Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 19
Further examples of use of lattice diagrams

by

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Hello and welcome to mook on applied electromagnetics for engineers. In this module we will continue the discussion of transient propagation, that is propagation of transient voltages in the time domain or the transmission line circuit. So consider the example that is given here, of course we will be using the lattice diagram that we developed in the last module, in order to solve the following problem.

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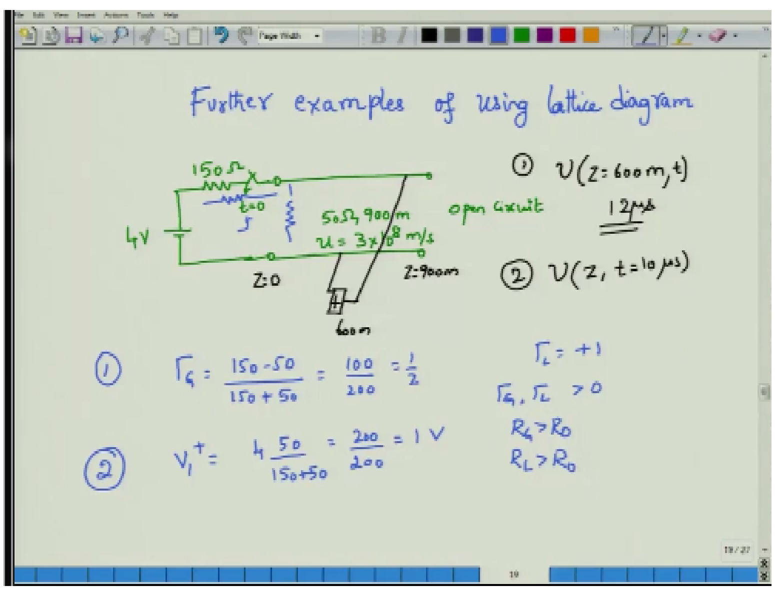

So we have a battery which is producing a 4 volt DC voltage okay. It has an internal resistance of 150 ohms, it is connected to a transmission line circuit which is 900m long and terminated in an open circuit okay. The characteristic impedance of the transmission line is 50 ohms and the velocity over which the waves would propagate the voltage would propagate on this transmission line is $3x10^8$m/sec.

So what we want to know is to find out what is the voltage at Z=600m that is we will imagine that we are keeping a oscilloscope here okay, this oscilloscope will be kept at 600m which is about two-third from the source side okay. And we want to find out what is the voltage that we would observe on the scope for let us say up to 12 micro second. So up to 12 micro second we want to know what is the voltage measured at this point on the transmission line which is about 600m, it may not be nicely written here, but this is 600m from the source.

So our coordinate system is z=0 at the source of the generator side and z=900m in this case, because the length of the transmission line is 900m on the load side okay. We not only want this, we also want to find out what is the distribution of the voltage across the length of the transmission line at t=10 micro second. So imagine that you actually are able to take a snapshot of the voltage on the transmission line at the time of 10 micro second and we would like to know what is the voltage at different points in the transmission line.

The voltage distribution on the transmission line at p=10 micro second. Now we will be using the lattice diagram that we discussed in the last module. Remember what the lattice diagram means on the one axis you had the length of the transmission line that was the coordinate or the ordinate there which was Z axis. And on the other axis you had the time axis. So this was also called as the space time diagram or also called as the bounce diagram, because voltage could be bounding back and forth.

And I told you that there are two initial steps to begin drawing the lattice diagram, the first step is to calculate what is the reflection coefficient at the load side, as well as because there is a reflection from the generator side, so calculate what is the reflection coefficient at the generator side. So calculate both γG and γL okay, and put that one down on the lattice diagram.

Then we need to calculate what is the first aptitude of the first forward voltage, and that will be obtained by the voltage divider that is formed by RG the generator impedance of the generator resistance, and the transmission line characteristic impedance, and whatever the voltage that you have connected.

So let us first calculate as part of that strategy, let us first calculate what is γG, γG clearly is given by 150-50, 150 being the generator impedance divided by 150+50, so this is 100/200 or equal to ½. Similarly, what is the load reflection coefficient by now you know that the open circuited load will have $\gamma n=+1$, just for our reference we will also note that γG is positive and γL both are positive numbers that is the sign of these two are positive numbers indicating that RG is greater than R0, RL is also greater than R0.

So this is the first step of calculating the reflection coefficient. The second step would be to calculate what is the initial amplitude or the amplitude of the first forward voltage which we denote it as V1+, V1+ in this case is the voltage divider that would exist between the generator impedance of 150ohm and the characteristic impedance of 50ohm. So V1+ is given by 4x50/150+50, so this 4x50 is 200/150+50 is 200, so this equal to 1volt.

So we have a 1 volt initial forward propagating voltage launched on the transmission line at the time t = 0 okay so now what will happen how do we write the lattice diagram so we will go to the lattice diagrams here.

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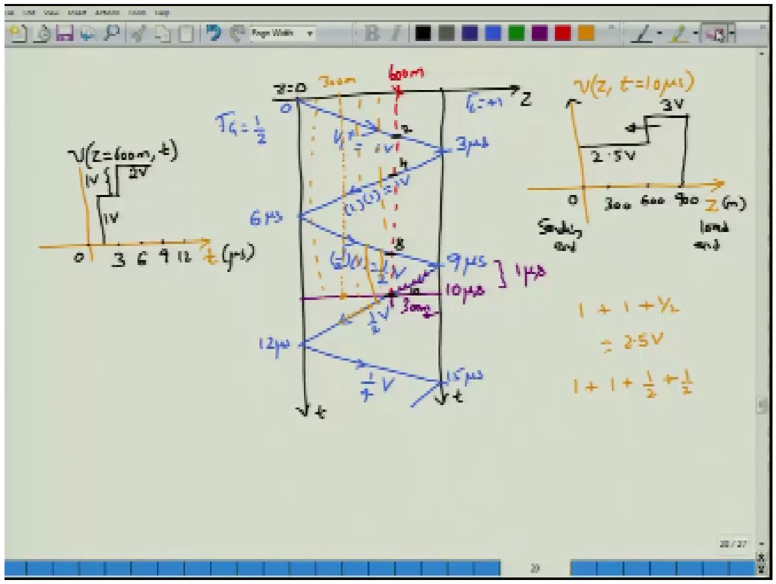

On 1 axis we draw the z axis and down here I will draw the time axis okay so I know what is the length of the or the time delay of the propagation in order to find the time delay I go back to this circuit I know that the length of the transmission line is 900m okay so time delay will be given by 900m and you know that the velocity is 3x 10^8 m/ s so this is about 3 micro seconds okay so what we have 3 micro second, so let us now bill the lattice diagram.

So first you would have a V1 launched okay V1 + we calculated was 1 volt so 1 volt is launched at the = 0 and it reaches z = 900 m that is the node side at 3 micro second, at which point it gets reelected back because there is an open circuit with a reflection coefficient of 1 so please note that I am actually writing both the reflection coefficient as well as the amplitude of the voltage that we have received okay.

So this is a good practice to remind our self that what is happening to the voltage as well as the reflection coefficients so we would not make any mistake so let me rewriter γ l = +1 here and γ G = + ½ on this side s0 1 x 1, 1 volts begins to propagate back this is true because a open circuit simply reflects the voltage back so at once it reaches the generator side at 6 micro second that will be further reflection so this was given + the next one was given – now you have V2 + with an amplitude of ½ x 1 which is = ½ voltage right.

So ½ will began to propagate at t = 6 micro second and reaches load at 9mirco second there will be one more reflection of ½ voltage because the reflection coefficient here is = + 1 when there will be further reflection the reelected ½ voltage will reach at 12 micro seconds where as the further reflection of ¼ voltage will began to propagate from 12 and reach at 15 micro second so let us stop here not because I just ran out of the space here but also because the voltage keeping reducing an d we also have enough information to solve the two questions that where post to us.

So first question was what is the voltage distribution along the entire length of the transmission line at a 6^{th} time of p = 10 micro second in order to answer that we need to look at out lattice diagram and first locate where n = micro second occurs, so let me locate 10 micro seconds so this is about 10 micro second now if you look at the transmission line on the transmission line if you so you want to find out the entire distribution right.

So this at 9 micro second the voltage would be reflected the reflected voltage of ½ volt as began to propagate right it as began to propagate and it has reached this particular point, so the total time elapsed here is 1 micro second after 9 micro second so it is a total is about 1 micro second and in 1 micro second what would be the length of the transmission line that the voltage wave form would cover.

We know that the entire length of the transmission line is 900 m and it requires 3 micro seconds for the voltage wave to go from 1 end to the other end therefore in 1 micro second you are able to go 300 m so in this case since it is 10 micro second the point where this you know the blue line which is how the voltages are propagating is meeting the times access which I have drawn a slice of time as I can all the slice of time which I have drawn is at 10 micro second.

So this point would be 300 meter always from the load but 300 meter always from the load is nothing but 600 m okay so we would have actually reached at this 600 micrometer I mean 600m please note that the voltage ½ as not propagated here so the voltage as actually not propagated to this side so this side voltage would still be seeing whatever the voltage that was there earlier and what is the earlier voltage to this that would be ½ volt and this would again be ½ volt and so on and so on, okay. So let us now plot what would be the voltage distribution that would be present throughout the section of a transmission line.

At a time of 10 micro second so let us see how this would be, so I need to plot the voltage as a function of z here because I am taking the snap short in time let me mark three points corresponding to 300 meters, 600 meters and 900 meters so obviously I am measuring z in terms of meters so this is the origin which is 0 meter this corresponds to the sending end, okay and this corresponds to the load or the receiving end or the load side or the load at the end at the connected.

So that we are connected, so if I am for example let us build up it in this way, suppose I am there at 300 meter okay, so 300 meter will correspond to some point over here and I keep moving forward in time so at some particular time I have a voltage incoming voltage of 1 volt then after a certain time there was one more volt that was coming in so therefore the voltage at this point of time on the 300 meter distance from the transmission line would be 1+ 1 that would be 2.

So initially it was 1 volt that jumps by a 1 volt when this reflected voltage 1 volt reaches from this side v2 – reaches, now v1 – reaches here continue along there would be one more jump of ½ voltage so now the total voltage is 1 + 1= 2, 2 + ½ which is 2.5 and then we keep coming in over here and we need to stop at this point this is very important I need to stop here because this is the time slice I have taken.

So upto 10 micro second this is what the 3 voltages that I have received and you can clearly feel that I can take slices along z at any point and cleanly the voltage that I would be obtaining would all be constant and all will have an amplitude of 1 + 1 +1/2 which is 2.5 volts only in this region beyond the red line region the total voltage will be 1 + 1 = 2, 2 +1/2 +1/2 because of the reflection so that would be 3 volts.

So you will have 1 + 1+ ½ + ½ voltage that would have propagated the half wave would have propagated from the load side until about 600 meter, so the voltage distribution if I plot the voltage distribution up to 600 meters it would be 2 . 5 volt however from 600 to 900 would be a voltage of 3 volt and a 0.5 volt step is actually propagating from the load side towards the generator.

So this is the answer to the pervious I mean one of the question that we have asked, what is the voltage along the pair transmission line what is the voltage along the entire transmission line at a snap short time of 10 micro seconds, so this is the answer. We also have asked another question which said that what would be the voltage distribution at some at not some at z = 600 meter so imagine that we have kept an oscilloscope there, right.

So we kept an oscilloscope and we want to find out what is the voltage and we want voltage upto say 12 micro second, so let us begin to first write down the voltage in a axis so I have T measured in micro second this is 3 this is 6 this is 9 this is 12, so let us that is what we need to come up to 12 micro second, how do we do that? Again look at the red line or 600 meter I have actually dropped this particular line right so you follow this through.

How much time would be required for the voltage that is launched at the source side that is at z = 0 to reach z = 600 meters well this has about 1 microsecond per 300 meters so there are distance that you need to calculate sis 600 meter so from the source it will take 2 micro seconds to reach 600 meter but from the node you will only take 1 micro second, therefore the factor the meeting point of the intersection point between the blue line and the red line is happening at 2 micro second indicating the voltage propagation from the generating end to this 600 meter section.

And then at 4 micro second because remember there is only one micro second delay from the load side for the voltage to be 600 meter it is very close to the voltage and the next will be 6 + 2 that would be 8 micro second and the next slice that we have will be at 10 micro seconds anyway nothing would happen at 12 micro second therefore we can even stop this one at 10 microsecond.

So at 2 microsecond we see a jump of 2 volt okay, so this is the 2 volt and at 4 you would see a jump of another +1 volt so there will be a further jump of 1 volt so this is 1 volt plus a jump of 1 volt makes it 2 volts so this is the jump 1 volt that we have and this would happen at 4 sorry, let me redraw this one in a slightly bigger scale okay, so let us draw here.

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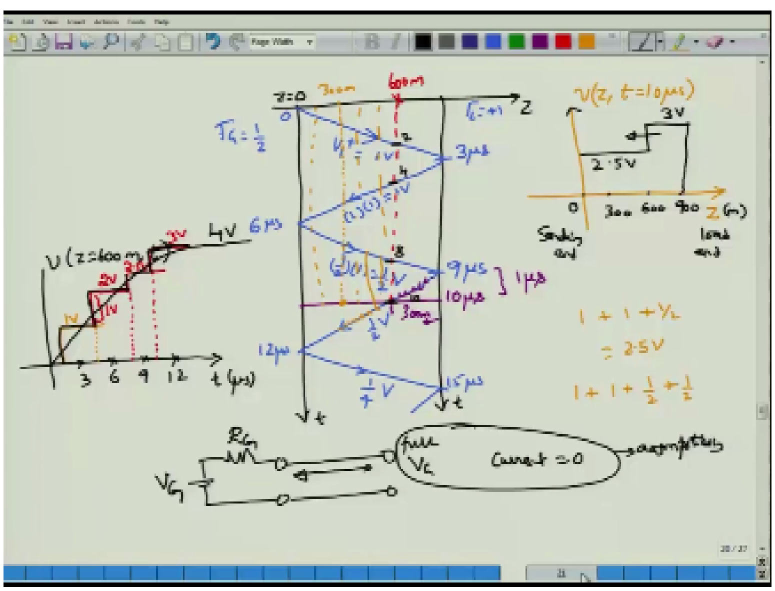

So what I am looking at is the voltage has a function so t is in microsecond, the voltage at z=600m as a function of time up to 12 microsecond but we anyway have seen that whatever we need to find that information that up to 10 microsecond is sufficient, so 3,6,9 and 12 because no change happens at 12, so what is the voltage so initially you have a voltage jump of 1 volt at 2 microsecond so this is the 1 volt pulse and at 4 microsecond there will be a further jump, okay.

And this further jump it also of the same magnitude 1 volt therefore you go to 2 volt so the jump here itself is 1 volt and this continues to be the case until you reach 8 microseconds so this was at 8 microsecond where you would reach one more jump and this time the jump is about half volt, so you will go to 2.5 and at after that 10 we will again reach a jump of another 2.5 so it becomes 3 volt so here there was a jump of 0.5 so this is 2.5 volt.

So I would suggest that you actually look at this diagram carefully and then convince yourself that the values that we have written is correct, if there is any mistake in the calculation please make correction accordingly. So you can see that the voltage is steadily increasing and eventually what would be the voltage that you should see on the transmission line.

Well, remember this is an open circuit right, and an open circuit would have a full voltage across it and the line is loss less, so no matter what line that we have connect as long as it is loss less the voltage that you would see on an open circuited end of the transmission line will be the full value of V_G, so here you have some R_G and after time where the transients have settle down and you have reached the steady state you should see a full voltage of V_G and that would be the voltage distribution along the entire length.

What would be the current distribution, well current would be equal to 0 in the case that you are taken an open circuit eventually we will see that the current is 0. But mathematically these values are reached at a asymptotically in time that is they will be reached eventually but if you wait for sufficient microseconds long then the voltage would have reached all most the open circuit voltage and therefore these values are alright, okay.

So you see a steady line or an oval damped at line towards 4 volt okay, so this is what is a solution for this problem. Before dropping this transient propagation let us consider one more case okay.

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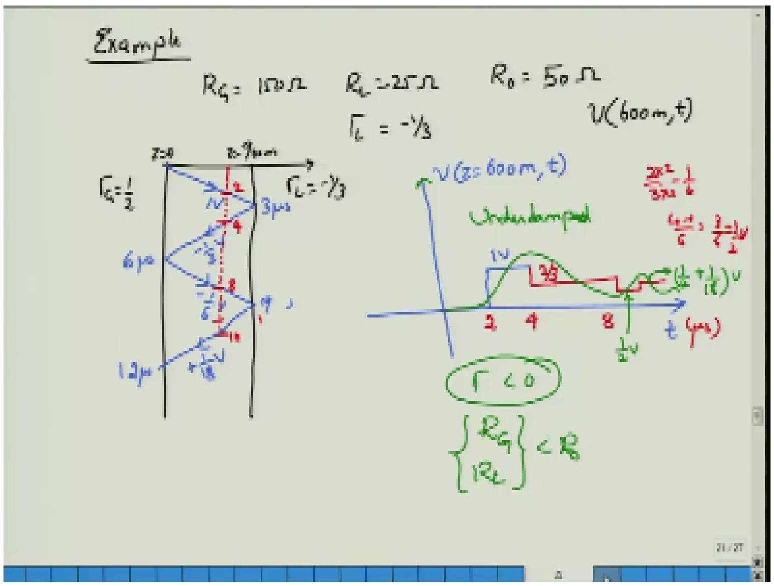

In this case we will consider at R_G of 150 Ω as we considered in the previous case so this is the other example that I would like to consider, so we consider R_G of 150Ω, R_L of 25Ω so you can now see that Γ_L will be negative and Γ_L will be equal tom -1/3 where I am assuming R0 is still equal to 50Ω, the length and other values are all the same and what I want is again to see what is the voltage at 600m as a function of time up to say 10 microseconds, okay 10 or 12 microseconds.

I will construct the lattice diagram over here first all the values that we will require are already here Γ_G=1/2Γ_L=-1/3 this is at 900m, this is at z=0m okay, so we begin with a v1+ which is the same as the previous case which is 1 volt so the 1 volt wave propagates from the receiving side and reaches the load side at which point gets reflected what is the reflected voltage amplitude -1/3 again there will be one more reflection of -1/6 volt because -1/3 gets multiplied by 1/2 therefore this becomes -1/6.

Now further reflection will be at +1/18 volt so you can see 1/18 is all most you know very small and just keeps on reducing and where that different times slides such that this voltage changes are happening at the lower side this would be at 3ms and there are the source would be changing at 6 ms, further change in the lower voltage happens at 9ms and the 12ms. So we want to find out what is the voltage as a function of time at a fixed distance along the transmission line which is at z = 600m okay.

In order to do that we go back to the same condition we drop a line okay or a slide at z = 600m and we need droop this line until 10ms so 10ms happens to be here somewhere so we can drop a line at 10ms so this is 10ms we will dropped it. Now it is easy for us to see what is happening at 2ms you will see a voltage jump of 1 volt okay, and at 4ms remember because this is so where are the slices located this is a 2 this is 4 this is 8 so this is at 10 I made a small mistake here so wherever we drop that would be the line which corresponds to the inter section point right.

So we have this line and we have a voltage going back which was 1/18volt okay. So we have changes happening at 2 4 8 and 10 at 2 you will see a jump of 1 volt okay and at 4 you will see one more jump but this time the jump is in the negative direction because the value of the reflected voltage is -1/3 so you will see 1-1/3 volt propagating. So what is 1-1/3 this is 2/3 volt okay, so you will actually see that the voltage has dropped from one volt to 2/3 volt the changes happening that two and jump happening at 4.

This continues until you go to 8 ms so this time is measured in ms of course at 8 ms you will see further jump of -1/6 so the voltage would drop to 2/3 -1/6 so I can multiply both sides here by 2 this becomes 3/6 3-1 /6 is 1/3 that is still positive but it would be 1/3 voltage at 8 and a 10ms you would see a jump and this time the jump is slightly positive because 1/18 volt. And this is how you actually start seeing so if I have got the numbers round please sorry this number is actually 4- 1/3 right so this is not 1/3 but this would be 4-1/6 which is 3/6 =1/2 voltage.

So this voltage value here let me draw it here, is actually ½ voltage and the voltage at this point is ½ + 1/18 volts. So this is the voltage as you would see at 600m and you can see that this is of the form which is oscillating okay. So this is an example of a under damped system, so why is this an under damped system? Because at least one of the γ's has become less than 0 so sign of γ is less than 0 indicates that one of the resistances it could be either Rg or Rl has become less than

R0 and this is the only condition that you should have we should not have both having less than 0. As final example let us consider the propagation in the same situation of a transient.

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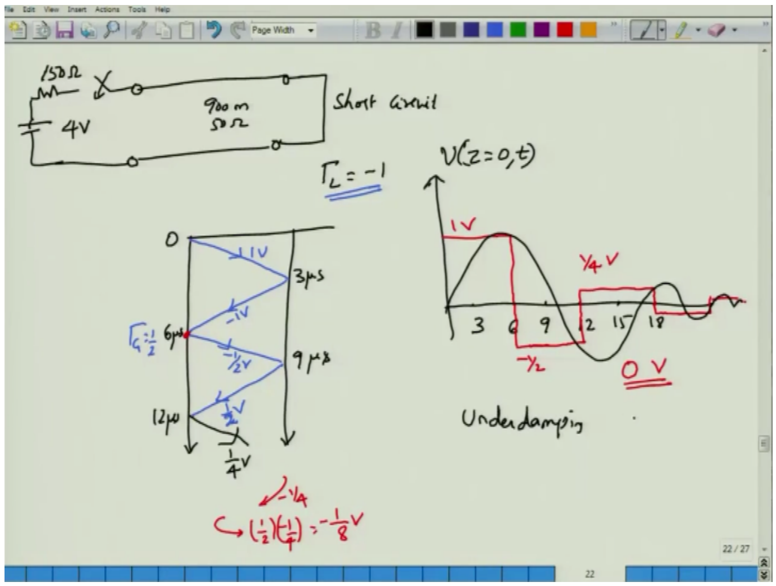

But this time we short circuit the load so let us short circuit the load so clearly short circuit in the load will giving γl (-1) I keep all the other parameters to be the same so I am still have a 900m 50Ω characteristic impedance transmission line connected okay to load of so connected to a generator of 150 Ω impedance and a step voltage of or a battery voltage of 4 volt okay. So I will not completely solve this one I will just write down the lattice diagram I hope you will be able to follow this because γ l is -1 whatever the voltage that we receive.

So initial voltage that would be launched will be 1 volt as in the previous case but this time there will be a reflection of -1 volt because the 1 volt that we receive at the lower side will be multiplied by γl = -1 and therefore the reflected voltage will be -1 volt, there will be one more reflection from the generated side because the generated is about ½ the γ g is ½ so the reflected voltage is -1/2 volt and there will be a further reflection for the lower side which would be ½ volt okay.

The sign here becomes as negative because γ L is negative okay so let us now plot and see what kind of voltage we would see but this time I want to plot voltage at Z=0 that is I want to plot the voltage at the source or generator site so where are the changes happening the propagation delay is 3 micro second.

For changes at the source happens at 0 6 micro second and 12 micro seconds so this is 6, 12 and at this point there would be another ¼ voltage going backwards so that I would need it right so 9 micro second so let us put down the numbers of 3,6,9. And 12 and see what kind of voltage changes will happen so at 0.

You have launched a 1 volt which changes to 0 volt at 6 right see you launched 1 volt which begins to propagate okay at 6 micro second you have received a -1 so 1-1 will make it 0 so that is what we should usually think however there is also -1/2 volt generated from the generator and that begins to propagate therefore the voltage at this point is -1,-1.2 + the previous 1 volt 1 and -1 will cancel and-1/2 would still be there.

So the voltage here would be -1/2 and this would be present until you get 12 volt at which point of +1/2 volt coming in from the low side would make this voltage go to 0 however there is 1.4 volt reflected symbol from the generator and which is begin to propagate and therefore the voltage here is about ¼ volt you can note there is clearly seen where this is going because after at 12 you will get the 15 micro second point at which there will be a-1/4 reflected of the load side okay.

And there will be one more reflection from the generator side it will not be at 15 sorry this would be at 12+6 that would be 18 micro second okay so let me redraw this one so it is 18 micro second so this is 12 this is 15 so this is 18and there was a+1/4 voltage generated and propagated at 18 because of the reflection at this point of -1/4 this would initially go to 0.

But then -1/4 would be reflected further with ½ or you know with a value of ½ multiplied by-1/4 which is equal to -1/8 volt so it would just go slightly below become-1/8 and eventually go up and then keep doing this reaching a steady state voltage of 0 volt does this make sense to steady state 0 volt yes of course it makes because you have a short circuit at a node side and the voltage of the short circuit will be equal to 0.

Again you will see that because one of γ is becomes negative there is an under damping or an oscillatory kind of a behavior so you would see that there is an oscillatory kind of a behavior okay so in order to obtain under damping situation you need to have one of γ go to 0 this both γ are less than 0 are greater than 0.

Then you go to what is called as over damping case in which there will be stair case kind of a thing it would actually keep increasing, increasing in eventually reach of the steady state value so this completes our discussion of transient voltages on the transmission line we have looked up important cases you know if you considering power transmission cable is a lighting hits that you know how this line would be propagate that depends of course on what is the transmission line to which it is connecting so thank you very much.

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