Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 26
Review of vector field and Gradient

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to NPTEL Mook on applied electromagnetics for engineers in this module we will study scalar but mostly we will study vector fields, what is this scalar field what is the vector field before that let me ask a you question what is signal we know that if I have circuit okay.

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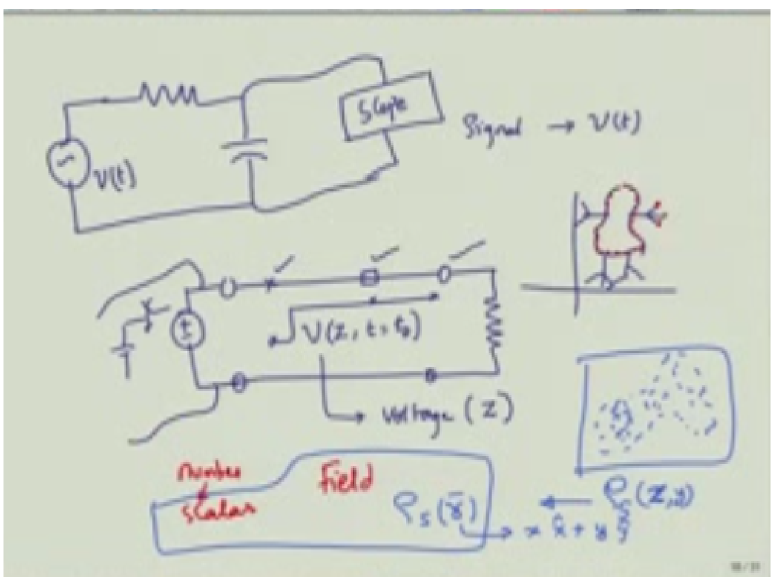

May be some complicated circuit but for now just imagine that there is a simple RC circuit and this RC circuit is connected to a signal generator and we connect an octillion scope across the capacitor well we cannot connect an octillion scope across the capacitor, so this is actually a virtual octillion scope that I have connected and if the voltage is changing with respect to time the octillion scope would also show us what is the voltage change that is happening across this capacitor so we give a name special name for quantities that vary with respect to time we call them as signals.

Although signals can also be used to describe variations of a quantity with respect to position or with respect to anything it is general understood to be a quantity that is varying with respect to time in electromagnetic we have seen situations where a voltage changes with respective time consider the familiar example of a transmission line problems so let the transmission lien be terminated with some impedance and then let us say we connect a certain voltage generator to this transmission line and if I keep an octillion scope or if I look at what is voltage od this particular point on transmission line.

This voltage will be changing and that could be corresponding to a signal which would be et voltage signal on this quantity would be a voltage quantity right now I also know that on a transmission line voltage at a point let us call this point at this across point will be not be eth same compare to a voltage at this square point because the voltage along the transmission line will vary and if this is not a signal generator but this a battery connected to be connected to the transmission line sum T = 0 then we can see that the voltage distribution along said will not exactly be the same as the input voltage.

The voltage of the point x will be different compare to the voltage at point which is denoted by the square we know and then the voltage will be different at the load side so right so these quantities if you where to freeze time essentially this voltage is not changing the battery is not changing with respective time but if I take constant time instant that I consider the voltage distribution along the transmission line I might see something like this the voltage at this point will be different the voltage at this point will be different right.

So if freeze time then the voltage of different points on the transmission line also is a function of the position at which I am measuring the voltage so the quantity that I am looking for is something that is varying with respect to the position on the transmission line and since we know that any position can be considered or respected by a vector what we actually have is some quantity which is varying in terms of the position vectors or different position vectors you have a different value for this particular quantity and the quantity that we were considering was the voltage.

So what we had considered was a quantity that was varying with respect to z when the times was considered to be some constant time t = 0 t0 so we have one example of a signal which represents the quantity such a voltage current or power or whatever that is changing with respective time and the we another quantity in this case it is the same quantity except now the quantity is depended not only on time of course because things are changing with time but it is also changing with respect to the position or with respect to the positions vector z can be associated with the particular position vector.

So what we have is a quantity that is varying with respect to positions you can even imagine scenarios where is this quantity whatever that quantity that we are considering cloud just be a function of position so if for example I take measurements on body if you know I take my temperature at different points on my body and then graph that one so this is my body let us this is the one that I am looking at this are my legs and these are my hands and if I look at the temperature at this point.

The temperature at this point the temperature here the temperature will not be the same the temperature at different points of my body will be different at different points so what we have is an example of a quantity that is varying as a function of position, and such quantity is called as a field in both examples that we considered that is in the transmission line example that we considered as well as in the temperature of the body that we considered the quantity that was varying was just a number.

Right so you do not specify temperature and then say temperature in the direction of 30^0 with respect to north you just say that it is just a number 30^0 C 31^0 C 32.3^0 C it is just a number and this number as we know is what is called as the scalar so what we have temperature at different points on the body as scalar field it is a field because it is varying with respect to the points on the body if it is varying with respect to the position and in the case of transmission line the quantity that we were interested goes a voltage again voltage is quantity that is scalar because you do not say voltage 30^0 with respect to north voltage is also a scalar and that is also an example of quantity that depends both on z and time right in case if freeze time at say some t = t0 then this is an example of a scalar field, so we have a correspondence of a quantity that is varying with respect to time we call it as a signal and then we have a quantity that is varying with respect to position we call it as a field.

Of course this fields can be scalar or vector in the case that we considered we have only considered the scalar fields which require just a number to be specified at every point in space and possibly in time as well, do come back to that electromagnetic example let me take some charge and then throw this charge at different points right, so I distribute the charge on a piece of paper okay.

So some places the charge density will be high some places the charged density will be low and the way these charged densities or the charges are distributed on this piece of paper forms a scalar field so if I denote the charge density as ρ and then this is a surface charge density we will talk about all this quantities later on so if I have this scalar quantity which I denote by ρ_s this will be a function of 2 points x and y.

Alternatively I can write this as ρ_s as a function of capital R where R is the position vector of this particular point on the piece of paper which is x, x^ + y y^ as we have seen in the previous class, so this is an example of a scalar field so please note the notation at every point at every position vector it is as though you are pined a particular number and that number is the quantity that is scalar field. How many a vector field look like?

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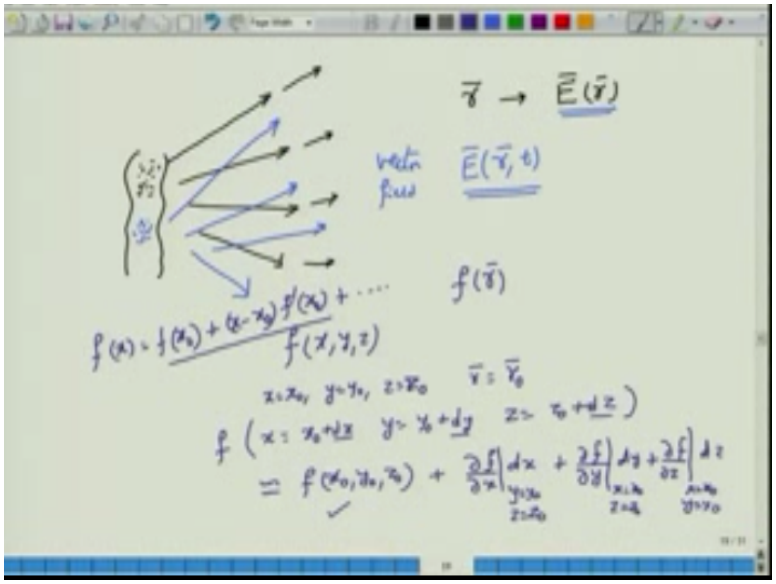

Suppose I take the same charge distribution out there right and I know that around charges there will be electric field this is something that we know from our high school physics classes that surrounding a certain distribution will be the electric fields and we know that electric fields are not just a number but they have to be associated both with the number as well as a direction, you do not say electric field is 10 volt/meter.

People will immediately ask you what is the direction of the electric field right, so a different points the electric field direction will be different we can also imagine that the amplitude of electric field to be different and they move away from the charge the amplitude will actually drop so it will be larger nearer and then the amplitude will drop according to what Colum's law that we are already are aware off.

But the point is at every position vector R I can associate a vector itself right, so I have vector associated at every position vector is corresponds to a vector field okay. We further can a have a situation where this vector field is changing with respect to time how might I obtained that I take this charged distribution and then I shake this charge distribution right I make this you know take it into a box and then just shake the box.

And for the charged which originally here they might just be displaced and correspondingly the electric field directions will also change right, it will not change immediately but it will act after a certain small amount of delay which we will talk about later but the point is that the corresponding electric field also has changed and if I keep doing this with respect to time what I have generated is a vector field which depends on both the position at which you are measuring this vector as well as with respect to time.

Not because of the shaking of the charge phenomenon. So this is a vector time varying field you can have a time invariant field such as this case E(r) and if E(r) is further dependent on time then what you obtain is a time varying vector field so you can have field which are time variant and time invariant. So this is all about a scalar field at and a vector field, okay.

Now let us see what kind of operations we can perform on this fields and what kind of operations are important for the study of electromagnetic, okay. The goal will be to understand these vector fields and the corresponding calculus that is associated with this vector fields in order to appreciate Maxwell's equations which form the basis of this Electromagnetics the entire subject of Electromagnetics.

With that goal let us look at what kind of calculus that these fields will obey, in our study both scalar fields and vector fields will be important let us first look at a scalar field, okay. So I have a scalar field and I will use the Cartesian coordinate system to develop most of the relationships here the relationship with circular, cylindrical and spherical coordinate systems are slightly more complicated.

So I would not develop them directly but you can look at the text book for the expressions okay to understand more about that one and you can also look at some of the exercises where we will derive this relationships for other coordinate systems so purely out of my convenience I am using Cartesian coordinate system but there is coordinate free interpretation of all these vector calculus operations.

Vector field calculus operations that we are going to perform, with that kind of caveat in mind let us look at the scalar field which we denote it by some F of x, y and z right. I could of course obtain an equivalent description of F with a position vector R but let us keep it here as f?(x, y, z) just to show you the Cartesian coordinate axis that I have taken, okay. Suppose I have this function I know the value of this function at a certain point at $x = x_0$, $y = y_0$ and $z = z_0$, okay.

In other words at a certain position vector $r = r_0$ I know what is the value of this function F, now if someone asks me what is the value of this function F the scalar function or the scalar field at $x_0 + dx$ at $y = y_0 + dy$ and $z = z_0 + dz$ this values dx dy and dz let us assume to be very small so that

I am not considering the point that you very far away located from x^0 y^0 and z^0 it is actually a very close point and answer to this question what is the function at this point the answer to this question is given by Taylor series right so f of x is given by f of x_0+x-x_0*f prime of x_0+ and so on if I retain only the first two terms then I know how to write this f of x at x^0+dx y^0+dy, z^0+dz oaky.

And that value is given by approximating with this one with Taylor series with only first term involved this is given by f of x_0, y_0 and z_0 this is something that I already know because there is the value that I already have measured and there is variable with me so this + the partial derivatives of f with respect to x multiplied by change in Δx itself when evaluate this partial derivative.

You have to assume of course that y is y^0 z is z^0 okay and then you have the other contribution because of the change in the y direction so that would be Δf/Δy again this Δf/Δy should be evaluated such that x is x^0 z is z^0 and finally you have a contribution because of the partial derivative of f with respect to z evaluated again at x is equal to x^0 and y is equal to y^0 multiplied by Δz okay or rather in this case Δx is dx Δy is dy and Δz is dz. So let me rewrite all that, okay so this is dx, dy and dz. You see that the change

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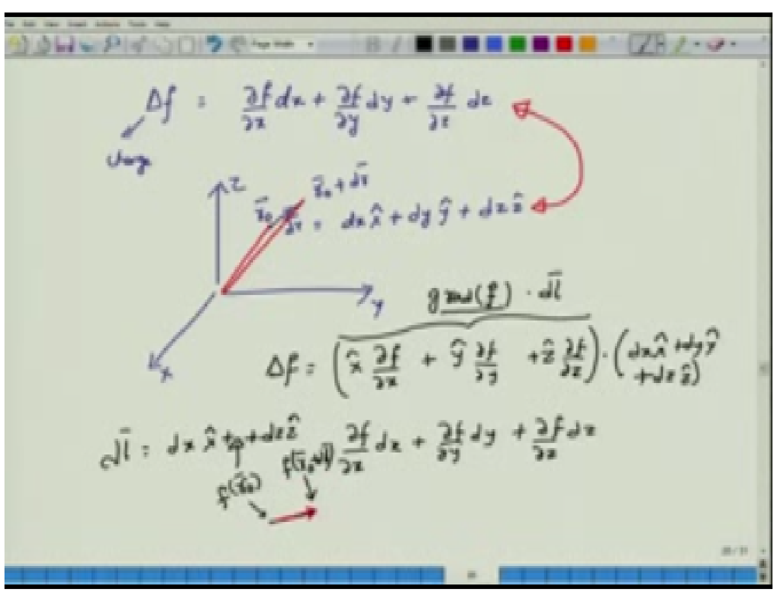

In the value of the function which we can denote as Δf if Δ denoting the change. Change in the f given the value of f at x0 y0 z0 if simply given by $\partial f/\partial x$ x dx + $\partial f/\partial y$ x dy +$\partial f/\partial z$ x dz. Why do I need to take partial derivatives? Because f is the function of x, y and z okay, it is not just the function of 1 particular variable, it is the function of more than one variable, therefore it is the partial derivation that needs to go in.

And when I am evaluating $\partial f/\partial x$ that is partial derivation of f with respective to x, I have to keep the other two coordinates constant. The other two coordinates are y and z and they have to be kept constant. So notice the way we have written here is that you have partial derivatives oaky, multiplied by the change. Now let us draw some pictures, suppose I go back to this three dimensional picture x, y and z and locate this point as the point r_0, r_0 is the point where I already know the function.

And now I have consider a new point, which I can call as r_0 + dr right, where there this dr this small displacement vector which is given by dx ^x + dy y^ +dz z^ right. So if I go along direction x by a quantity dx along y by dy, along d/ dz and actually going from one vector which is the position vector at r0. To the new position at r_0 + dr. something that we have already covered here. Now these two equations if you look then you might come up with a very interesting way of combining the equations.

And the way I combined the equation, you can observe here I will first define vector associated with this partial derivation, so I will cal this as x^ $\partial f/\partial x$ + y^ $\partial f/\partial y$ + z^ $\partial f/\partial z$ right all these partial derivatives being evaluated at the point r_0 and then it take the dot product of this with respect to the displacement vector dr, which is dx x^ + dy y^ + dz z^. the net result is the same because x. x will give you $\partial f/\partial x$ x + $\partial f/\partial y$ dy because you have y.y x. y and x.z are 0 similarly y.x and y.z are 0 and so on right.

So what I obtain is the same expression but what I have done is to define a vector okay and I call this vector as gradient of the scalar field f okay, so I cal this as the gradient of the scalar field f okay this is connected or this is dot product or you have to take the inner product of this one with the corresponding displacement vector dr and it is one of the convections that instead of talking about the displacement vector dr we talk about the change of a function along a particular line and therefore we talk of the line element dl.

Remember from the Cartesian coordinates that dlф where the line element that we had pick which was a vector line element and this is given by dx xф+dy yф+ dz zф right, so you put in the remaining term over here. So what you have seen is that if I pick up any direction as along as a direction is reasonably small, then the change in the function so I know the function value here at some position r_0 and along this particular line, so let me draw that one in a red color so along this line element the corresponding value of the function f(r0+dlф) right dlф3 being the line element. I can obtain if I know the corresponding value of the gradient and we have a special notation.

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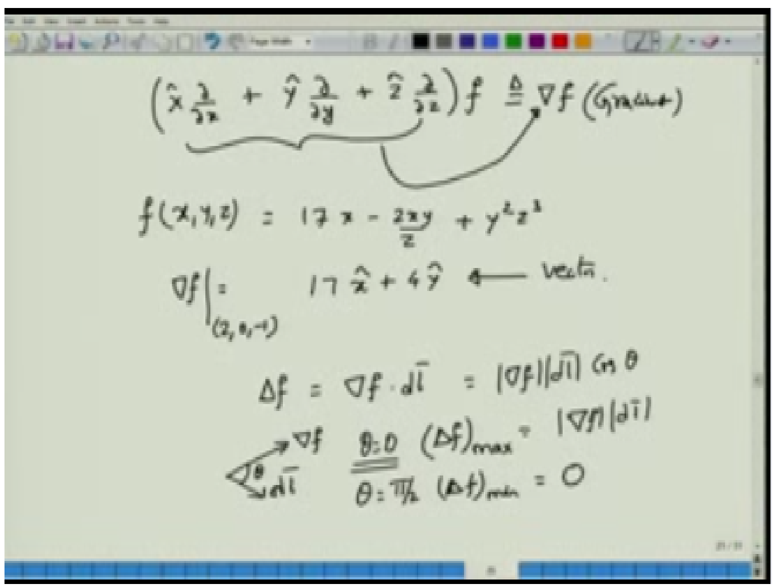

For this particular quantity which we call as the operator, we call this as the ∂ operator the reason for this calling this as the ∂ operator is very clear because these are only partial derivatives only this will make sense only when we take this and operate it on some quantity, in this case we operate this quantity on the scalar field f, in order to obtain what is called as the gradient of f and this gradient of f is a vector quantity which can be used to dot with any other quantity.

And the way we denote this entire thing that is in the brackets is by writing the inverted triangle and calling this as a napler or the ∂ operator. So this is an example of one of the vector calculus or the operation that you are going to do with the scalar quantity called as the gradient. Let us look at the example a numerical example and then discuss what is the importance of this one, okay.

So let us say I know that f(x,y,z) you know is given by sum $17x-2xy/z+y^2z^3$ okay, this is some function that has been given to us and we want to find out what is the gradient of this, the gradient of this is of course from the above formula that we already know so is that $\hat{x}\partial/\partial x+y\phi\partial/\partial y+z\phi\partial/\partial z$ operating on the scalar field f which I already given here and what I will do I will just give you the value of this ∂f at 20 and -1.

So I need this as an exercise you just have to complete this partial derivatives and evaluate this partial derivatives at 2,0 and 0-1 and the value that you are going to get will be something like $17\hat{x}+4\hat{y}$ as a result this is a vector, we already know that this is a vector, okay. So might ask what actually are we doing here, I mean why did I introduce a gradient operator observe something that is very interesting.

The change in the function which is given by the symbol that you know it is a proper triangle Δ, Δf as you move along a particular line is given by ∂f.dlϕ, now I know that this ∂f.dlϕ is a vector or is a dot product so I have the magnitude of ∂f right, which is a vector now and then I have the magnitude of dlϕ vector and then cosθ, where θ is the angle between so let us say this is ∂f and let us say this line is the angle dlϕ or rather this line is dlϕ and this angle θ is the angle between these two quantities.

If you move along the direction of ∂f cosθ will be equal to 1 and then what you obtain will be the maximum change right, so the unction undergoes maximum change if you move along the direction of the gradient for which you know cosθ will be equal to 1, so this will happen when θ=0 so that cos0 will be equal to 1 and the maximum change if you move a unit of dlϕ is given simply by taking the magnitude of the gradient and then multiplying it by the magnitude of dlϕ, this is of course just the magnitude right.

And if you move in the direction where the function is actually constant then θ=π/2 then Δf change okay, the Δf change will be equal to 0 because in that case θ will be equal to π/2. To give you more understanding of that one let me give you a simple picture out there imagine that I have this you know surface palm of y hand and I place this particular object okay this is not a very nice object okay but let me place this object.

Now what I do is I take a needle okay and I mark the needle here okay I mark the needle and then I paint some ink on the needle and then I push the needle on to this okay I push the needle until it hits my palm of course it will be little bit of paint for me but I am able to with stand that one then I pull the needle back.

So I say push it and pull it back I know if I have color needle and let say that color is removed from this object you can imagine a potato for example okay so you can imagine a potato being kept on a nice you know nice surface over here and then you plunging the needle and then remove it and whatever the marking that you would see on the you know this ink is gone so that ink would represent the height of this particular object with respect to this flat surface at that position.

So I have place that object here so at this point I plunging the needle pull it back I know what is the height, so I can actually create a different points you know I can measure this needle and then create a height and then you can see here that the object is curved a little bit so the height that you are going to measure at this point will be different from the height that you are measure at this point.

In fact we call this in the topographic class that this surface is slightly you know there is a bulge in this surface there is a technical term for this which elevated right. So this is actually an elevation, a small elevation is what you can actually see here right if you imagine a potato being kept on a flat surface and then you can now measure this needle height and then actually jogged down you know you make a 2D picture of that one what you obtain is what people obtain when they actually created maps.

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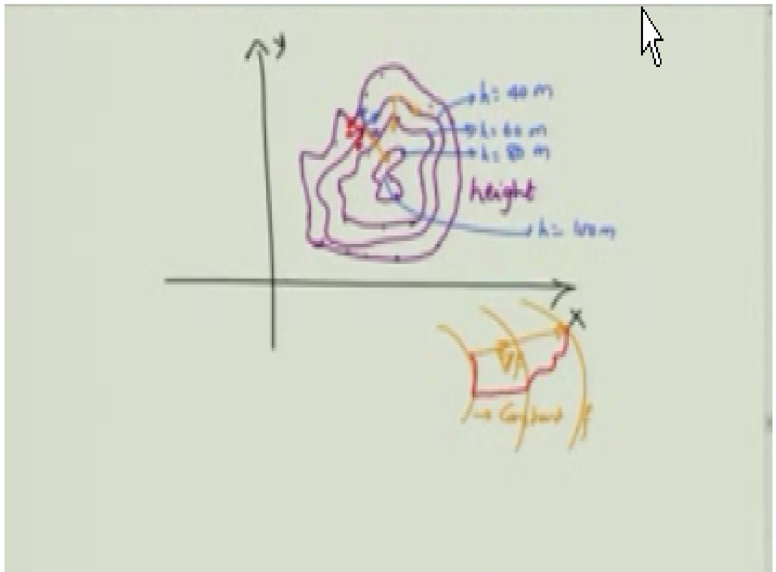

So on the piece of surface that you have which we will call as x and y axis at different points you measure the height and then let us say you connect all the points for whose height will be the same. So at different points might actually have same height right? So if you connect all those points which have the same height you end up obtaining what is called as the contour map okay, so you can see that this is the contour map of a hill because at the center you know the height would be maximum and all these lines that I have drawn in this purple color corresponds to a constant height.

And along this constant height, so this is the constant height map that I have drawn so at different points on the x and y place, so it is imagining like a potato kept on a flat surface or a mountain that is kept on a flat surface and then measure the height at different points by plunging the appropriate measuring tools and then calculating the height and then creating the contour. Now let me show you something suppose I am at this particular point okay and there is a certain contour along this point okay?

What I do now here is from this point I just you know imagine taking short steps okay by my stick or something so I just imagine taking s short step along this. Now observe along this blue line right and actually moving along the contour of a constant height, so let us say this height is equal to 40m this height h is equal to 60m and this height h is equal to some 80m and this point let us say is equal to 100m.

So you have a hill which is about 100m height okay. So along this blue line I am actually moving on the contour itself I have taken a short step and I have seen that there is no change in the function that is because I am actually moving on the contour right. If I move here I see some change but not much if I move here I see some change but not much but if I move perpendicular to this contour right so let me go back to this one.

If I move perpendicular to the contour then I experience maximum change right, so from one surface to the other surface if I move perpendicular surface of constant heights or contour of constant height I actually experienced maximum change okay. And this maximum change that I am going to obtain will be the change along the δ f if I move on the same contour of course I am not saying any change in the height because these are the constant height contours.

So from this you can imagine that or you know you can observe that the gradient that you are going to find out will always point it will be perpendicular to the constant f value, so these are the contours of constant function values and this would be the gradient which would always be perpendicular and as you go from one contour to the other contour you know of different values of f if you keep moving along the gradient you will actually be able to reach the maximum point you know much faster than you take any other point any other paths.

So if you work to move along this path okay then you would take you know longer time because here as you move you do not see any height and then as you move along here you see a maximum change of the function and then as you keep moving along here and reach the point which is the top of the hill you would have take a lot more time than what you would have taken. So this is the physical significant of gradient.

Gradient always represent the maximum change and in fact if you start if you want to decent from the top of the hill you have to just find out what is the maximum gradient or maximum slop and then dissenting the opposite direction this is called as the steepest decent and it is one of the very useful signal processing operation that you are going to do okay. So this complete discussion on gradient we will see couple of other operations that we are able to perform with the Del operator in the next module, thank you very much.

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NPTEL Team
Sanjay Pal
Ashish Singh
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Ram Chandra
Dilip Tripathi
Manoj Shrivastava
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