/m2 x m2 should give me the total charge you know in Coulomb's right. And in the corresponding point from the equation becomes okay the corresponding point form the equation becomes $\nabla. D = \rho v$ where ρv is called as the electric charge density or simply charge density okay so these equations are not new to you, we have seen the previous module how to interpret this equation right.

Then I write $\nabla.D=\rho v$ I simply mean that if I take divergence of D which is the very valid basically operation that I am going to do right so you take a piece of you know closed surface and then evaluate the quantity D that would be normal to the surface and then if I integrate over the entire surface area if there is a source such as a positive charge you know or a tub with a water kind of analogy that we are discussed that source would be a a charge density so the source of D is a charge density.

Similarly if I take you know a closed path or a contour and then integrate H over that contour and then divided by the surface area and take a limit of the surface area that is going to 0 I obtain the rotation so in a sense that what we are seeing is that source of the magnetic field which would be this rotation kind of a thing or a curl of the magnetic field will be precisely equal to the contribution or it deservedly equal to the amount of the current that is particular and closed side.

On the right hand side is giving the total current because the quantity current density okay is measured is the you know usually humpier per meter in general way so if you integrate this one over the open surface then you obtain what is the total and enclosed current so this is what you have a current density this is what a charge density, charge density can be come in three varieties if I take the charge and sprinkle it all over the space I have the volume charge density.

If I spread it around in a thin layer of this one then I will be obtain in the surface charge density which is measuring in meter square and then finally I have a point charge density which is the just a point charge you know of vanishing location, mathematical little bit of complicated stuff and then finally I have line charge density in the form of charge occupied in the line. Single for the current density the most common current density would be something that would be spread over the surface.

Changing along the entire surface or it might be spread over the sheet of a plane okay, so this would be you know you have to imagine a 3d picture but this could be a 2d picture. So on a piece of plane like thing of I start putting in the current vectors or the current density vectors that become the sheet current. In general it is measured in an ampere per meter square. Here it is measured in A/m okay. That couple of addition laws which I have not discussed.

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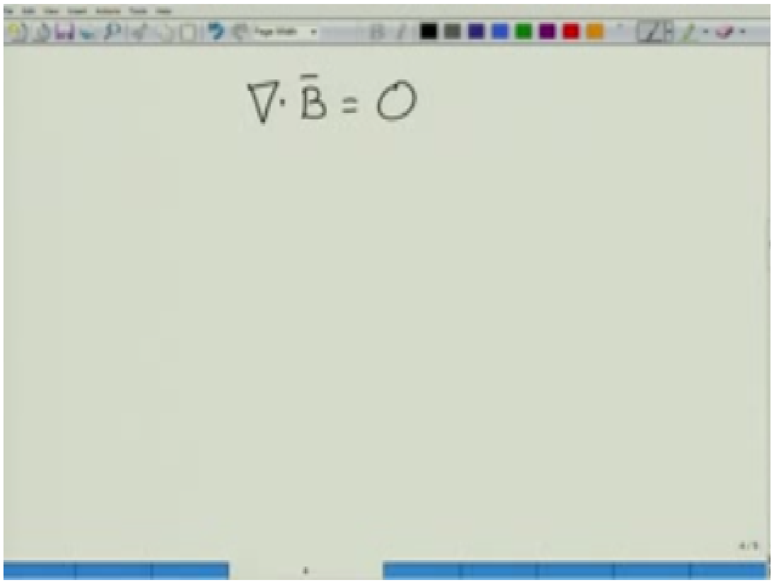

Is this law which tells you that the diversion of B = 0 indicating that b lines don't have any source, in other words there are no magnetic poles are like poles are whatever south poles isolated once which can give you this particular source for the B. hence ΔB is always = 0. So we have discussed 3 Maxwell equations. Although we have not discussed one of the most important equation, which we will leave it to the next class okay, this is called as paradise law and once we have paradise law that will complete Maxwell equation, thank you very much.

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