Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 18
T-lines in time domain: Lattice diagrams

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Hello and welcome to applied electromagnetics for engineers. In this module we begin study of transmission line circuits in time domain, we want to understand how when I apply a pulse or a step voltage or some complicated voltage on the transmission line, then how would the voltage at the lower side would look like in the time domain.

So far what we have considered is the transmission line behavior in the frequency domain, because we had applied a sinusoidal voltage, we had assumed that there were no transients in the circuit, all the transients had died down and whatever that we were obtaining at every point the voltage and the current both were sinusoidal okay.

So this is what is called as a frequency domain analysis or sometimes called as the study state sinusoidal analysis. But frequently and these case increasingly digital designers are interested in how the transmission line would behave in the time domain. Why is it that they are interested, because look at what the digital signals are. Suppose there is a IC module which is driving another IC okay. And there is a wire between them and so far we have seen that a wire is not just a wire, but it acts like a transmission line.

So there is a transmission line which is terminated by one IC that driven or the load IC and then there is a driving IC. The signals that are generated from the driving IC this could be a NOT gate, this could be a NAND gate, or it could be any other kind of a digital device, it could be clock which is changing and distributing the clock on to another chip, it could be the address, decoder line which is, this is an address decoder which is actually trying to change the address, or address a particular memory module or a particular chip on the microprocessor or on the micro controller boards.

So it could be any of these scenarios in which there is a digital signal which is changing in time and it does not remain like a pulse. For example, if you are transmitting a pulse it is not a pulse for all the time, it is the pulse for a finite amount of time. And then the value would again change, so it is necessary to especially for digital design is to understand how this digital signal is actually propagating on an analog transmission line okay.

So the analog aspects of the digital voltage is how, you know the voltage pulses are changing or how the step voltage is changing if very particular and very important to minimize distortions and to better understand the propagation of pulses and propagation of transient to voltages on the transmission line. In the power transmission line cases, there would be a moment surging the voltage, because either the load have been momentarily short circuited or there was a lightening which struck the cable.

So even though there the signal would be more or less a sinusoidal signal, it is also the surge or a transient that has happened because of the lightening and one would like to understand how far this lightening would propagate or the lightening induced voltage would propagate before it could die down. So for all these cases it is advantages to work in time domain rather than the frequency domain okay.

Technically it is possible for you to work in the frequency domain by transforming everything into Fourier transforms or more evenly into lap lass transform, but such an approach kind of masks what if happening in the time domain. So what we will do in this and couple of modules that would come down is to try and understand the behavior of step voltages first on resistively terminated transmission lines.

And then explain this step voltage analysis on to a reactively terminated transmission line. And then finally consider how pulse would propagate on a typical transmission line circuit that could be terminated either by a reactance or reactive elements such as the capacitor or an inductor, or would be terminated by a pure real node okay. So let us begin by first understanding how a step voltage would propagate on a transmission line. Towards that, let me first discuss the problem with you, let me first give you the basic idea of the problem.

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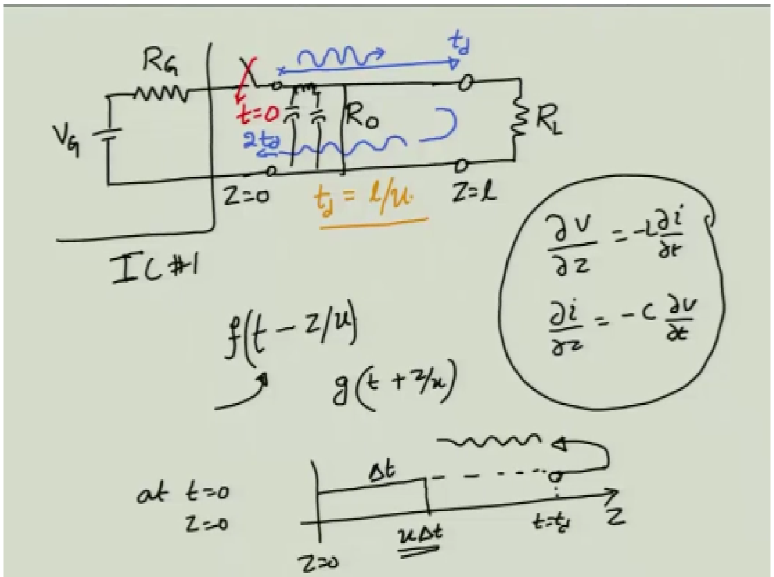

So we have some battery voltage okay, any transient of some voltage can be modeled as a battery with some resistance, keeping in mind the terminology that we have introduced in the previous cases I am going to write the impedance or the internal resistance of the battery as RG remaining myself that G is the generator. So the voltage can also be written as VG just to show that this is a generator or the force okay.

Now to this we connect a transmission line or actually transmission is already connected, if you want you can think of this as a IC okay. So one integrated circuit module or one chip is the particular thing okay, which is now equivalently represented by a voltage followed by a resistance okay. So this transmission line of course is not intended, but it is actually the artifact of being a wire not being a wire actually.

So this would be let us say terminated thing and load of some value RL. We will also assume that the transmission line has a certain characteristic impedance which is R0 okay. Now let us watch what happens, we will not connect this one to the transmission line, what we will do is we consider a switch because when the voltage at the IC output or, you know before the lightening it would be some steady state voltage.

Suddenly because of an action whether the IC voltage line is changing the address line is changing the voltage would increase and then remain constant or it would be of this form or it would be of this form or it could be of this ringing form so any kind of a voltage wave form is allowed of course it is rather easier to analysis this case compare to these two cases right so we will consider initial the idealized step voltage which will be 0 before t = 0 so I where to draw the time axis's okay so this would be the generator voltage so this could be the voltage at the input terminals of the transmission line.

So initially this is 0 and the movement you turn on that is turn the switch on so the voltage jumps at this point the voltage jumps to a value of V_G okay or rather the source will be connected this voltage will not be exactly V_G at the input terminals of the transmission line we will see what it could be so this voltage but it is achieved in the voltage okay and this the switch that would be closed at t= 0.

So what we want to do is to understand what happens when the switch is close so that this voltage would now be in connected will be connected to that transmission line before connecting this voltage we will assume that the transmission line is uncharged that is there was no remaining charges and therefore this transmission line was simply to wire which were completely in active, now let me ask you once I close the switch do you think that this whatever voltage that should be available at the input terminals of the transmission line the imminently available at the output terminals of the transmission line.

Of course it would not be available at the to reach from the generator side all the way up to the load side and how much is the delay that is that is caused because of the presence of transmission line that depends on what is the safe velocity of the voltage along that I would like to mark that the term phase velocity is not you know a very good term over here because safe would give you a convection of a sinusoidal source but there is no sinusoidal source here, so rather than talking about the phase velocity I should be more careful and talk about the dealing.

Deal that a particular point of the voltage a particular voltage value would take before it appears at the output terminals of a transmission line okay so that is the time delay of the transmission line and I would like to actually talk about that but occasionally I might slip of by using the term phase velocity and that case you please correct yourself okay this time delay is given by l/ u okay I mean by writing this I mean desisted writing the subscript of p for here in the fear that I would be denoting this several phase velocity.

Any way so the delay is length by velocity the velocity with which every voltage value move on the transmission line okay so that is first thing that you have to observe or understand the second thing that you have to understand is that when connect the battery on to the input terminals of the transmission line until the voltage that is connected here reaches the transmission line M or reaches the load which will happen after a time delay of td they will be only be a forward going voltage okay.

So form the time when the switch is closed and the generator is connected to the transmission line until the time delay td the source does not see the load in fact the source does not see the load until it goes here and the voltage there will be because refracted R_L will not be equal to R_0 there will be a refraction the reflected voltage would propagate okay and the reflected voltage would propagate and arrive at a time of two time delay.

So until the time 0 to t to td this load will not change to load will not see the voltage okay and from time 0 to 2 times time deadly the source will not see the voltage okay for the changes whatever that could happen RG would not see the changes so whatever the changes that have to happen have to wait for the total time delay of td for the lower to see the source and for the source it will take a team delay of $2t_d$.

But once the reflector voltage reached back to generator or the source the forward going voltage is still present so that is still propagating but if R_G is not equal to R_0 that would be further reflection that the source side or the generator side so there will be a second reflector way that if propagating and that reflecting way again reflector from the lowered come back after 4 t_d then again it keeps happening until you reach a site of steady state behavior okay.

So in the steady state behavior you would see whatever the condition that would be necessary so after all this are die down okay how to be analyze how to be understand the propagation of step voltage here there are various methods that have been followed to understand this voltage propagation in the transient of time domain behavior what I would like to just remind the simple factor we started off when we discussing the transmission line or the solution.

We said that nay voltage of the form f of T-z/u should be the solution of the voltage equation if you remember for the case that of $\partial v/\partial t$ right so $\partial v/\partial z$ is equal to $-\partial i/\partial t$ with L and similarly $\partial i/\partial z$ was equal to $-\ \partial v/\partial t$ correct so these are the time domain equations and any functions of the form F of –t-z/u or g of t+z/u will satisfy this two set of equations okay. Therefore these are the solutions. Now if the source voltage is applied at t = 0 at z = 0 which is where the source or the generator located for me please note the coordinate systems here now the coordinate system is z = 0 at the source silence z = l at the Load side, okay.

So when I connect battery to the transmission line at the input terminal of a transmission line at z = 0 and wait for a time of Δt I do I expect that the voltage change would have happened throughout the length of the load or towards the length of the transmission line on the line would have been only charged up to a distance of Δt x u okay. So this is along the z axis it would have been charged only up to this u Δt.

Because there is a capacitance here there is a capacitance here there is an inductance here I mean conceptually of course not an actual way so each of this elements take some time to charge and by time Δ t only the section up to u Δt would have been for this is z = 0 only up to the section u Δt would have been charged this will continue and at a time of t = td the entire section would have been charged.

And this voltage when it sees the load would start to reflect okay, so when this seize the load it starts to reflect and this reflected voltage would propagate backwards, okay. So there is a simple way to understand how this voltage propagation happens.

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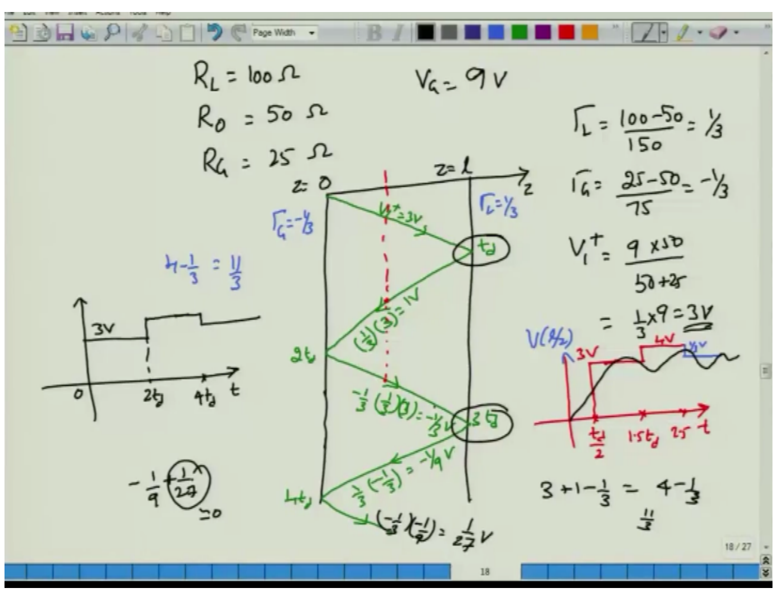

And that understanding comes from what is called as a lattice diagram or the bounce diagram okay which is what is a popular method to understand how transients are propagating and I would like to discuss that one with you, okay. So we are going to understand how the step voltage propagates by looking at a lattice or a bounce diagram, let me first give you the construction of this, okay.

You get facilitated or you get understanding of how to use the lattice diagram and once you have understood if there is any I mean there is any doubt with regard to how the voltage would propagate we can resolve them, first learn the tool then we understand how the tool works for this particular case, okay. So the bounce diagram or the lattice diagram begins by first drawing two straight lines, okay.

And these lines are labeled along so this lines are actually the time access okay and the length between the two or the distance between the two will denote the source and the load points or the load reference planes on the transmission lines circuit, in other words this L is the length of the transmission line, okay. So this is along the z axis because there is a space and there is a time axis both on the same graph in which we are plotting.

Sometimes it is also called as a space time diagram okay sometimes this is called as space time diagram okay, the first step is to understand what is the initial voltage value that begins to charge the transmission line, so would it be equal to VJ well unfortunately that could not be equal to VG because RG is present if the generator resistance where to be equal to 0 then the moment you connect T = 0 at T = 0 you connect this source to the transmission line.

The voltage usually launch would be equal to VG because there is RG and there is an regular characteristic resistance of R0 the voltage gets divided between these two between RG and R0 in the appropriate voltage dividing ratio, so the initial voltage amplitude let us call this is as V1 + is given by and this + indicates that this begins to propagate from the generator side all the way to the load side, okay.

So v1 + will be equal to whatever the generator amplitude VG that would be there times RO/R0 + RG so when I write this I am actually assuming that or I am actually telling you that the initial voltage VG that is applied okay through the resistance RG sees a transmission line characteristic impedance okay. It will not see the load of course it cannot see the load because first the v1+ voltage has to go.

Reach the load get reflected back and only then the affect of RL; would be visible to the generator, okay and because there is a two time delay of this happening until the time this would be the valid transmission line circuit at the or the equivalent transmission line circuit at the load sorry at the source or the generator point, okay. So initially we have v1+ which would begin to propagate until a time delay of TD so we will denote that by writing down an arrow okay, and this arrow you know reaches this point at t_d this is the time access please remember this is the time access that we have written, on this I would like to mention what is a value of v_1^+ okay, that is important that you mention the values so that when you add a platter on it, it becomes very easy.

So this point it is v_1^+ but now what happens at t_d well, you cannot just have v_1^+ and then the entire circuit stops unfortunately because R_L is not equal to R_0 there will be an amount of voltage that would be reflected, how much would be reflected first find out what is the load reflection coefficient from this expression R_L-R_0/R_L+R_0 and Γ_L times v_1^+ will be the one that is reflected, so let us call this as v_1^- this is our first reflected voltage.

So this v_1^- reflects and reaches the source at a time of $2t_d$ okay, and then this begins to propagate again so I will come back to this one what it is but first let me write v_1- which is equal to $\Gamma_L v_1^+$ you can also note down what is the value of Γ_L here, similarly note down what is the value of the generator reflection coefficient why do I need this generator reflection coefficient here, because once v_1^- reaches you can think what is v_1^- has the source voltage that is now hitting this generator side, okay it is carried by the transmission line and it hits the generator because V_G is already sending out a positive voltage when this v_1^- hits you do not need to consider V_G okay.

V_G can be considered as a short circuit, however because of this reflection there will be a reflected voltage back, so what could that be that would be v_2^+ okay, this is the first forward wave, this is the second forward wave by now you know what should be v_2+ this would be $\Gamma_G v_1^-$ this would reach the load at $3t_d$ which will then be reflected back, you know by the load side with v_2^- given by $\Gamma_L v_2^+$ okay, and this again for the reflects reaching here at $4t_d$ and so on until that would happen okay.

So this is the lattice diagram and this lattice diagram contains all the information that is necessary for you to actually write what is the voltage at different points on the transmission line. Suppose I setup a hypothetical oscilloscope well, now the hypothetical but at real oscilloscope okay, I setup this oscilloscope at the midway between the transmission line okay, so the transmission line is of this length I setup my oscilloscope at this point okay, and I launch a step voltage here right, so now I am looking at the oscilloscope well until half time delay I do not see anything there is no change that would be happening until half time delay right, if $1t_d$ is the 1 you know time delay then until half the time I would not see anything right.

Where do I see some voltage, I would actually see some voltage because this is my oscilloscope right, oscilloscope which is kept at l/2 and any voltage that is launched from the source or from the load side would take about $t_d/2$ to reach the center of the transmission line so I would not see so if I keep moving along like this, so if I keep noting down what is the, this one I see at this particular time which happens to be $t_d/2$ or rather $t_d/2$ in this case the voltage continues to travel, travel and reaches the center of the transmission line at a time of $t_d/2$.

At which point the oscilloscope suddenly jumps and starts to show v_1^+ and you would continue to show v_1^+ because any change to this voltage you know v_1^+ at the center of the transmission line will not happen until this v_1^- comes and hits on to this right, or this point the oscilloscope will now jump to $v_1^+ + v_1^-$ okay. So if I were to see this on the oscilloscope in terms of the time okay, at z=l/2 I will not say anything up to time $t_d/2$ then I will see at jump of v_1^+ that would continue until a time of so what is the time here, this is td as already elapsed this is $1.5t_d$ so until $1.5t_d$ or $3t_d/2$ we would see anything at this point again I see a jump okay, the jump will be having a value of v1- okay.

The jump will have a value of v_1^- so that the total voltage of this point would be $v_1^+ + v_1^-$ okay, this is what would I see at z=l/2. Suppose I interested in saying what happens to the source, so the source is at z = 0 obviously right, so initially when the switch is closed the source voltage jumps up and it would be v1+ and then if it continue to be V1 + until it is start to receive a reflected voltage which happens at 2 times td right.

So when that happens the voltage actually jumps up or upper down depending on the sign of V1- so the jump will be of the magnitude V1- but would have a sign up appropriately so the total voltage at time 2 TD here will be whatever the voltage has been launched so there is change in here that is V1 – but please remember as for as the voltage reach as the voltage actually changes that v2 + therefore here sorry I have spoke earlier the magnitude of HMP is not just V1- but V1- + V2 + okay.

And this continues until you go up to 4td at which point the jump would be of V2- + v3 + okay so there would be another jump, jump or could be dip depending on whatever that you are looking at, this would happen at 4td okay. So this is how we would see up the source, similarly at the load the total some of the voltage here is 1or rather V1 + γ l v1 + the voltage here will be v1+ +γl v1+ + v2 + +γl v2 + okay so this is the voltage at changes or jumps that are happening at 3td and at td okay.

So you can write down this voltages at the function of time and then actually setup an experiment in case it is possible for you to do so and you can track down and observe that these are the voltage jump that are happen okay. Let us consider an example that will make lot of these concepts little clear up okay.

Let us consider the case were Rl is about 100 Ω okay at R_0 which is the transmission line characteristic impedance is about 50 Ω, I consider Rg to be 25Ω okay and Vg to be about 10v now let us try and find out what will happen to the spaced on diagram and I would like to see what happens to the voltages at the source voltages at the load and voltages at the middle of the transmission line okay.

The first step is to calculate what is value of the reflection coefficients, so the lower reflection coefficient is 100 – 50/ 100 + 50 so this would be about 1/3 right so this is 1/3 and γ g is 25 – 50/ 75 this is – 1/3 so we have γ l γ g differing a sign. Let us start with the lattice diagram so this is the lattice diagram that I would like to start this is the z axis this is z = 0 this is z = l okay here I note down what is the value of γ l which is 1/3 I also note down what is γg which is -1/3 okay.

What is the initial voltage amplitude that needs to propagate that is V1+, V1+ is given by 10V divided between R0 and R0 so this would be 50+25 so this is 1/3 of 10v so let me go back and correct this one instead of 10volt I will make it nine volt so that all the multiplication become slightly simpler okay. So this would be nine so this would be 1/3 times nine this is 3 volt okay. So initially we launch a 3 volt you know 3 volt voltage step okay so V1+ = 3 volt okay.

And this would reach at sometime td I am not specified time td once a voltage step of 3 volt reaches at time td here it would be reflected back how much would be reflected back 3 if the initial voltage times whatever is the γ l value so that would be 1/3, so this would be 1volt so one volt will be reflected back and then at the source side it would again be reflected back at td how much would be reflected when we started off with 1/3 3 this is what k or one volt Kelvin that need to be multiplied by -1/3.

So this would actually be equal to -1/3 volt okay at which point at 3td when it reaches the source side there will be further reflection this term it would be -1/3 which is incident and 1/3 which is reflected this would be -1/ 9 volt and you can see whatever the other changes that are happening, you can see that the magnitude of the voltage keeps on reducing because both γ g and γl are less than 1 when this tart multiplying this quantities then they would start to go down to 0 okay λL into λg and so on them the product of the advantage start to go to 0 and this changes happens as 4Td.

Now we have ready to complete the problem we want to sketch what happens at the centre of the transmission line so let the centre of transmission line at half of Td/2 or you know Td/2 there will be a voltage jump of 3 volt so this is3 volt it continues to be 3 volt until we hit the next voltage value which is +1 volt at a time of 1.5Td.

So at 1.5Td we need or we have a voltage which will jump by 1 volts so total voltage will be 3+1 therefore there will be jump of 1 this becomes 4 volt okay and it continues to be the same case until we get a 2.5Td so at 2.5 Td what would happen there will be a dip now because 4 for this total voltage the jump voltage is -1/3 so this would be 4-1/3 how much is 4-1/3 there is about 11/4so there is a.3 volt drop so that would be a small jump here okay.

So this is a small jump not a very large jump so let me go back here and correct it and so there is small jump of about 1/3 volt and it continues to remain as it is and dip keeps on you know it just goes on like that so this is the voltage at L/2 now what would be the voltage at the source side well initially you started of step voltage of 3 volt which would persist so this is 3 volt at a time T=0 but at too time delay you would see that the total jump will be a 1-1/3.

So that could be 3anyway it is transmitted one would arrive so there will be jump of 1 but there would be -1 /3, so 1-1/3 is again 4-1/3 kind of scenario so this would again sorry this would be

3+1-1/3 this is 4-1/3 which is 11/3, so 11/3 is still slightly up compare to 3 volt so it could continue to be in this way and when it goes to four times Td this would be but the 4 times Td even now have at voltage of -1/9 coming in and a voltage change of -1/3*-1/9 which is 1/27 volt change.

So the total jump will be -1/9+1/27 so clearly 1/9 is larger in magnitude 1/27 this can be approximately equal to 0 so there is a small drop in the voltage and then it continues to be as it is so this is how the way forms would look the way form here would be here except that this way form would happen after the time delay of 2Td I mean time delay of Td/2.

What I would leave as a exercise is what would be a voltage at the low side again you will have to add up a total voltage here to that keep doing it but you will quickly feel that the circuits are kind of settling down to a steady state values right so this voltage would be shifted to low side and they would see some kind of a ringing effect okay.

So this is the transmission line which can be taught of as and under transmission line okay so this is the example of using the lattice diagram in order to understand how the step voltage would propagate on the transmission line in the next module we will talk about more topics on this one so we will first consider terminating the transmission line with a inductive or a capacity oriented see how the voltage changes and then we will consider the propagation of lattice so until then thank you very much.

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