Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 02
Introduction to Transmission lines

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to the mooke, NPTEL mooke on applied electromagnetics for engineers. This is the first module where we properly begin the study of electromagnetics but we will do so in a slightly different manner than what we usually are comfortable with or usually taught with. Usually you start with some mathematics of vectors, and vector analysis calculus, vector field then move on to electrostatics, magnetostatics then introduce Maxwell's equations rather by Faraday's law and then introduce Maxwell's equations and then start looking at the applications of Maxwell's equations.

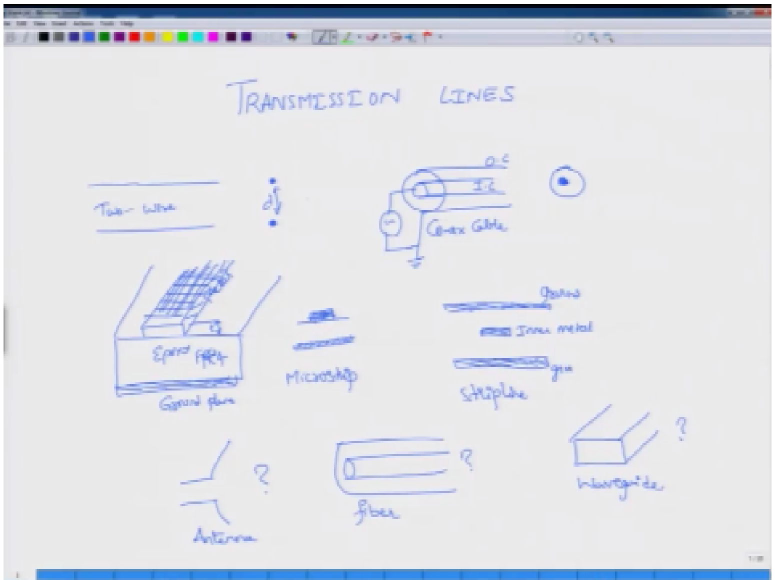
We will not take that route we will take a slightly different route which I believe is much more interesting we start with transmission line theory. This transmission line theory is although can be directly derived from Maxwell's equations which we will do so later on okay, and therefore has a proper electromagnetic basis can also be thought of as an empirical or a kind of an ad hoc solution which allows us to utilize the concepts of circuits, you know the same quantity such as voltages, currents, resistance, capacitance, inductance and conductance to apply to this analysis of these type of structures called as transmission lines.

Strictly speaking transmission lines are part of electromagnetics because they support what we call as transverse electromagnetic waves or transverse electromagnetic mode of propagation of electromagnetic waves, but that kind of a electromagnetic study we will postpone for some time later on okay. The subject of transmission lines is actually very important it was developed earlier to deal with long lines, you know he power lines that you might see if you go outside you will see that those power lines are carrying AC voltages at about 50hz or 60hz and these lines are actually really long, you know there are really transmission lines in the sense that they take this AC waveforms from one end of the country to all the way to the other end of the country right.

So these are low frequency transmission lines and this subject originally started to understand how this AC voltages would travel along from one point to the other point is a very crucial thing AC voltages are actually traveling along as though they are waves okay. So this was the origin of transmission line theory, but today transmission line theory is extensively used to not only analyze those low-frequency transmission lines but also very high frequency effect that would invariably crop up whenever you take a high-speed behavior or a high-speed signal propagation on various devices it could be a printed circuit board, or it could be a processor, it could be communication inside the BJT is inside a chip IC integrated circuit and so on okay.

So what are transmission lines basically transmission lines are devices that allow you to physically connect two elements one element presumably being the source which is what is wanting to send some information out and a load which is taking that look okay. So in that context anything can be considered as a transmission line, however the electromagnetic community considers transmission lines as those structures which are made out of metallic conductors okay.

(Refer Slide Time: 03:47)



So the common examples of transmission lines include this wire which is actually the small, you know like which is the stranded wire that I have picked up from this one and of course these wires cannot be used to carry high frequency signals they can be used to carry low frequency signals. So this is a primitive transmission line and if I start increasing the length of the transmission line or length of this wire then we need to understand the behavior in terms of a transmission line I also have another type of transmission lines over here this as you know is a printed circuit board and if you look closely you will be able to see these lines okay if you look very closely you will be able to see these lines which we call as the micro scrip plants.

These are nothing but you know metallic lines or metallic strips of very small thickness and these metallic strips are separated from the ground plane which I have not shown in this one this is a four layer printed circuit board and there is actually aground plane in between so these metallic wires are all reference to that ground plane and you can see that this is a piece of wire or a copper.

That we have drawn and this acts like a wire this is connecting one IC which you can see here this they have not populated the I see on this printed circuit board but we have left a slot here you can see that there are lot of wires coming out which actually are connecting to different or coming out from different pins of this particular IC.

So you can see that this line for example is connecting the output of an IC integrated circuit with this connector okay if you look at the back side we can also see some more lines which are slightly thicker okay but these again are you know similar in the characteristic of the other one so these type of lines on a printed circuit board are called as micro strip lines okay.

If you look at here the standard or a very common form of transmission line happens to be a two wire line right which will be just two long parallel wires sometimes these lines are connected by some dielectric okay, so there is some dielectric out there but otherwise these are just two long parallel wires that you consider, so these are called as two wires and this is the prototypical example of a transmission line.

So all the different structures of transmission line can be in somehow related to this two wire line this is the cross sectional view cross section shows that these are two conductors okay which are separated by a certain distance let us call the distance SMD apart another common example which I have not brought here.

But you can see it in your lab would be a coaxial cable this coaxial cable is a very interesting transmission line here again you see a two wire kind of a structure except that the wire is an inner conductor that is shown here and an outer conductor when you want to connect a voltage source we normally connect the voltage source from inner to the outer conductor and this outer conductor is usually grounded okay.

So this will also be canonically equal to this wire line situation because there is a inner conductor which is wire and an outer conductor which is another wire and this is the cross sectional view of course this cross sectional view is not very nice because I have not drawn it very nicely but there is a circular cross section there is an inner conductor and an outer conductor okay this is the micro strip line that we have been talking about so this is a piece of metal or a metallic strip okay I should have probably written a metallic strip like this.

So this strip will have a certain thickness steel and it would be separated from the ground plane by an insulator right so in the example of the printed circuit board that I showed the dielectric or the insulator might be epoxy or it could be the dielectric called s fr4 okay and there is certain distance between these metallic peels and there has to be a certain thickness the width and all this will change the properties of the micro strip line another line that is closely related to micro strip line is what we call as the strip line in a strip line what happens is that you have a inner metal conductor and you have a top ground plane.

And a bottom ground plane the inner metallic conductor is also called as the signal conductor because that is what carries the signal, signal is reference with respect to reference to the ground planes okay so this is one more structure there is also a coplanar waveguide or rather which is another transmission line structure which will have two micro strip lines and these two micro strip lines can actually.

You know one of them will be used as a reference the other one will be used to carry the signal okay so these are all different type of transmission lines as I said each transmission line the word transmission line is very specifically used by electromagnetic community to refer to anything that is only metallic okay so although antennas do fall in that range so antennas are also made out of metals but we do not call them as transmission lines because it does not connect physically source and the load okay.

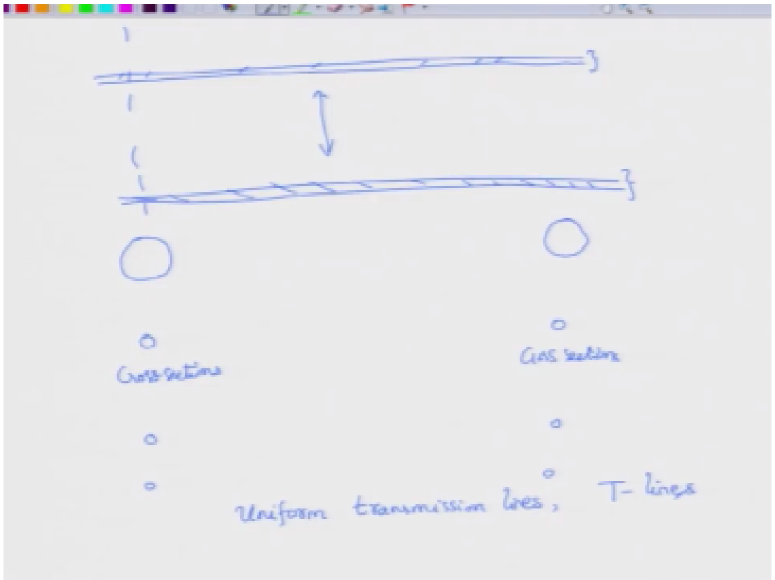
Similarly an optical fiber is another transmission line in our opinion but electronic magnetic community does not recognize the fiber as a transmission line simply because this is made on using a dielectric material itself and the working principle is slightly different from that of the transmission lines okay similarly a waveguide is a special kind of a transmission line because it is made out of a single conductor and therefore cannot be modeled with the two wire type of a behavior.

So this is also not a transmission line for us wave guides need not be only rectangular they can also be circular they can also be spherical you can find lot of variations in that so these are different transmission lines and what we want to study is to kind of characterize these different transmission lines in terms of a few parameter now this is something that is that is some that that would be interesting for us because whether I am considering a two wire line or a coaxial line or a micro strip line.

If I am able to model each of these transmission lines with a few parameters then I can analyze the transmission line behavior without really bothering which transmission line I am applying it to otherwise I should apply the theory to each of these transmission lines separately and that kind of an analysis can quickly become problematic to us okay so our goal would be to understand the two-wire line because all the other transmission lines can in some sense be reduced to two wire line so we will analyze this transmission lines in the form of a two wire behavior in order to do that let us actually write down the two wires okay.

Of course physically these wires cannot have 0 thickness there has to be a small amount of thickness.

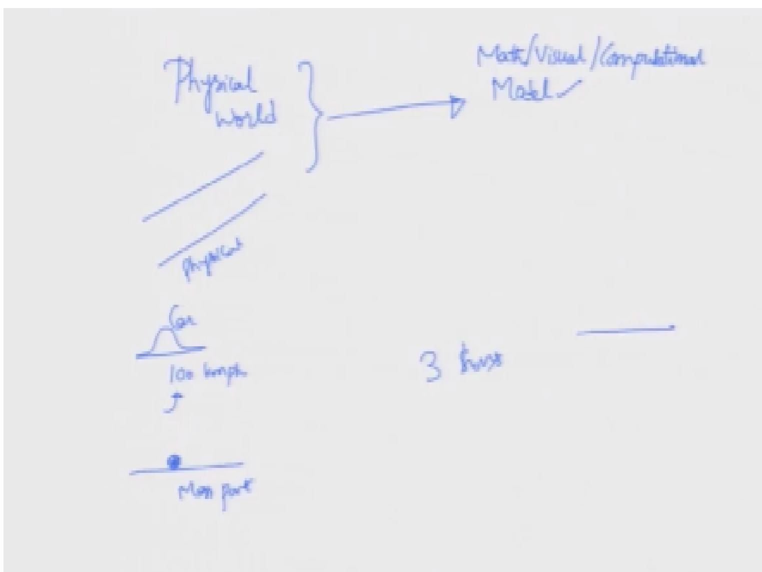
(Refer Slide Time: 10:43)



So let us say I write these wires with this much of a thickness but I understand that this thickness is actually very small compared to the spacing between these two wires okay so I am considering two wires of very small thickness okay and the spacing between them is very large compared to the thickness okay there couple of additional things the transmission line theory demands although that is not shown here it is not necessary that the thickness of this upper transmission line be the same as the thickness of the lower transmission line okay it is also not necessary that the shape of the upper wire be the same as the second wire the first wear and the second where however there is an important assumption that we are going to make in the initial stages that no matter what the cross-section that we choose.

For example this could be the cross section of the upper wire and this would be the cross section of the lower wire no matter where along the length of the wire that I go I must have the same cross section. So the cross sections here at one end of the wire or at one point of the wire now along this particular plane must be the same cross section here as well for simplicity and for reasonably accurate of description of reality we consider the cross sections to be of equal shape and equal size and we also assume that the cross sections are essentially same at all points on the transmission line okay such transmission lines are called as uniform transmission lines and instead of talking about transmission I'm instead of saying transmission lines transmission lines I will now use a shorter this one for this called as t lines, a t line is a transmission line okay.

(Refer Slide Time: 12:36)



So this is a transmission line and what exactly is the model for a transmission line you have to understand something there is a physical world right so there is a physical world in which things happen so you have a actual wire okay these are physical wires out there but then when I capture them I capture them using the mathematical model so I capture this physical world using a mathematical model a model is a description which need not be mathematical it could be visual or graphical.

It could also be computational okay so a model is our picture of what clearly is happening in the physical world a simple example is this suppose there is a car which is moving along the highway at about say 100 kilometer per hour right when you want to describe this movement of a car and calculate its position or its velocity you do not normally worry whether the car is a Fiat or a Hyundai or some other car right.

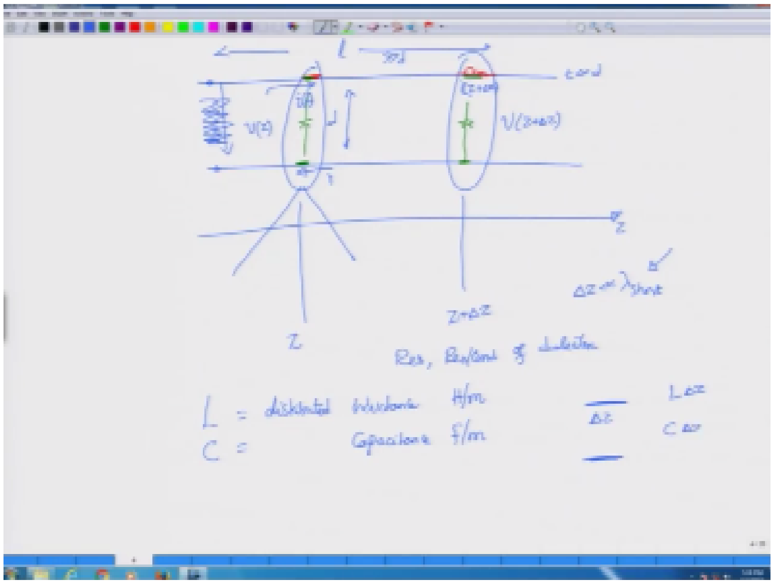
So you kind of approximate all of that and say okay I have a car which is moving with this speed and I want to know how much distance it would cover in a given span of say three hours or something right I am just giving you some arbitrary numbers they do not really mean anything what I want to say is that a car is a physical you know system but when we model this car we model this as a small point particle.

All though its entire mass is concentrated at a particular point which is called as the mass point or the point mass approximation so my car, track, scooter including ourselves will all be modeled as a mass point this kind of just simplifies the behavior you know like to the equations that turn out to be the model that turns out to be simple and it is also reasonably accurate if you are not happy with this point mass approximation you can then go to a slightly different model or a more involved model and so on.

So modeling is an art in which you describe or abstract out the behavior of the physical world in firsts to generate a reasonably simple model okay characterized by certain parameters which you can then measure and keep track of it okay if you want more detailed model you of course can going to very detailed physics or detailed models of this physical world but then the complexity of the model would also increase.

So it is always a trade-off between how simple and accurate your model is that is the best solution or how complex and complex and accurate your model is depending on the situation you might be able to get away with simple and reasonably accurate or you might want to do a complex very accurate models so having said what that model is.

(Refer Slide Time: 15:20)



What we are interested is to get to the model of a transmission line so we will imagine that I know we have an infinite length of transmission line which of course is not impossible but we will just say that the length is very, very large compared to the thickness I have compared to the distance okay first of all we had assumed that the thickness T was very small compared to D which was the spacing between the models I mean spacing between the wires and now we will assume that the length of the wire or the two wire transmission line that we are considering is much, much larger than the spacing between the two wires.

So in sense I am considering a very long wire okay and therefore I can kind of ignore this spacing okay but that is what it is but now if I look at one section of a transmission line okay. I might identify two things, first from my physics knowledge I know that if there is a current flowing here right it will then in this short piece of wire it will then induce a certain inductance because a current carrying wire has a certain amount of self inductance associated with it in this case there is also mutual inductance from the other one.

But let us not worry about it at this point and by the way if there is some current flowing into this point here or this piece of wire here there has to be a reverse current also flowing of the same magnitude okay because current has to be closed but right now do not ask how this current can be flowing when you have an infinite wire just take this as on a faith at this point things will become clearer later on. So I have a current flowing here generating a piece of inductance here or generating inductive effect okay.

And similarly when I take any two pieces of wire right between the two then there has to be a capacitor in between that right these two pieces of wires are simply two metals I am pushing charges on one end and I am pulling charges from the other end right as the current flows away from it is pulling the charges and the current flows into this one from the top where it is pushing the charges so anywhere where you have two pieces of conductors being charged and you know by being pulled and putting the charges it will need two capacitive effect right.

So there has to be a small capacitance here and there has to be an inductive effect now this picture would be the same no matter where I go on that mission line, okay. Let us say I am here at a plane called said I take this axis to be the z-axis I am at some particular plane called z and I am also at a particular plane called $Z+\Delta Z$ okay, this ΔZ is actually very small this is especially small compared to the shortest wavelength of the source that I am connecting.

We will see this rule later on clearly when we talk of high-speed signal propagation but for now just assume that this ΔZ plane which I have considered or $Z+\Delta Z$ plane that I have considered is actually very, very close to the other plane and that distance between the two planes is very small compared to the shortest wavelength of interest. Again there has to be a inductance here right, because there will be some current flowing into it but it would not be the same current because the current here flowing would be $I(Z)$ correct and the current that is flowing here would be $I(Z+\Delta Z)$ right.

Similarly there will be a capacitance here okay, but that capacitance or the voltage across this capacitance will not be the same so the voltage across this capacitance at Z will be $v(z)$ the voltage across this capacitance will be $v(z)+\Delta Z$ this is just to denote that voltages can change along the piece of wire which is what a transmission line does similarly current at one point or one plane will be different from current at the other plane.

But what is interesting is there is an inductance associated and a capacitance associated with this current okay, you might also ask have we considered the wires to be ideal that is have we not neglected the resistance that the wires offer you are right, they have neglected resistance of the wires because that makes our equation simple and once we understand this regime of operation it is very easy to include the resistance and conductance of these wires.

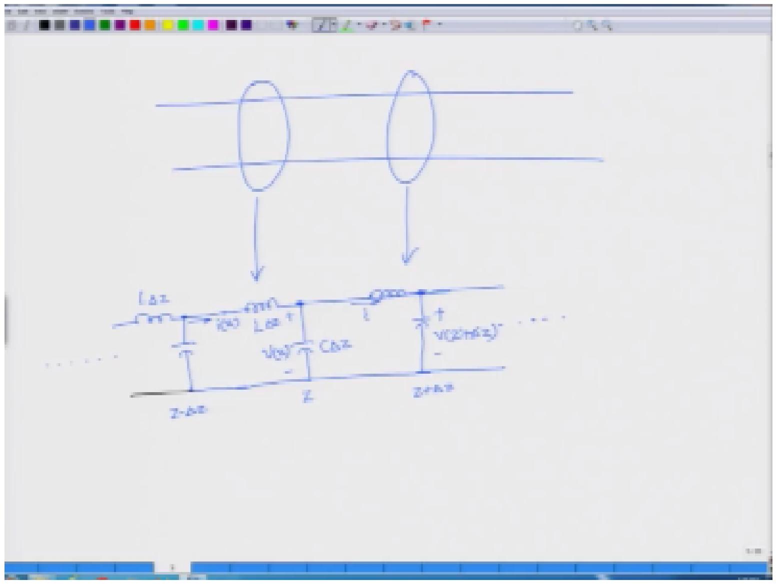
Why would a conductance be present, because there is a dielectric here right and when you were whenever you charged a voltage here ideally if the dielectric is ideal you do not see any leakage current in practice you would actually see some small amount of leakage current okay this leakage current can be modeled by a conductance. So I am considering ideal lossless transmission line okay uniform ideal lossless transmission line or uniform lossless transmission line in which I am going to ignore the resistance of the conductor that make up the wire as well as the resistance or the conductance of the dielectric.

So both dielectric is assumed to be ideal as well as the wires are assumed to be ideal so this is my transmission line okay. What is the idea behind this one now considering at two different planes the point is that there is an inductance here and there is a capacitance and these inductance and capacitance are associated with every point along the transmission line. So these are actually termed as distributed inductance okay.

And this distributed inductance is measured in the units of Henry per meter okay these are not point inductances these are not inductance s of say 10Henry or 20 Henry these are inductances which are distributed at every point along the where every short section of the wire will have an inductance associated with it and between every two soft short sections of a wire there will be a capacitance so these are not localized but they are distributed if precious conductance is inductance and capacitance s are localized then we call them as lumped inductance and lumped capacitance they are kind of lumped together but these are not lumped they are there at every point on the transmission line.

So I have L and C one is distributed inductance and the other one is distributed capacitance okay capacitance is measured infrared but we have distributed capacitance which are measuring flow rate per meter, so if I consider a short piece of length of a wire which is δZ long then the total inductance associated with this one will be $L \delta Z$ total capacitance associated between two sections of wires which are δZ will be $C \times \delta Z$ okay.

(Refer Slide Time: 22:14)



Now we proceed to look at the equations that describe this behavior, so as I said you have a transmission line and at each section I have an inductance and a capacitance so I consider a particular section where I have some inductance okay the total inductance is L x δ Z because L is the inductance per unit length and the total capacitance is C x δ Z I am putting all my inductances and all my capacitances only from top wire to the bottom wire actually I should also include the inductance of the bottom wire as well as capacitor has to be split between the two.

But this equation that you get will be the same equations that you would obtain if you associate all of the inductance to the top wire and all the capacitance to the bottom wire, so there is no problem if you do this particular way okay. So at every node sort of thing you will have so this is the plane that I have considered and said there will be one more plane here okay and this plane will be at Z + δ Z.

So this is at the plane that Z + δ Z this is at the plane Z this clearly is a plane at Z - δ Z and this structure does not end it keeps on going forward in this particular way okay there is a current flowing through this which is I of Z okay and the voltage across these elements would be some D of Z okay. So this is V of Z and there will be a voltage here which is V of Z + δ Z and there will be a current which is flowing through this one oh can I just rewrite this one the model would be slightly better if we interchange the positions of inductance I mean--can if we change the positions of the capacitors okay.

So the proper model will be something like this. So I consider this place at Z and Z = Δ Z so this is my plane ad said this is my plane as that + Δ Z and this is the plane at Z + 2 Δ Z and so on the first course keeps continuing the voltage here will be D of Z + Δ Z and this current will be I off grid the current through this one will be I of Z + Δ Z. So this is the current that is flowing into the inductance that is between Z and Z +Δ Z and I offset + Δ that is the current that is flowing from Z +Δ Z onwards to the next section of the wire.

The voltage across this capacitor is V offset. Okay now I know from circuit theory that I can apply KVL and KCL write kickoffs voltage loss and current loss. I consider this particular node okay into this loop if I apply the Kirchhoff voltage law I know that this would be - V of width as I go from bottom wire to the top where this would be –V of Z and there is a current flowing through the inductor.

If there is a current flowing through the inductor what would be the voltage across the inductor that would actually be L x DT but in this case my current is also dependent on set because I am considering that current will change from one set to the other Z therefore I cannot use di by DT I have to use del I by DlT.

So that would be+ L Δ Z del I by Del T remembering that I will be function of both Z and T + there will be a voltage at Δ Z. This is at the same time T so I have not shown that time dependence here I will show that one later. So this sum of voltages must be = zero if you rearrange this equations and then divide everything by Δ Z.

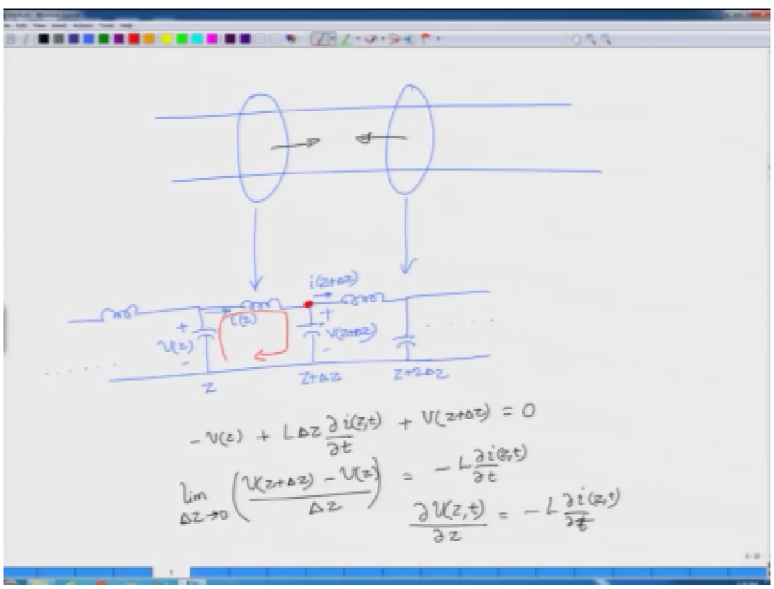
What you get here is this equation which is an interesting equation because I can then go to the limiting procedure of making these sections to be as close as possible or as close as possible that I want and as I shrink them that is as I bring these sections closer and closer together. Right what happens is that the inductance there becomes again as is distributed but the sections start to become very small.

But there is no problem with this limiting procedure because I will be adding more and more sections here right as I bring these two together I will have more and more sections within the same earlier distance. So this limiting procedure is very well done and for physical reasons the voltages and currents are not running off to infinity therefore the left-hand side of this equation the right-hand side I do not need to limit anything.

Because there is no Z dependence there so on the left-hand side I have as the limit Δ Z goes to zero this equation is nothing but the partial derivative of V with respect to Z and therefore I can replace that 1/ that = - L del I by del del T I being a function of Z and T similarly and I leave this as an exercise to you.

I would like you to know obtain an equation by applying KVL okay to one of the nodes that we considered in the previous case so you can apply KCl to this particular node okay which will have some incoming current and an outgoing current the difference between incoming and the outgoing current will be the voltage across this one. So if you apply the Kirchhoff's current law at this node which I have shown here with the red dotted color you see that there is an incoming current I offset and there is an outgoing current.

(Refer Slide Time: 28:19)

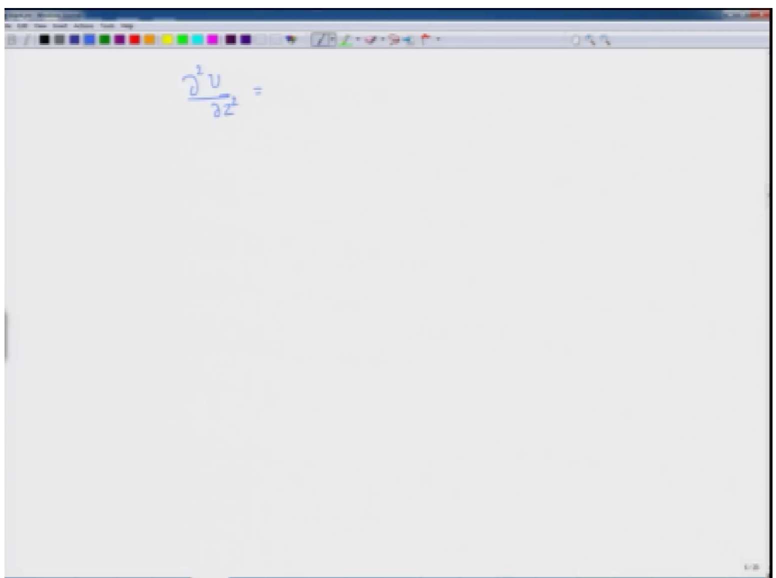


I of Z plus Delta Z right and the difference between these two currents must be used to charge the capacitance so you will have minus I of Z plus I of Z plus Delta Z my notation is that incoming current is negative and outgoing current is positive right and this difference between the two must be because of the current that is being used to charge the capacitance.

And that charging current across through the capacitor will be Canto Delta Z because remember Cascara cadence per meter Delta Z is multiplying that so this term must be equal to zero as before you can do getting procedure by dividing on both sides I mean dividing everything by Delta Z and then adjusting the current sat Z plus Delta Z and Z and to end up with an equation that would be like an analogous equation to this fellow rights you will have an equation in which the rate of change of current with respect to Z will be given by minus Cell V of Z T by delta T okay.

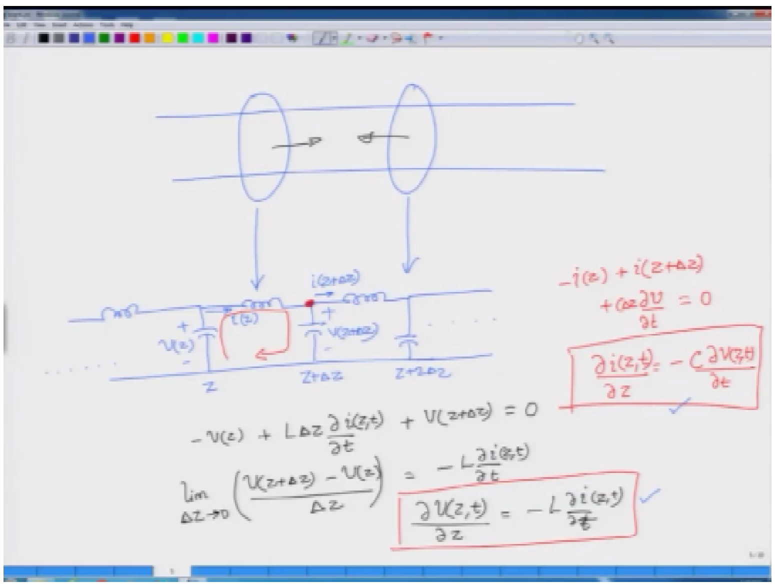
Now you can actually consider these two equations and then try to solve them okay normally what we do is we consider these equations in pairs and then we reduce this first-order partial differential equations into second order partial differential the way to do that one would be to differentiate the voltage equation with respect to Z okay.

(Refer Slide Time: 29:46)



And then utilize the other equation okay in order to show that you can compress to first order partial derivative equations or partial differential equations into a second-order partial differential equation of this particular form okay I of course derived it only for the voltage this consists of actually differentiating this one with respect to Z and then differentiating this one again with respect to time and then recognizing that there will be term of del I by Del Z del square I by Del Tell Z on both sides .

(Refer Slide Time: 30:14)



And then substituting one into the other it is fairly simple exercise please follow it up and then show that the voltage will satisfy this equation you can also show that the current equation will be the same it will be setsquare I by Del Z square is equal to Lingo C del square I by Del T square in both equations you should remember that voltage is changing both as a function of Z as well as a function of time similarly the current is changing as function of Z and T in the next module we will look at the solution of these two equations thank you very much.

<u>Acknowledgement</u>
Ministry of Human Resource & Development

Prof. Satyaki Roy
Co-ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan

Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K. K. Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production