Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module-24
Rectangular coordinate systems

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Hello and welcome to NPTEL mook on applied electromagnetics for engineers. In this module we will briefly recall coordinate systems, and to introduce coordinate system let me ask you by a question. Let me begin by asking you a question, suppose you have a long road okay, the road spreads all the way of the - ∞ to + ∞, well I gave with a game a little bit, but anyway. So there is a long road okay.

On this road there are houses which are spreading all over the road and these houses are kept at a certain distance. So you have your house, then you have a next house, next house, next house another house behind, beside you, beside you and so on okay. Suppose you want to invite your friend to lunch on a particular day, how do you go about inviting your friend. Let us say you can call up your friend each friend has a phone, so you can call up your friend, and invite this person to lunch okay.

Remember it is a long road, there is nothing to distinguish your house from your friends house or your house from a different house. So how do you go about inviting your friend, you might try a simple idea, you might say that well, I know which house I am in. So I am going to call my house as a reference house okay. And then I count the house in which my friend lives, and then pickup a phone, tell me him or her that I am at the reference house, please come.

However, the friend would immediately say well, no, no I am actually at the reference home, because one reference point could be ask you any other reference point in a infinitely long road right. So any point could be reference. So therefore, these two friends do not even agree upon which one is the reference road. On the other hand, you could say something like this, you can

say okay, I am going to make my house as a reference house. But I am going to put a number to my house okay.

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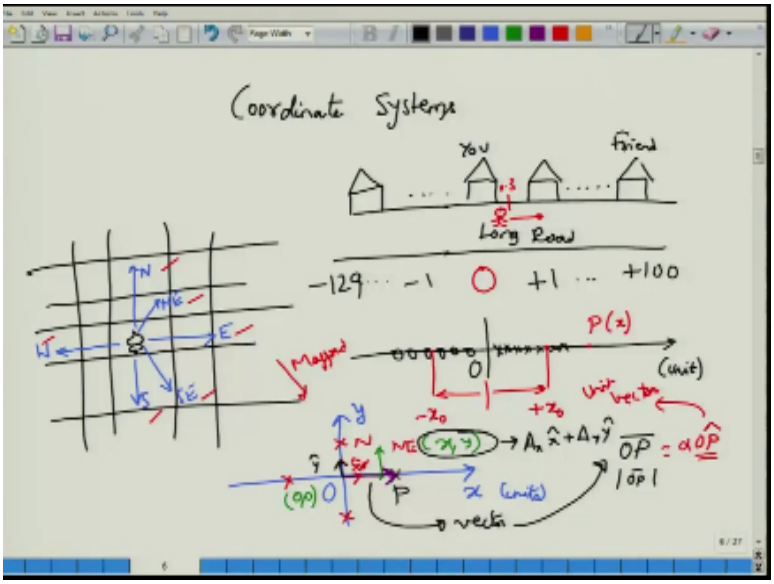

I am going to say a number 0, let us say my friend is present as I start counting, I will find that my friend is actually at the 100th house okay. Also I note if I come outside and I am looking at this one from here, this corresponds to the sense of right and this would be the left okay. So I am actually looking this is me, who is looking, you know this is the eyes at which I am looking. And then you see that to this I have right houses to the right, and houses to the left.

And at 100th house to the right is my friends house okay. So I pick up a phone and then say, well you come out of the house, look to the left and then come to the 100th house. What would that person do, he would come out or she would come out your friend and then they would look at their house, their house will carry a number 0, and since it is to the left that has been mentioned that person would walk all the way to your house which corresponds to 100 left to that person okay.

So this way you have actually solved the problem, otherwise you would never be able to invite your friends nor you would be getting an invitation to your friends house for lunch or a dinner. So this seems to be very simple way of representing locations okay, by making one of them as a reference. Now it could be even made even more simpler, I know that there are only two directions here right, it could be right or left direction.

And every direction right or left there is a certain house along the direction right. So I can begin to actually label this in a slightly different way, I call all houses to the right as plus and all houses to the left as minus okay. So this next immediately next house to me on the right side will be +1 and so on my friends house which is now +100. Similarly, the houses to the left will be -1 immediately, and then there is another house at -129 okay.

So these numbers along with the sign would indicate particular locations, but there is nothing specific about, you know having only houses. Let us say I have a chicken okay which can walk at any place okay. This chicken can go along this road either to the right or to the left and it can stop at any point, and lay an egg on that. So if I want to describe the position of this egg right, I cannot just say +1, +100 because this could be at some unit distance 0.3 from 0.

So I need to actually go to the real number system in order to specify every point on this long road and I can do so by the simple number line arrangement that you are all familiar with. I call my house as a reference house call it as 0, and then the increasing distances from my house will be measured by a real number, you know it is like a scale and I am measuring that one by a real number there will be some unit which I would have chosen you know I could have measure all my distances in terms of meter kilo meter or centimeter and so on no matter what distance I have choose any point on to the right or on to the left can be specified by a real number.

The only distinguishing point so this two points are actually are the same distance right so this two points are at same distance but then they are separated or distinguished by the fact that one happens to be to the right of 0 the other happens to be left of 0 so this would be +x0 this would be –x0 okay.

Any point on the line can be specified by a single number let us called point itself as P and the value of x of that particular point will essentially be the coordinate or the location of this particular point I can extend this idea of 1 dimensional coordinate system that is on a road to a plane okay imagine I want to you know I am actually able to go every were on the plane you know on a field or on a road that I am going.

Then let us say my car is going around like this I do not know whether this is nice picture of a car us let us assume this is a car okay the car could be travelling any day along the north

direction could be travelling in the east could be travelling along the south or could be travelling on the west okay.

Now the car could also be travelling along the north east or it could be travelling long south east okay instead of talking about a north east west and south I can go back to the number systems and then obtain a simpler representation of this, this is called as cartation representation I consider again any point of infinity spread over plane as appoint 0 or the point origin okay and if I move along this line which is shown to be increasing along this way if I move along this line and increasing the distances along this road okay which I will call as point x okay.

There is a label which I am going to make x and this would be a units which have I am going to assume it to the same units of the distance for both x as well as another direction which I would call as y okay just this two directions are sufficient because north would then happen to be +y okay at a certain distance south would happen to be –along the y direction east would happen to be + or positive x west would happen to be negative x.

And if you want to specify north east you can specify this north east by asking someone to move a certain distance along east and then move certain distance towards east what have actually done is to create for every point okay two numbers which would be x and y and this is also giving algorithm to move along and can reach different points.

I can move long one axis as would call it as x axis and then move along the y axis alternatively I can move first along the y axis and then take right turn left turn whatever the turn that I want to take I can then move and approach the other point the direction which I followed the path which I follow does not matter to the location the location if exists by the two numbers x and y that I provide of course the origin will be denoted by 0 and 0 okay.

Again you will notice that x itself can be positive and negative y itself can be positive and negative in the previous case you just had one number which could be the positive or negative which could be denoting this particular location of the house here note to specify a location you have chosen two numbers and look at that you have chosen you have two blue lines which we have drawn to arrows at the end.

This arrows simply signified that there increasing in this particular direction but if I now stop at this red point which I would originally the right point okay and then draw one arrow and terminated at this point which was the red point earlier so all this points as some points p okay if I draw an arrow from the origin and stop at this point what I have done is to essentially describe the vector okay let us call this vector as vector OP where O is origin of the vector P is the ending point of the vector okay.

The length of this vector OP will be denoted by this you know putting the vector in between this parallel lines okay and you can also notice that I can define a unit vector along increasing direction towards x and a unit vector along the direction of y and then I can describe OP vector as some alpha times the unit vector along OP which I will call as $OP^{\wedge}$ okay of course if I take instead of OP if I take x or if I take the x axis itself then I can define a unit vector along the x axis which I will further denote as $x^{\wedge}$ so here you can see that $OP^{\wedge}$ is essentially the same $x^{\wedge}$ that we are using and these hat vectors are called as the unit vectors.

So this is something that we already are familiar with so the same grid that we had which had north, south, south east, east north east west and so on as been now mapped as we will call as now been mapped on to a 2 dimensional grid in which you have placed x real numbers and real numbers y in the two increasing orders you can also note one more thing I have x hat vector I can similarly defined a $y^{\wedge}$ vector which would be just one unit vector that is the length of this vector would be just one unit.

And it would began at the origin and terminate it at 1 unit along the y axis okay so just as x started at the origin and was terminated at 1 unit away from the origin along the x axis I can do the same thing along the y axis then any point x and y can be associated with a vector which can be returned as $A_x x^{\wedge} + A_y y^{\wedge}$ what is this $A_x x^{\wedge} + A_y y^{\wedge}$

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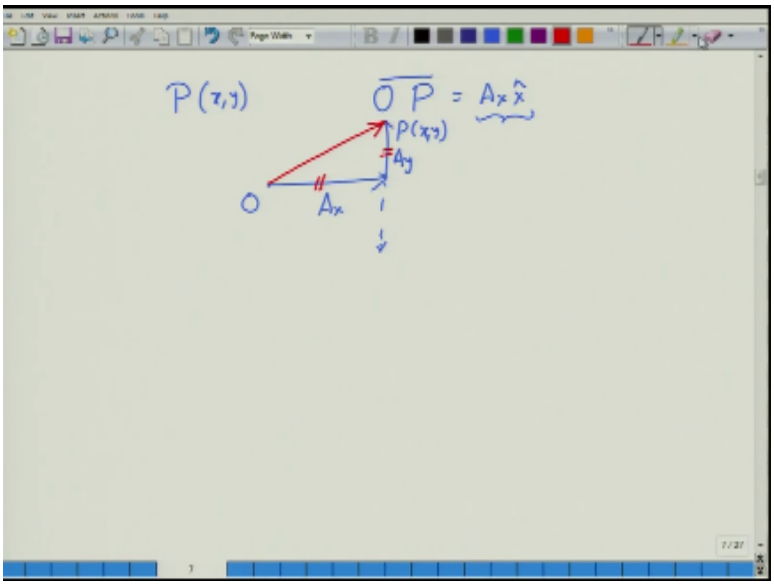

The point P which was you know presented x, y okay this gives you any point on the grid that we had considered then if I draw an arrow from the origin point all the way up to this point p the I will be defining a vector which we can call as vector OP and this vector can be given in terms of so how do I move in order to go from point O to P first I have to take A_x units along the x axis which is along this no right direction or left direction depending on the sign of A_X so I move along this axis by an amount of A_x so by itself $A_x\,x^{\wedge}$ is a vector who is magnitude is x and angle is along x axis right.

So magnitude is of course magnitude of A_x it cannot be +1 and then I move either up or down so I could move in any way I want so I can move either up or down if the point P happens to be located here then I have to a distance of Ay and I would have arrived at point P although actually started at origin O and arrived at point P okay I did so not by moving directly from O to P I did so by moving along x and y okay but no matter what I did weather I move directly from O to P or I move from A_x and then you know move to and intermediate point along axis and from there I moved along to the y axis have arrived at the same point.

So insisted if I draw a line which is a red line that we have drawn here this corresponds to the actual vector origin here and it as a tail here okay you can see the disk vector as been returned as the sum of these two vectors right so this is geometrically how you add two vectors you can take the first vector and to the tail of the first vector you put the head of the 2^{nd} vector and then you

draw from the head of the 1st vector to the tail of the 2nd vector to geometrically complete this triangle.

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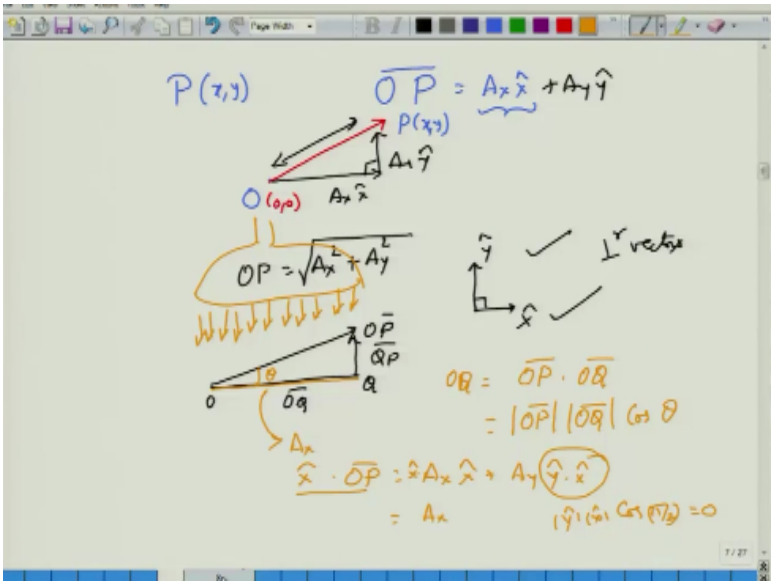

And you would obtain the sum of these two vectors and if you now remove these A_x and A_y this would simply be a vector that begins at the origin which is described by the points or coordinate 0, 0 and terminates at P(x, y) right so this an example of a vector the way this vector can be obtained is to add two vectors okay 1 vector will be along the x axis A_x $x^{\wedge}$ A_y $y^{\wedge}$ what would be the length of this vector OP or the magnitude of the vector OP the length of vector OP is very simple right.

And it can be related to this A_x and A_y how so I know Pythagoras theorem tells me that if I tackle this right angle triangle then this distance that I have is = to $A_x^2 + A_y^2$ alternatively length OP of this vector okay is given by square root of $A_x^2 + A_y^2$ finally notice one thing we had a vector along x axis 2which would be any vector but for now consider the unit vector x and then you had one more vector along y direction which was the unit vector so let us make it unit vectors here it is not looking unit.

So if I now make it into a unit vector right or consider two units vectors I see that geometrically they are meeting in such a way that the angel between them is 90^0 rights, so these two vectors are called as orthogonal or perpendicular vectors oaky so these are perpendicular vector. However if

I consider the vector op so this is a vector op this vector op is not equal to the vector let us call this a sq okay.

So there is a vector q here and a vector qp here so you can clearly see that op is not perpendicular to oq or oq is not perpendicular to or other sorry qp is not perpendicular to op. in fact there is an angle between the two and if you now imagine that someone is shining light from the top you actually constructed this one in someone is shining light on to the top what would be the shadow cost by this line that shadow will have a length given by this vector oq you can imagine someone you know has a torch and then you know shining light of this one and you can immediately see that the length of the vector op along this direction in which the light is shadow.

So which is along the direction x will be exactly equal to the magnitude of the vector oq so in fact ion mathematics we can find out the vector or the length oq by performing what is called as the dot product between the original vector op and the vector oq. So you can take any two vector and perform a dot product which is the result which will give you a number which is given by the length of op multiplied by the length of oq and cos θ where θsit he angle between the two vectors.

And in fact what would be this length this length you already have seen that it is equal to ax so in fact we can take the original vector op = ax x hat + a / y hat and then dot this vector op with x hat okay, which would be a unit vector along which I am going to dot okay. And then I see that and sorry I forgot to mention is that it does not matter in which length you know perform the dot operation you can switch oq and op orders it does not matter is I can either do x hat or op op dot x hat I get the same regard.

So if I do this or I take the dot product on both sides I know that this left hand side would be the component of op along the x axis okay, and this would be given by the component ax itself because y and x are perpendicular to each other and dot product will obviously be equal to 0 for this case simply because the magnitude of y magnitude of x are one but in the angle between the two is 90^0 which is cos π/2 therefore this is equal to 0.

So this is the dot product which we will now use in the next module to go from one coordinate system to the other coordinate system. So I hope you understood the two dimensional Cartesian

coordinate system as this is called we not move on to three dimensional Cartesian coordinate system.

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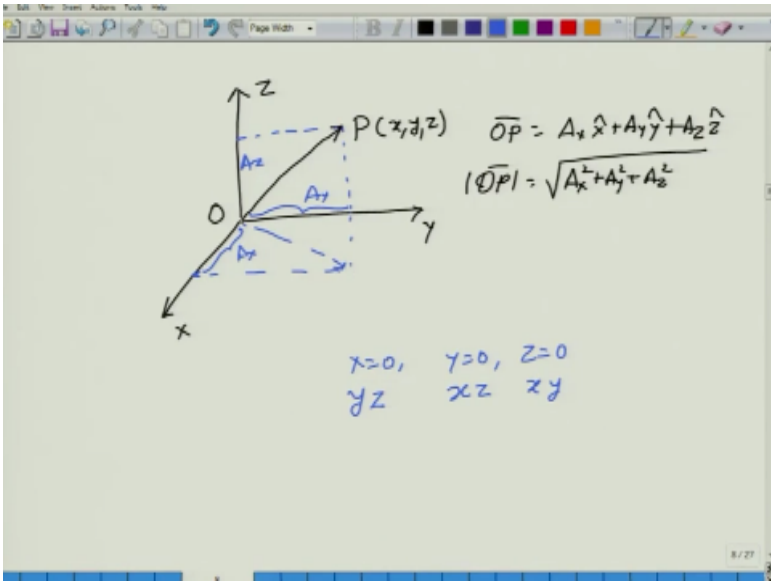

The three dimensional coordinate Cartesian coordinate system I imagine that I have three axis x, y, and z okay so I have three axis over here and any point p on this axis can be located you know by specifying three number x, y, and z at before I still have a vector op which can be written some ax, x hat + a/ y hat + az, z hat and the length of op which is the vector length is given by $\sqrt{ax^2 + ay^2 + az^2}$ what are the ax^2 and az these are the components of the vector op along x and y direction okay.

So if go to this way and then see this is the component that you would obtain as Ay this is the component Az and this is a component Ax, so you move along Ax then move along the y direction by amount of Ay and then go up to get to Az or if z is negative you go down okay. You

can also see that this three dimensional system that we have consider is actually created by three different plains x = 0, y = 0, and z = 0. X = 0 plan is where you specify both Y and z, y = 0 is the plain where you specify x and z and z = 0 is the plain where you specify x and y okay.

We will talk more about all this coordinate system and then we introduce other type of coordinate systems in the next module and till then, thank you very much.

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