Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 21
Transient analysis with reactive termination and Time-domain reflectometry

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Hello and welcome to the NPTEL Moore called applied Electromagnetics for engineers, here in the last few modules of the transmission line theory that we have been discussing, in the previous module we were discussing the time domain behavior of the transmission line where the source impedance was in general not equal to the characteristic impedance and the characteristic impedance of the transmission line was not equal to the load impedance.

This resulted in multiple reflections with the output approaching the steady state value in an asymptotic manner that is it does not approach an single time duration what could be the final value or the steady state value but it approaches in the series of stare cases or ring kind of a approach so it is the transient corresponds that we were interested in, in this module we will consider a general type of a termination.

Specifically we will be interested in those terminations which present capacity or inductive type of loads for the rest of the transmission line circuit, we will see that the concept of reflection coefficient and the lattice diagram that we studied in the last module will not be directly applicable here because the reflected voltages will not be or reflected currents will not be in the same shape as the incident voltage and the current.

Both reflected voltage and the reflected current will be functions of time even then the incident voltages are not functions of time or their constant with respect to time okay, so we will see how to solve such problems we will consider the simplest case of an inductive termination first then go to capacitive termination. So what is the problem that wearer discussing.

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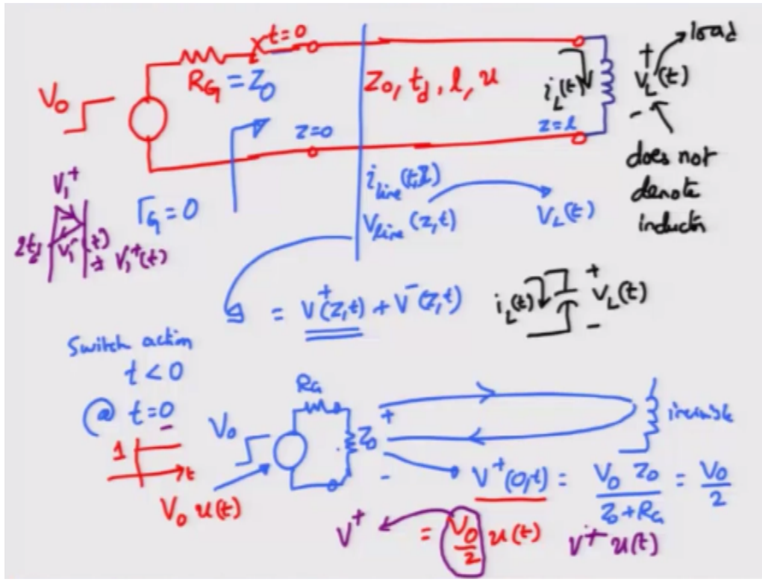

Remember we had a source somewhere right this source we had considered to be a step kind of a source we are considering the idealized step voltages the amplitude of the step voltage let us say is equal to V0 and this was connected via resistance or the internal impedance of the source which we called as RG in the previous module we let RG to be whatever the value it was and therefore there was reflections at the generator side as well.

However to consider that approach with reactive terminations will be little complicated one as to perform Laplace transform on the signals on the functions that we are considering in order to solve those problems where RG is in general different from Z0 so what we do instead is to assume that RG is equal to the characteristic impedance of a transmission line which immediately Γg = 0 and therefore there is no reflection.

When the reflective voltage comes from the load travelling towards the generator there would not be further reflection from the generator side as before we will turn on this transmission line circuit or the source so we will turn on this source at a time T = 0 okay, connecting to the rest of the transmission line which will have the characteristic impedance of Z0 and at time delay one way propagation time delay of TD.

Which comes because of the propagation length L of the transmission line and the velocity of the wave voltage wave on this one will be given by the velocity U not UP velocity U okay, so we connect this transmission line to a reactance as I said we will consider first a case of a pure inductance, if we label the voltage across the inductance as V subscript l(t) please note that this subscript Ls does not denote inductor, okay.

Because later when we consider the load to be a capacitor we will again label the voltage across the capacitor as VL(t) okay and the current through the capacitor as IL(t) indicating node okay, so these are not indicating the inductances they are simply indicating then this is actually a load voltage so as I said there will be a current through the inductor which we will label as IL(t) at any pint on the transmission line circuit, right.

At any point on the plane on this transmission line circuit there will be some you know line voltage and this line voltage at any point on the transmission line of because of the continuity must be equal to the node voltage at the load side we can choose $z = 0$ as the source and $z = L$ as the load or we can choose $z = 0$ as the source and then $z = -L$ as the generator does not really matter which one we choose.

Let us arbitrary go with this coordinate representation so $z = 0$ denotes the sending wave or the generator $z = L$ denotes the load, so there is a certain line voltage which we recognize must be composed of the incident voltage which is changing along the line plus the reflected voltage which is $v^-(z, t)$ so this is in general the line voltage okay, so we have seen this expressions right from our second or third module where we were discussing the line voltage.

Similarly we will have the lien current as well, the line current again will have both incident current as well as the reflected current this line current will also be changing along the length, okay. What is the incident voltage at this point, or what is the incident voltage v^+ let us look at the circuit here, before just which is closed that is $40 < 0$ we assume that the entire circuit is not energized which means there are no initial current or no initial voltages at any point on the transmission line.

So at $T = 0$ when we turn on the switch the generator as still not seen the load here the load is invisible to the generator this is because the first and the incident voltage has to go get reflected and then come back right only when it comes back then the load which is inductor, capacitor or it could be anything that becomes visible to the source here. So we have an amplitude of V0 there is an interval resistance of RG.

So as for as the source is concerned at time T =0 you actually see the characteristic impedance of the transmission line has the input impedance. So that impedance seem looking into the input terminals of the transmission line will appear as the characteristic impedance z0 because that is what the source can actually see at t =0, consequently it will generate an incident voltage which we will label as v+.

So at the source side it could be v+ (0, t) which would be v0 divided equally between z0 and RG in this case we have considered RG to be equal to z0 therefore the voltage amplitude will be v0/2, however please note that we can consider the switching actions so this was the which action, right. The action of a switch is to connect the generator to the list of the transmission line at t =0.

We can model that reaction by modifying the voltage source as v0 u(t)where u(t) is a unit step n function, okay which will be 0 initially for t <0 so if you plot these are the function of time this would be 0 initially and it would have an amplitude of 1, so obviously multiplying this unit step function by as voltage V0 we get the step voltage, so what we actually launch as the initial voltage v+ at the generator side is actually v0/2 times u(t).

Let us further introduce a simple or other simplified constant instead of carrying this v0/2 all the time let us call this as v+, v+ denotes the amplitude of the initially launched voltage, if you remember our and hence you know the lattice diagram this step is similar to the first lattice diagram where you are calculating v1+ so fortunately that step is still the same, however the rest of the step will not be the same, okay.

And instead of V1+ we call it as v+ because we know that there will only be single deflection so this v + s goes all the way to the load and from the load gets reflected and comes back to the generator and will be observed later on, so v1+ would travel v1 – would reflect back okay, however v1 – will be in general not equal to some constant times v1+ (t) okay so it would not be exactly of the same shape.

Just changed in the amplitude whether it is positive or negative amplitude change it will not be equal to the incident voltage it will be different from the incident voltage however once it comes back at a time 2 times steady this would be completely observed there would not be further reflections from the generator n, to summarize at T= 0 we launch a voltage whose amplitude is let us say v+ and it is u(t) that we have launched. At what time does this initial incident voltage reach the load.

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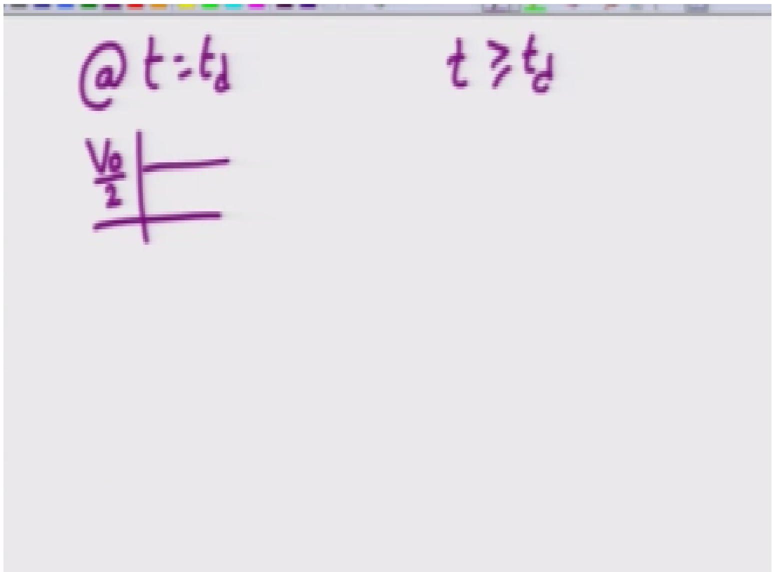

It will reach the load after a time t = td thus a time t = td this voltage would have reached which is having an amplitude of v0/2, right. So this voltage reaches the load side so which means any operation at the load side must begin only for t ≥ td right, so we now have an incident voltage what we want to do, is to find out what would be the reflected voltage and then you know add the incident and the reflected voltage in order to calculate voltage at the load and voltage at any point on the transmission line.

We can do so if we remember that the line voltage consists of incident + reflected voltage line current consist of incident current and the reflected current and there is a relationship between these quantities with respect to load voltage, what is the relationship? Let us look at that relationship now.

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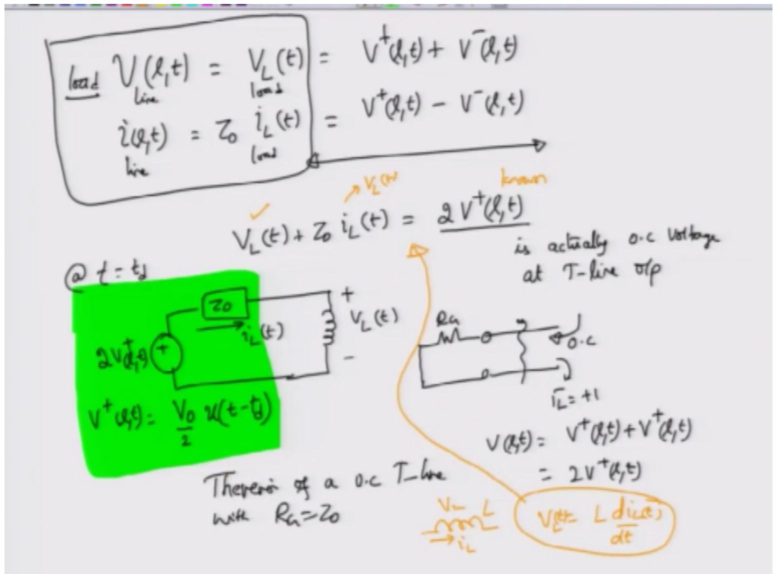

So at t= td and thereafter we know that line voltage that we considered let us call this as some v(l, t) where v(l, t)will be the total voltage I have dropped the subscript line here into this one I have dropped the subscript line here and this is the load side voltage correct, so v(l, t) which is the load voltage or this would be the line voltage evaluated at the load terminal must be equal to the load voltage vl(t) and this is further equal to v+(t) which is at the load side okay.
So let us so v+ of the load side + v- of the load side okay the + and – of course indicating the forward and backward traveling waves, similarly z0 times i_L(t) I have just moved z0 onto the left hand side of i_L(t) just to avoid writing a fraction here but you can clearly see that this is given by z0 i_L(t) is given by –v + -v^- all evaluated at the load side, correct? This of course is the line current i(l, t) so this is a line voltage this is the line current.

This is the load voltage this is the load current and these quantities are obviously equal when you evaluate this one at the load side and of course they are equal to these right hand side quantities, let us now add both equations and eliminate v^-(l, t) although our ideas is to find v- let us noe follow this procedure let us add eliminate what we will do is, we will find one unknown from that unknown we will find out the reflected voltage.

So adding these two we get V_L(t) + z0 I_L(t) that must be equal to 2v^+(l, t) is there a way to understand this equation better, well this is an example of a KVL type of a equation right, so here you have a source which has an amplitude of 2 or which has a this one of 2 v+ (l,t) if you remember further the v+(l, t) is nothing but v0/2, u(t, -td) because we are considering this one at t = td correct.

So beyond that only your voltage has arrived at the loads and hence from the load side it will be reflected so because of this one way propagation delay v+ the incident voltage at the load side will be equal to the step voltage of amplitude v0/2 that have been delayed by td, so go back to this one, so this must be equal to z0 let us say the current flowing through this one must be $i_L(t)$ right and then you must have a load voltage which is $v_L(t)$.

What is the load here, it is the inductor that we have considered does this you know if by applying KVL you can show that this equation is actually represented by this further very curiously this value of 2v+ (l, t) is actually the open circuit voltage, correct. The open circuit voltage at the transmission line output, correct? Because you take the transmission line over here and then connect this RG to whatever the VG you wanted to connect here, right?

If you leave this one open circuit then clearly γl will be equal to +1 which implies the total line voltage evaluated at (l,t) must be equal to the incident voltage but because γ l is +1 the reflected voltage will also be the same, so if this will be v +(l, t) which is equal to 2v + (l, t) okay and further if I remove this VG that is if I short circuit this VG and look from this side what is the load input impedance over here.

I feel that the input impedance for a open circuited load for this one would actually be equal to z0 or other I should look at the input impedance from this side so I see that the input impedance in looking at this point will be equal to z0 because this RG is now equal to z0. So essentially what we have written this portion of the circuit okay, so this portion of the circuit that we have written here in this green color thing that I have written in that box.

That is actually the venin equivalent of a open circuited transmission line with RG = z0 okay, so this is the open circuited equivalent circuit and from there we have simply attaching the load and then proceeding now to find out the load voltage and the line current, how do I do that well this is an inductor, what is the relationship for an inductor, in terms of the voltage and the current? For an inductor we remember that VL is the voltage across it Il is the current through it and the inductor has a value of L inductance you know measured in Henry.

Then the voltage VL will be email to LD iL /dt so I can substitute this expression VL(t),IL(t) so I can substitute this expression into this equation right and then try to find out VL because this is unknown this can be expressed in terms of Vl(t) or its derivative of that one as such this is known to us because this is the input that we have considered so this is known to us, right. So this 2v + l(t) is actually known. Because v+ the incident voltage at the load is given to us v0/2 u(t – td) so let us do that one.

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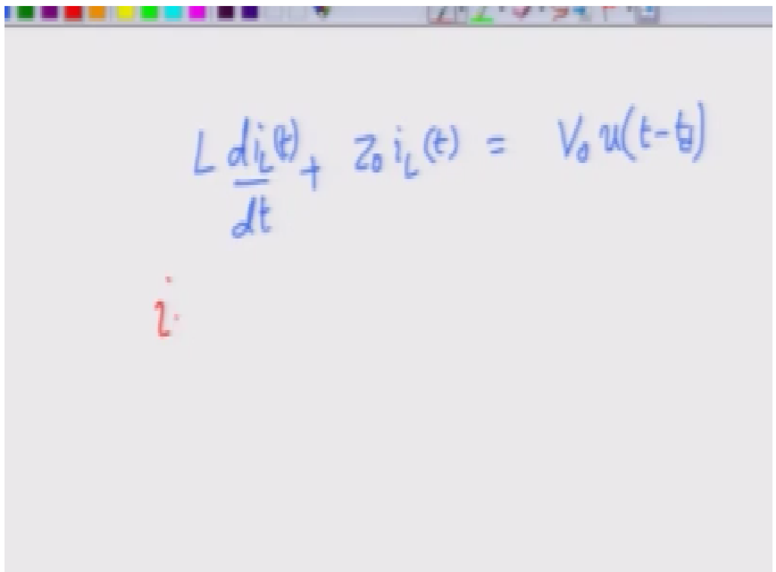

So we substitute this as L diL/dt + z0 iL of course these are all functions of time I do not have to specify small L which denotes the load here because this L itself is denoting the load, so this must be equal to v0 u(t – td) this problem is very well known to you this is actually a step response of an RL circuit right, so a step response of an RL circuit except that the step is applied at t = td instead of the usual t = 0 it is now applied t = td.

But is simply move the reference 0 to td = 0 then the solution of this one you know must be in the form of an exponential initially what will be the response of the inductor is the inductor is not connected if not charged initially it will act like a open circuit so which means initially there would not be any current finally the inductor will act like a short circuit for a DC voltage.

Which makes the current would have gone to its maximum value what is the maximum value of the current?V0/z0 will be the maximum value of the current so in-between it has to actually slowly build up to it, with the time constant of L/Z0 remember for an RL circuit the time constant is given by L/R so that's all so you can go back to the differential equation apply the boundary conditions solve it you can do all that but the net result.

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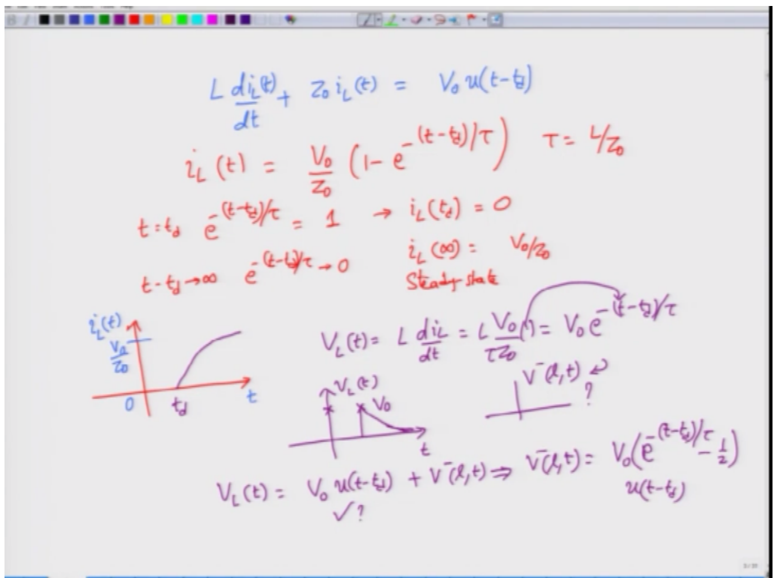

That you are going to obtain is very simple from the physical arguments and from a previous experience we can write $i_L(t)$ as v_0/z_0 which is the final current okay, which it must charge exponentially with a time constant τ, where $\tau = L/Z_0$ does this equation make sense let us evaluate at t=td $e^{-(t-td)/\tau}$ will be equal to 1 because t=td so you exponential of 0 will be equal to 1 and therefore i_L just when the step is arriving will be equal to 0, this clearly is the condition that we want because we had not charge the inductor before.

Now when t-td tends to infinity that is when you wait for sufficiently large amount of time right, then what happens to this exponential, the exponential term $e^{-t-td/\tau}$ will tend to 0 right, so it will simply draw of and i_L at infinity that is in the steady state will be equal to v_0/Z_0 okay, so the current eventually goes up to v_0/Z_0 in between how does this charging process look like we are very familiar with this I know the exponential.

So initially so you are starting at 0 and then you slowly go up you know exponentially or rather 1-exponentially with a time, the time constant of τ so this is with respect to the time axis, this is the current $i_L(\tau)$ so initially 0 to a final value of v_0/Z_0. Now it is very easy for us to find out what is the load voltage, load voltage is simply L di_L/dt so differentiate the above expression multiplied by L you will get the load voltage what would be the load voltage across the inductor here.

So if you differentiate the above expression you get v_0/Z_0 1/τ here so I know that 1/τ is Z_0/L so this will cancel out to Z_0 will cancel from numerator and denominator L will cancel and you should get this as v_0 and of course there is an exponential right, so I do not write the exponential which is written here t-td/τ it is a same process initially because the inductor is acting like an open circuit the full voltage appears across the inductor okay, and no current will be flowing in that loop.

But from that value it will keep on dropping eventually going to 0, so if you sketch the load voltage okay, this is the load voltage that I am sketching initially we begin with, with V0 that graph we have drawn there alright except this graph should have begin at Td and this graph should begin at or other exponentially going down with a value of V0 so it begins at V0 and then gradually or eventually goes out to 0 here so this is node voltage but what is my reflected voltage if I ask you what is V- so – is an numerator and it is V-that I reflected voltage at the load what it would be it is very easy to find out remember Vl of t is the load voltage which is given by V0 ut-td

You remember where this is coming from I hope you remember this my plus V- of L,T so solving this equation you obtain for the reflected voltage V0 e^{-t-td}/-1/2times of t-td okay so this expression were start at t=td and then you go all the way of infinity how does that look we can actually sketch that one s let us sketch that here so the reflected voltage at the lo volt so this is the reflected voltage at the load begins at t=td with a value of so this would be 1.

The exponential factor will be 1 and 1-1/2 will be equal to 1/2 , ½ times 0 right it will actually start at V0/2 and what will happen eventually when t-td becomes much larger this goes off to 0 and then you get -1/2 times V0 so eventually it would have dropped down to -1/2 Vo okay so this is the reflector voltage that actually starts to propagate in the backward towards the generator at the source this reflector voltage would actually propagate and add to the outgoing voltage.

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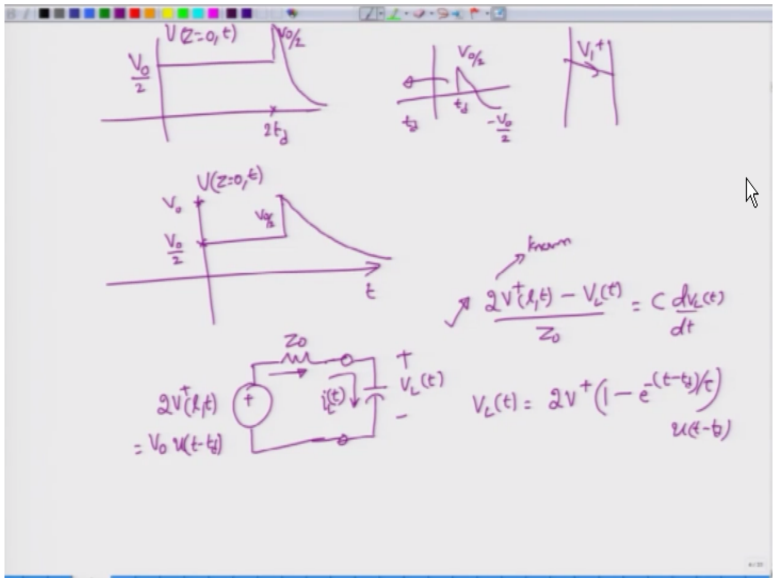

So outgoing voltage at if I along want to plot that total generator voltage at Z equal to 0 I know that I am already sending V0/2 this V0/2 is the initial V1+that we have right so this v0/2 is travelling at a time of 2 times of too time delay the reflector voltage which would be off this for me started of in this way at time t=td would now arrive after the time delay of td seconds.

And therefore it would initially add up so you initially see a jump in the voltage of V0/2 and eventually this value because this is –V0/2 is the final value here that gets added to the outgoing voltage of V0/2 therefore you will see that this drops of considering let me redraw that one here so this is the time at 2 d up to 2dyo have V0/2 amplitude at the generator site.

Then there is a jump of V0/2 so that the value here is actually equal to V0 okay and then it slowly drops off eventually going down to 0 so you see this is the voltage that you see at the input terminals of the transmission line so this completes our problem with respect to the inductor let us now look at the problem with the capacitor what I will do is go over or sketch the solution rather quickly.

And I will leave the steps for you to verify okay so in place of the inductor is I put the capacitor what would happen the equivalence circuit will change okay the equivalence circuit that I had in mind derived earlier is still which we have already seen connected through the characteristic impedance of Z^0.

Now connected to a capacitor okay across the capacitor is the lower voltage and through the capacitor is the load current the corresponding differential equation must now be return in terms of the lower voltage why because if this is a lower voltage that current through this branch current through Z^0 will be given by the voltage of left side – Vl of t/Z^0 and this current must be equal to the current through the capacitor.

The current through the capacitor is this is the equation you need to solve with the lower voltage here solve for the voltage Vl and you can see this now after solving this will again be the exponential right it would be 2V+1-E-t-td/2 times u of t-td okay why this making sense it makes sense because capacitor which is initially un charged will appear like a dead shot here.

It will appear like a short circuit which carries the entire current but the low of voltage will be = 0 the capacity voltage initially 0 and eventually as we go along now with respect to time it will be completely charge to the voltage of V_0, so if I quickly sketch what will be V_1 of t I started td with 0n voltage and then eventually go out to v_0 okay.

From the same equation vl you can find out what will be v – I will not derive it but I will leave this as an exercise, so v – at the load side would be something like this so with td you start with an amplitude of -30/2 and then gradually keep going on the way upto v0 / 2 okay. If you now combine the incident and the reflected voltage.

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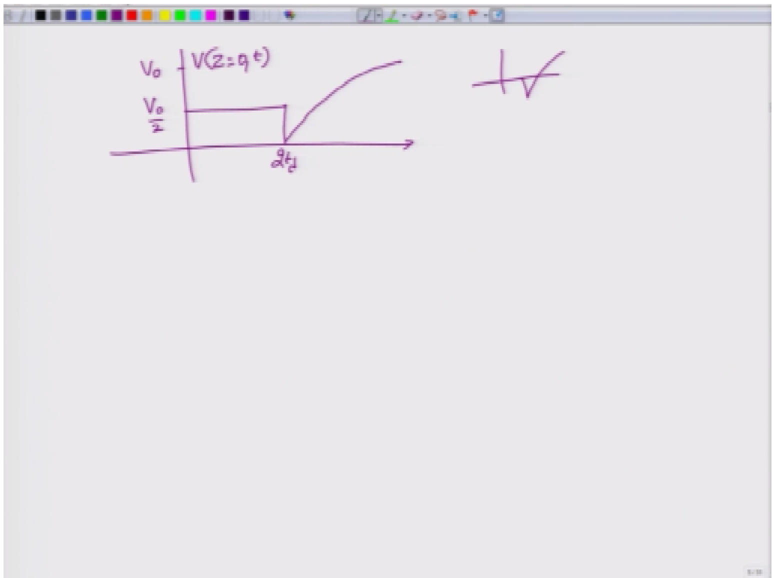

You can draw the voltage at the load side and that load voltage will be initially an amplitude of v0 / 2stepis propagating and when you receive now a voltage of the form this way initially negative and then going towards positive that step would be equal such that the you know enquiring magnitude but opposite in sign 2 v0 / 2 such that it will pull down the voltage completely to 0 at this point and then begins to raise to a final or in assume total value of v0.

So this is how you can actually distinguish if your monitoring the voltages at the input terminals of the transmission line distinguish between an inductor and a capacitor in fact the instrument that does so is called as a time domain reflect or meter which is the study for out next module. Thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

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Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi

Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
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